Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/195-7.3.5-u-a+b-arctanh-c+d-x-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [62]. This is test number [195].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (62)	0.00 (0)
Maple	98.39 (61)	1.61 (1)
Mathematica	96.77 (60)	3.23 (2)
Maxima	54.84 (34)	45.16 (28)
Fricas	27.42 (17)	72.58 (45)
Mupad	27.42 (17)	72.58 (45)
Giac	27.42 (17)	72.58 (45)
Sympy	25.81 (16)	74.19 (46)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

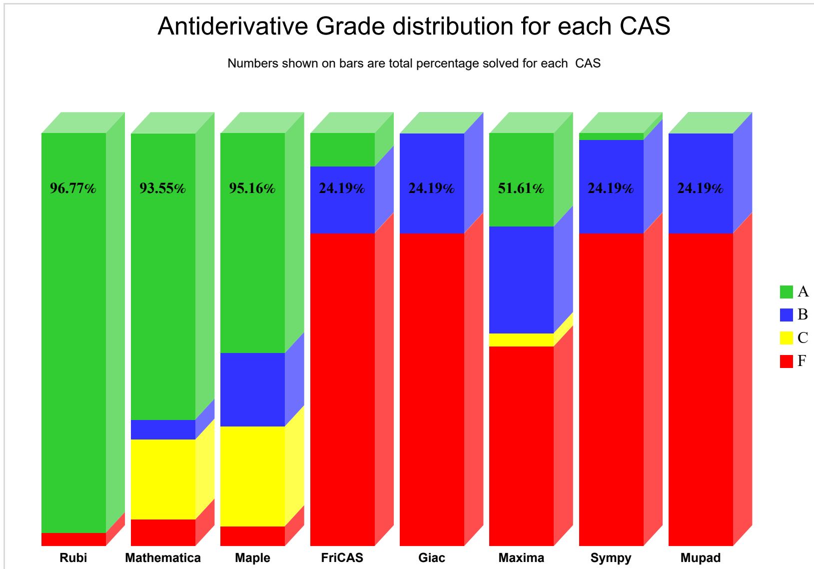
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

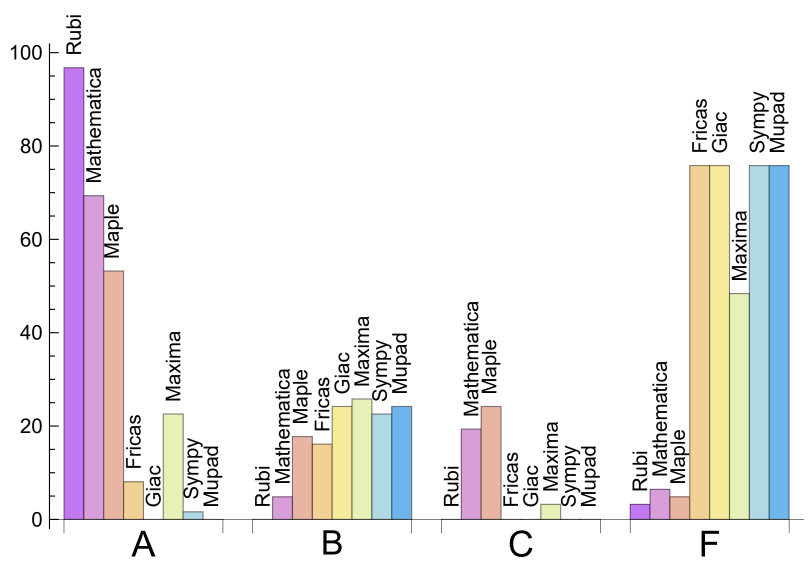
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.774	0.000	0.000	3.226
Mathematica	69.355	4.839	19.355	6.452
Maple	53.226	17.742	24.194	4.839
Maxima	22.581	25.806	3.226	48.387
Fricas	8.065	16.129	0.000	75.806
Sympy	1.613	22.581	0.000	75.806
Giac	0.000	24.194	0.000	75.806
Mupad	0.000	24.194	0.000	75.806

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maxima	28	96.43	0.00	3.57
Fricas	45	100.00	0.00	0.00
Mupad	45	0.00	100.00	0.00
Giac	45	100.00	0.00	0.00
Sympy	46	82.61	17.39	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.31
Giac	0.31
Maxima	0.42
Rubi	0.77
Maple	0.94
Mathematica	4.30
Mupad	4.30
Sympy	7.14

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Fricas	232.24	2.20	159.00	2.09
Rubi	240.32	0.97	148.00	0.99
Maxima	352.03	3.11	276.50	1.93
Mathematica	436.87	1.71	213.00	1.22
Mupad	501.76	4.00	237.00	2.50
Giac	608.12	5.11	351.00	4.12
Maple	986.26	3.62	301.00	1.59
Sympy	1824.38	12.24	275.50	3.34

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

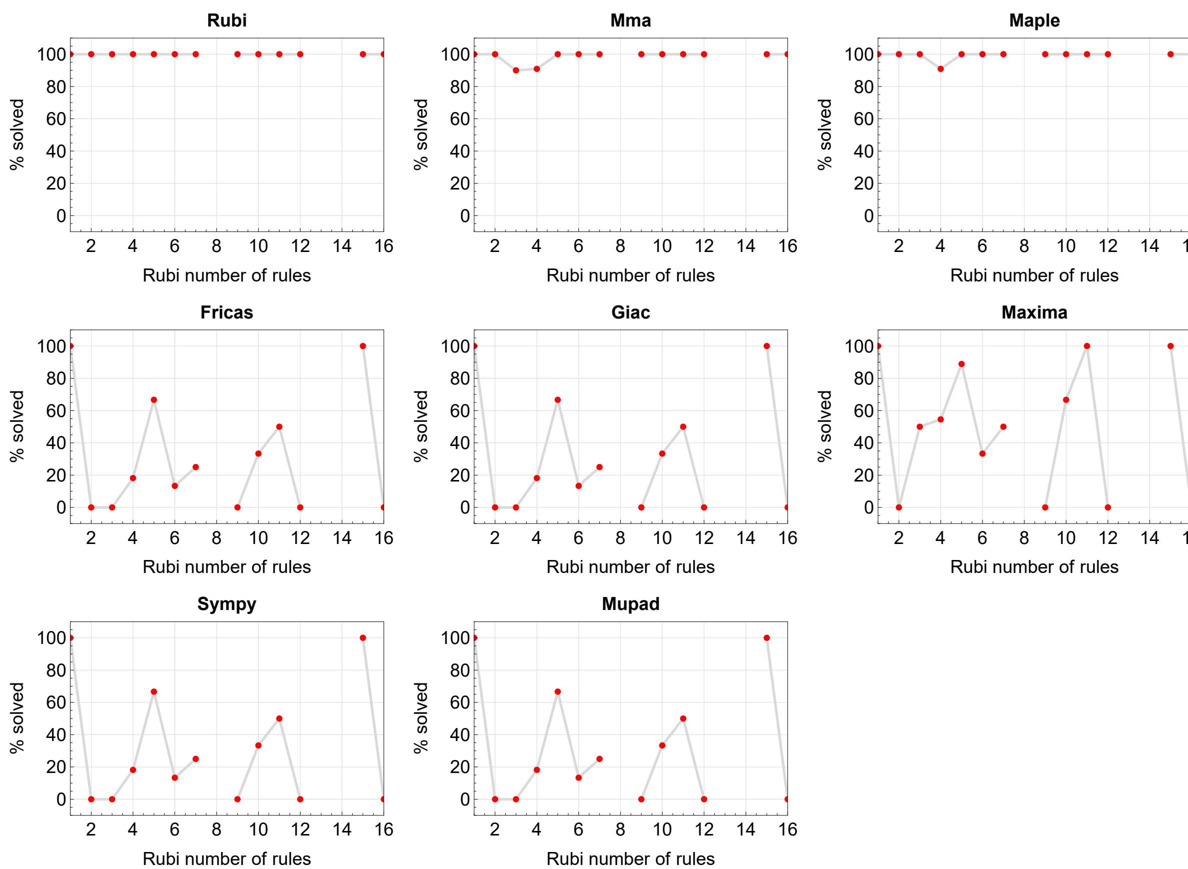


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

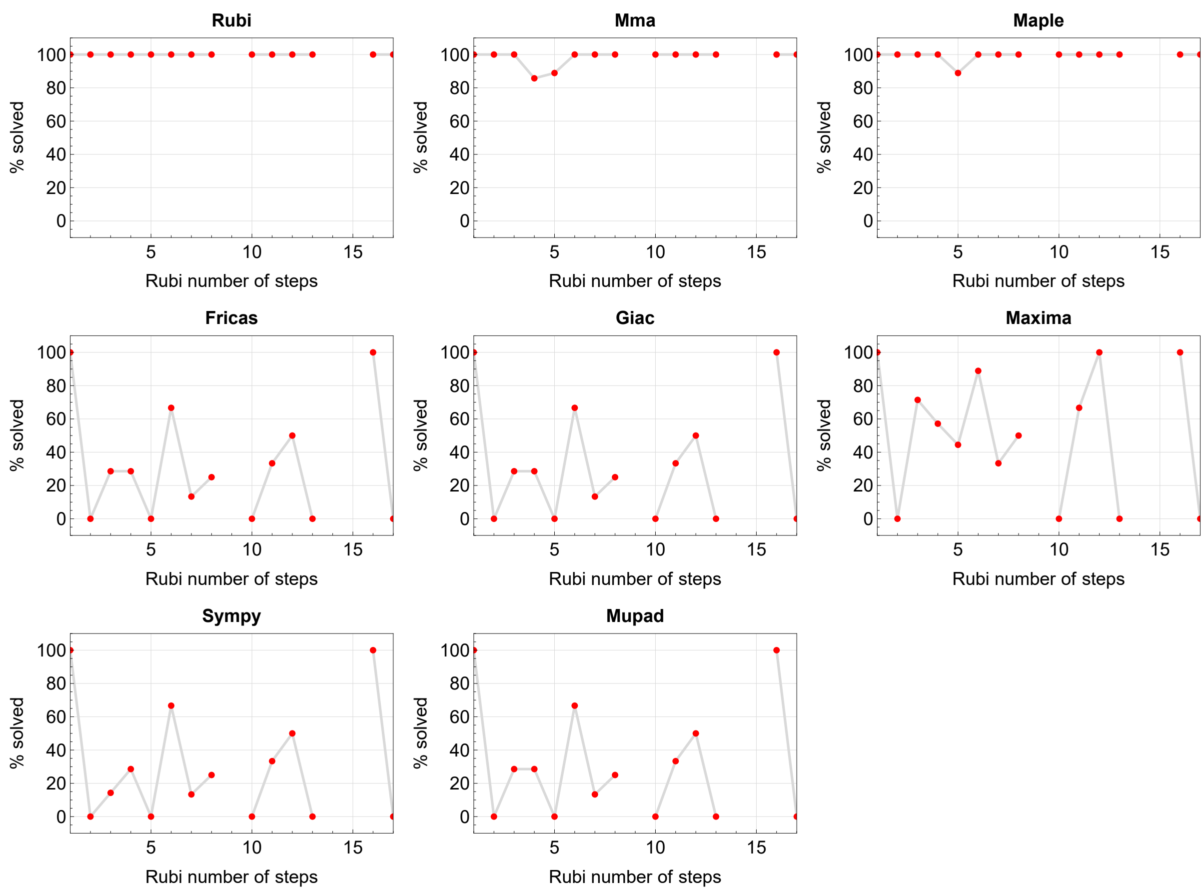


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

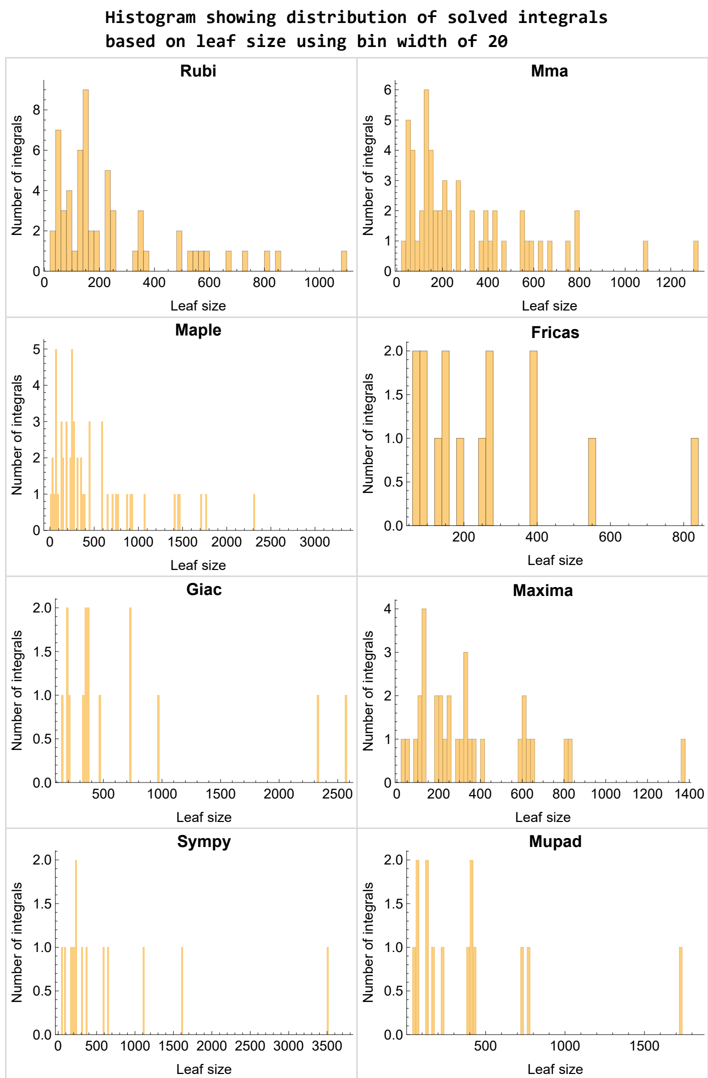


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

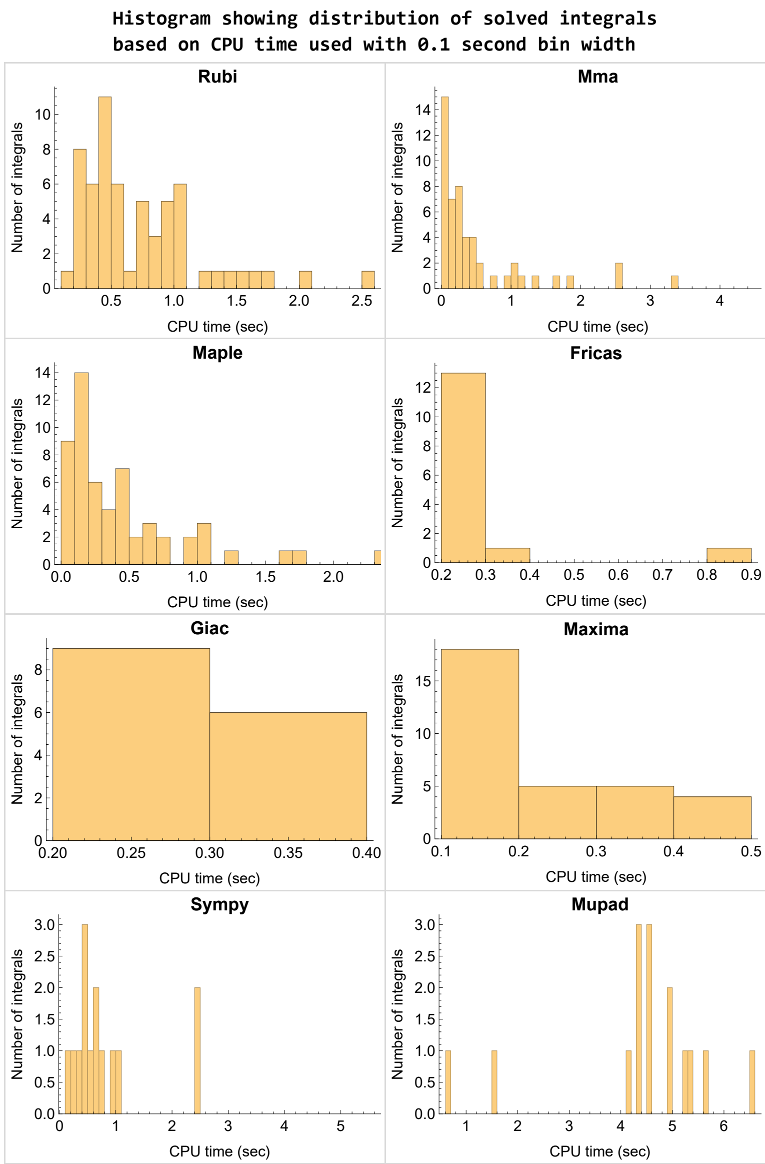


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

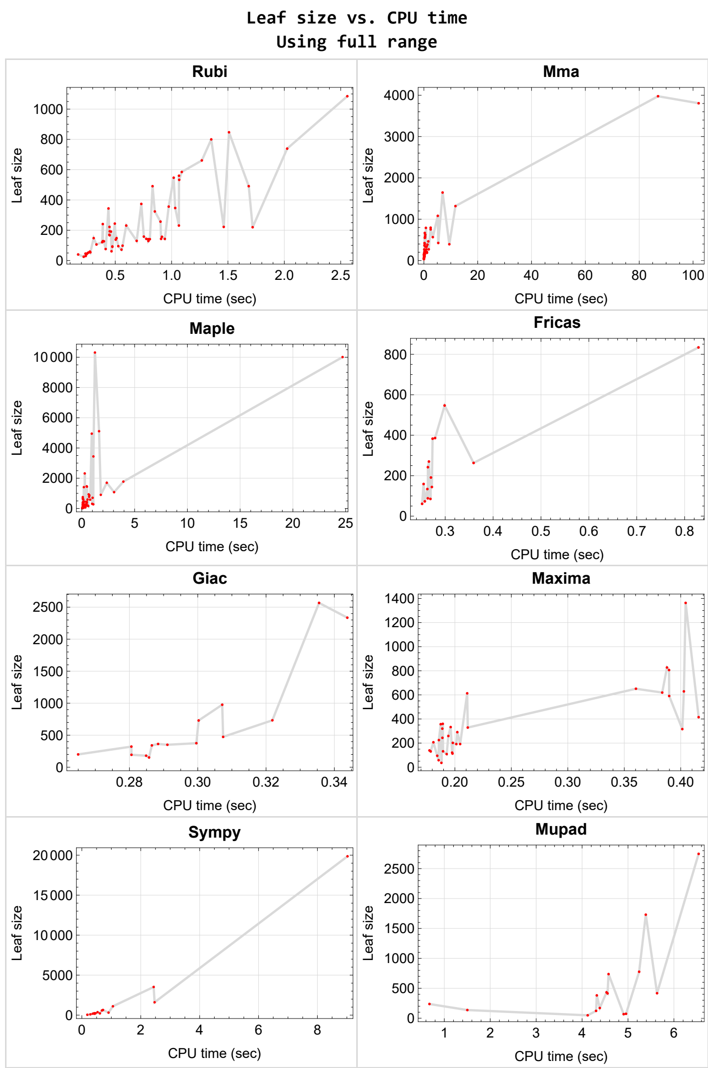


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{50, 51\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{10, 13, 15, 20, 22, 28\}$

Mathematica $\{42, 44, 45, 49\}$

Maple $\{5, 8, 18, 23, 25, 26, 27, 28, 42, 45, 46, 48, 49, 53, 58\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

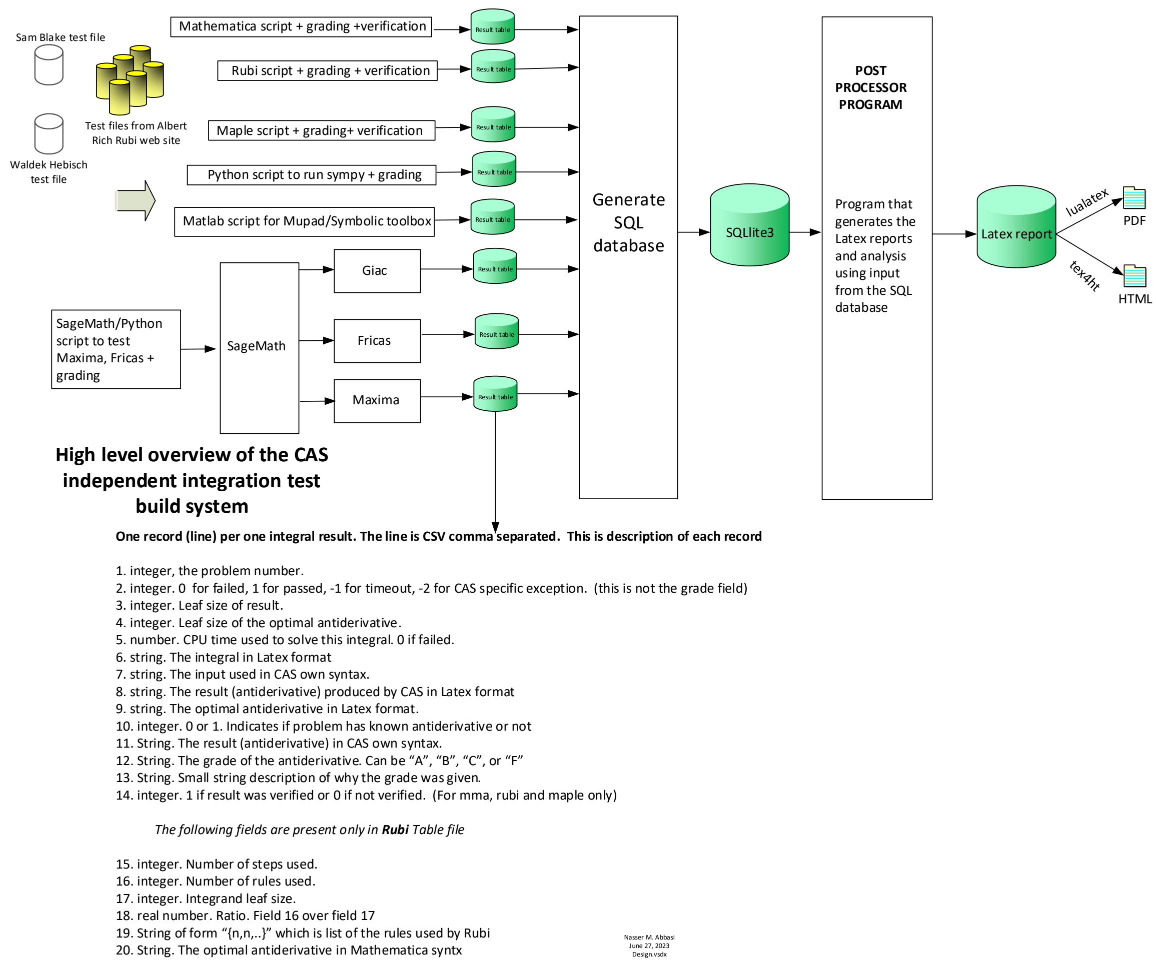
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 46, 47, 53, 54, 55, 57, 58, 59, 60, 61, 62 }

B grade { 2, 39, 45 }

C grade { 5, 6, 7, 18, 25, 27, 28, 42, 43, 44, 49, 56 }

F normal fail { 48, 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 29, 30, 33, 34, 35, 36, 37, 41, 43, 44, 54, 55, 56, 57, 59, 60, 62 }

B grade { 17, 19, 20, 24, 31, 32, 38, 39, 40, 47, 61 }

C grade { 5, 8, 18, 23, 25, 26, 27, 28, 42, 45, 46, 48, 49, 53, 58 }

F normal fail { 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 11, 13, 14, 33, 34 }

B grade { 9, 10, 15, 17, 20, 22, 31, 32, 36, 37 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 6, 7, 13, 32, 33, 34, 36, 37, 55, 56 }

B grade { 9, 10, 11, 14, 15, 16, 17, 20, 22, 24, 29, 30, 31, 38, 39, 40 }

C grade { 54, 57 }

F normal fail { 5, 8, 12, 18, 19, 21, 23, 25, 26, 27, 28, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 58, 59, 60, 62 }

F(-1) timedout fail { }

F(-2) exception fail { 61 }

2.1.6 Giac

A grade { }

B grade { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 34, 36, 37 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 34 }

B grade { 9, 10, 11, 13, 14, 15, 17, 20, 22, 31, 32, 33, 36, 37 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 55, 56, 62 }

F(-1) timedout fail { 50, 53, 54, 57, 58, 59, 60, 61 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	244	187	449	320	0	0	0	0
N.S.	1	0.93	0.71	1.71	1.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.538	1.153	0.275	0.189	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	193	463	350	259	0	0	0	0
N.S.	1	0.95	2.27	1.72	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	1.642	0.098	0.194	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	127	98	244	202	0	0	0	0
N.S.	1	0.93	0.72	1.79	1.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.426	0.269	0.078	0.198	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	76	55	91	139	0	0	0	0
N.S.	1	0.94	0.68	1.12	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	0.072	0.121	0.178	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	753	901	0	0	0	0	0
N.S.	1	1.00	5.09	6.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	2.531	1.796	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	257	208	301	244	0	0	0	0
N.S.	1	1.02	0.83	1.20	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.970	1.093	0.122	0.189	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	347	271	385	360	0	0	0	0
N.S.	1	0.94	0.73	1.04	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	1.071	1.808	0.089	0.189	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	61	75	158	0	0	0	0	0
N.S.	1	1.09	1.34	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	0.112	0.609	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	59	78	74	357	159	231	363	414
N.S.	1	0.82	1.08	1.03	4.96	2.21	3.21	5.04	5.75
time (sec)	N/A	0.292	0.088	0.230	0.187	0.255	0.464	0.288	4.555

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	54	59	67	225	144	180	322	237
N.S.	1	0.78	0.86	0.97	3.26	2.09	2.61	4.67	3.43
time (sec)	N/A	0.297	0.074	0.131	0.186	0.272	0.373	0.281	0.677

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	46	77	61	113	73	95	180	73
N.S.	1	0.96	1.60	1.27	2.35	1.52	1.98	3.75	1.52
time (sec)	N/A	0.251	0.031	0.079	0.198	0.257	0.289	0.285	4.957

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	43	54	54	0	0	0	0	0
N.S.	1	0.80	1.00	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.022	0.103	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	53	69	64	95	85	219	152	122
N.S.	1	0.84	1.10	1.02	1.51	1.35	3.48	2.41	1.94
time (sec)	N/A	0.279	0.081	0.186	0.184	0.269	0.612	0.286	4.305

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	50	100	67	131	88	313	194	67
N.S.	1	0.79	1.59	1.06	2.08	1.40	4.97	3.08	1.06
time (sec)	N/A	0.272	0.077	0.339	0.179	0.264	0.906	0.281	4.905

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	141	148	261	827	383	581	733	1730
N.S.	1	0.89	0.93	1.64	5.20	2.41	3.65	4.61	10.88
time (sec)	N/A	0.847	0.152	0.363	0.388	0.274	0.681	0.322	5.388

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	143	150	251	619	0	0	0	0
N.S.	1	0.80	0.84	1.40	3.46	0.00	0.00	0.00	0.00
time (sec)	N/A	0.831	0.359	0.357	0.384	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	91	134	221	316	191	238	351	432
N.S.	1	0.96	1.41	2.33	3.33	2.01	2.51	3.69	4.55
time (sec)	N/A	0.506	0.102	0.137	0.401	0.270	0.421	0.291	4.533

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	158	424	705	0	0	0	0	0
N.S.	1	0.94	2.52	4.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.813	0.252	1.022	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	95	126	252	0	0	0	0	0
N.S.	1	0.91	1.21	2.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	0.260	0.290	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	98	136	238	329	270	1102	375	776
N.S.	1	0.82	1.14	2.00	2.76	2.27	9.26	3.15	6.52
time (sec)	N/A	0.607	0.197	0.338	0.211	0.266	1.054	0.300	5.243

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	141	218	307	0	0	0	0	0
N.S.	1	0.78	1.21	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.866	0.449	0.978	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	157	218	273	613	547	3516	730	2746
N.S.	1	0.91	1.27	1.59	3.56	3.18	20.44	4.24	15.97
time (sec)	N/A	0.986	0.259	1.088	0.211	0.299	2.438	0.300	6.538

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	222	336	1079	0	0	0	0	0
N.S.	1	0.84	1.28	4.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.582	0.547	3.056	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	143	220	932	629	0	0	0	0
N.S.	1	0.89	1.38	5.82	3.93	0.00	0.00	0.00	0.00
time (sec)	N/A	0.986	0.298	0.648	0.403	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	231	581	1442	0	0	0	0	0
N.S.	1	0.90	2.26	5.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.143	0.464	0.485	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	143	129	232	1466	0	0	0	0	0
N.S.	1	0.90	1.62	10.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.861	0.498	0.461	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	166	143	335	4949	0	0	0	0	0
N.S.	1	0.86	2.02	29.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.024	1.038	0.941	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	220	385	1776	0	0	0	0	0
N.S.	1	0.82	1.43	6.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.843	1.341	3.936	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	25	31	14	58	0	0	0	0
N.S.	1	1.19	1.48	0.67	2.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.005	0.036	0.186	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	30	52	29	132	0	0	0	0
N.S.	1	0.94	1.62	0.91	4.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.008	0.115	0.190	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	167	270	599	333	386	644	2336	737
N.S.	1	0.99	1.61	3.57	1.98	2.30	3.83	13.90	4.39
time (sec)	N/A	0.466	0.148	0.138	0.196	0.279	0.718	0.344	4.577

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	174	351	207	242	369	976	381
N.S.	1	1.07	1.45	2.92	1.72	2.02	3.08	8.13	3.18
time (sec)	N/A	0.403	0.088	0.121	0.181	0.264	0.534	0.307	4.321

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	106	138	122	109	134	173	341	136
N.S.	1	1.09	1.42	1.26	1.12	1.38	1.78	3.52	1.40
time (sec)	N/A	0.354	0.042	0.078	0.193	0.263	0.435	0.287	1.502

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	37	36	61	46	200	48
N.S.	1	1.00	1.20	0.92	0.90	1.52	1.15	5.00	1.20
time (sec)	N/A	0.185	0.010	0.037	0.188	0.252	0.186	0.265	4.122

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	148	148	192	0	0	0	0	0
N.S.	1	1.14	1.14	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.038	0.289	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	121	125	137	121	263	1605	474	170
N.S.	1	1.05	1.09	1.19	1.05	2.29	13.96	4.12	1.48
time (sec)	N/A	0.409	0.122	0.236	0.198	0.359	2.471	0.307	4.386

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	170	174	197	291	834	19859	2567	417
N.S.	1	1.02	1.04	1.18	1.74	4.99	118.92	15.37	2.50
time (sec)	N/A	0.487	0.218	0.555	0.202	0.829	9.022	0.336	5.633

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	547	1082	2318	1363	0	0	0	0
N.S.	1	0.97	1.93	4.12	2.43	0.00	0.00	0.00	0.00
time (sec)	N/A	1.110	5.178	0.275	0.404	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	795	1411	806	0	0	0	0
N.S.	1	1.00	2.13	3.77	2.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.803	2.517	0.193	0.390	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	231	271	453	415	0	0	0	0
N.S.	1	1.05	1.23	2.05	1.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	0.947	0.141	0.416	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	93	103	134	0	0	0	0	0
N.S.	1	0.96	1.06	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	0.149	0.109	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	214	241	3806	1700	0	0	0	0	0
N.S.	1	1.13	17.79	7.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	102.080	2.365	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	491	425	592	0	0	0	0	0
N.S.	1	1.02	0.89	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.760	5.345	0.539	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	750	739	1318	870	0	0	0	0	0
N.S.	1	0.99	1.76	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.070	11.736	0.714	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	533	1646	10013	0	0	0	0	0
N.S.	1	0.98	3.01	18.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.149	6.957	24.709	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	326	324	566	10314	0	0	0	0	0
N.S.	1	0.99	1.74	31.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.901	3.322	1.243	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	130	194	265	0	0	0	0	0
N.S.	1	0.98	1.47	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	0.207	0.168	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	308	344	0	3441	0	0	0	0	0
N.S.	1	1.12	0.00	11.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.000	1.097	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1089	1085	3976	5109	0	0	0	0	0
N.S.	1	1.00	3.65	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.773	86.998	1.633	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	432	52	0	22	22
N.S.	1	1.00	1.10	1.00	21.60	2.60	0.00	1.10	1.10
time (sec)	N/A	0.298	3.540	0.207	4.000	0.270	0.000	0.385	3.699

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	262	36	19	22	22
N.S.	1	1.00	1.10	1.00	13.10	1.80	0.95	1.10	1.10
time (sec)	N/A	0.302	0.310	0.188	2.316	0.265	93.581	0.348	3.704

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	162	222	0	0	0	0	0	0	0
N.S.	1	1.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	780	800	623	259	0	0	0	0	0
N.S.	1	1.03	0.80	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.478	0.527	0.424	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	491	365	444	591	0	0	0	0
N.S.	1	1.02	0.76	0.92	1.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	0.206	0.467	0.390	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	138	138	181	192	0	0	0	0
N.S.	1	1.15	1.15	1.51	1.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	0.011	0.408	0.205	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	191	394	259	192	0	0	0	0
N.S.	1	1.03	2.12	1.39	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.487	9.458	0.410	0.201	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	560	555	581	651	0	0	0	0
N.S.	1	1.03	1.02	1.07	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	1.132	0.402	0.791	0.360	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	832	847	791	366	0	0	0	0	0
N.S.	1	1.02	0.95	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.549	0.751	0.474	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	549	648	0	0	0	0	0
N.S.	1	1.00	0.94	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.172	0.375	0.091	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	668	751	0	0	0	0	0
N.S.	1	1.00	1.01	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.345	0.345	0.091	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	356	403	768	0	0	0	0	0
N.S.	1	1.06	1.20	2.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.047	0.363	0.692	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	72	150	149	0	0	0	0	0
N.S.	1	0.87	1.81	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.073	0.192	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [4] had the largest ratio of [.7500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.93	12	0.417
2	A	5	4	0.95	12	0.333
3	A	6	5	0.93	10	0.500
4	A	7	6	0.94	8	0.750
5	A	5	4	1.00	12	0.333
6	A	8	7	1.02	12	0.583
7	A	7	6	0.94	12	0.500
8	A	7	6	1.09	12	0.500
9	A	6	5	0.82	21	0.238
10	A	7	6	0.78	21	0.286
11	A	6	5	0.96	19	0.263
12	A	4	3	0.80	21	0.143
13	A	8	7	0.84	21	0.333
14	A	6	5	0.79	21	0.238
15	A	12	11	0.89	23	0.478
16	A	12	11	0.80	23	0.478
17	A	7	6	0.96	21	0.286
18	A	7	6	0.94	23	0.261
19	A	7	6	0.91	23	0.261
20	A	11	10	0.82	23	0.435
21	A	11	10	0.78	23	0.435
22	A	16	15	0.91	23	0.652

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	13	12	0.84	23	0.522
24	A	11	10	0.89	21	0.476
25	A	8	7	0.90	23	0.304
26	A	8	7	0.90	23	0.304
27	A	10	9	0.86	23	0.391
28	A	17	16	0.82	23	0.696
29	A	4	3	1.19	12	0.250
30	A	4	3	0.94	19	0.158
31	A	6	5	0.99	18	0.278
32	A	6	5	1.07	18	0.278
33	A	6	5	1.09	16	0.312
34	A	1	1	1.00	10	0.100
35	A	7	6	1.14	18	0.333
36	A	4	4	1.05	18	0.222
37	A	4	4	1.02	18	0.222
38	A	5	4	0.97	20	0.200
39	A	5	4	1.00	20	0.200
40	A	5	4	1.05	18	0.222
41	A	7	6	0.96	12	0.500
42	A	4	3	1.13	20	0.150
43	A	7	6	1.02	20	0.300
44	A	7	6	0.99	20	0.300
45	A	5	4	0.98	20	0.200
46	A	5	4	0.99	18	0.222
47	A	7	6	0.98	12	0.500
48	A	4	3	1.12	20	0.150
49	A	7	6	1.00	20	0.300
50	N/A	3	0	1.00	20	0.000
51	N/A	3	0	1.00	20	0.000
52	A	5	4	1.37	18	0.222
53	A	3	3	1.03	16	0.188
54	A	3	3	1.02	16	0.188
55	A	7	6	1.15	14	0.429

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	3	3	1.03	16	0.188
57	A	3	3	1.03	16	0.188
58	A	3	3	1.02	16	0.188
59	A	5	4	1.00	18	0.222
60	A	6	5	1.00	18	0.278
61	A	2	2	1.06	19	0.105
62	A	7	6	0.87	32	0.188

CHAPTER 3

Listing of integrals

3.1 $\int x^3 \text{arctanh}(a+bx)^2 \, dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 263

$$
\begin{aligned}
\int x^3 \text{arctanh}(a+bx)^2 \, dx = & -\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}+\frac{a\text{arctanh}(a+bx)}{b^4} \\
& +\frac{(1+6a^2)(a+bx)\text{arctanh}(a+bx)}{2b^4} \\
& -\frac{a(a+bx)^2\text{arctanh}(a+bx)}{b^4} \\
& +\frac{(a+bx)^3\text{arctanh}(a+bx)}{6b^4}-\frac{a(1+a^2)\text{arctanh}(a+bx)^2}{b^4} \\
& -\frac{(1+6a^2+a^4)\text{arctanh}(a+bx)^2}{4b^4}+\frac{1}{4}x^4\text{arctanh}(a+bx)^2 \\
& +\frac{2a(1+a^2)\text{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b^4} \\
& +\frac{\log(1-(a+bx)^2)}{12b^4}+\frac{(1+6a^2)\log(1-(a+bx)^2)}{4b^4} \\
& +\frac{a(1+a^2)\text{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b^4}
\end{aligned}
$$

output
```
-a*x/b^3+1/12*(b*x+a)^2/b^4+a*arctanh(b*x+a)/b^4+1/2*(6*a^2+1)*(b*x+a)*arc
tanh(b*x+a)/b^4-a*(b*x+a)^2*arctanh(b*x+a)/b^4+1/6*(b*x+a)^3*arctanh(b*x+a
)/b^4-a*(a^2+1)*arctanh(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*arctanh(b*x+a)^2/b^
4+1/4*x^4*arctanh(b*x+a)^2+2*a*(a^2+1)*arctanh(b*x+a)*ln(2/(-b*x-a+1))/b^4
+1/12*ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*poly
log(2,(-b*x-a-1)/(-b*x-a+1))/b^4
```

3.1.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int x^3 \text{arctanh}(a+bx)^2\,dx =$$

$$-\frac{1+11a^2+10abx-b^2x^2+3(1-4a+6a^2-4a^3+a^4-b^4x^4)\,\text{arctanh}(a+bx)^2-2\text{arctanh}(a+bx)\,(9$$

input
```
Integrate[x^3*ArcTanh[a + b*x]^2,x]
```

output
```
-1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4
- b^4*x^4)*ArcTanh[a + b*x]^2 - 2*ArcTanh[a + b*x]*(9*a + 13*a^3 + 3*b*x +
 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 + E^(-2*ArcTanh[a
+ b*x])]) + 8*Log[1/Sqrt[1 - (a + b*x)^2]] + 36*a^2*Log[1/Sqrt[1 - (a + b*
x)^2]] + 12*(a + a^3)*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/b^4
```

3.1.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6661, 25, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{arctanh}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int x^3 \text{arctanh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x^3 \text{arctanh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -b^3x^3 \text{arctanh}(a+bx)^2 d(a+bx)}{b^4}$$

$$\Big\downarrow \; 6480$$

$$-\frac{\frac{1}{2}\int\left(-\text{arctanh}(a+bx)(a+bx)^2+4a\text{arctanh}(a+bx)(a+bx)-\left(6a^2+1\right)\text{arctanh}(a+bx)+\frac{(a^4+6a^2-4(a^2+1)(a}{1-}\right.}{b^4}$$

$\downarrow$ 2009

$$-\frac{\frac{1}{2}\left(-\left(6a^2+1\right)(a+bx)\text{arctanh}(a+bx)+2a\left(a^2+1\right)\text{arctanh}(a+bx)^2-4a\left(a^2+1\right)\text{arctanh}(a+bx)\log\left(\frac{2}{-a-ba}\right.\right.}{}$$

input
```
Int[x^3*ArcTanh[a + b*x]^2,x]
```

output
```
-((-1/4*(b^4*x^4*ArcTanh[a + b*x]^2) + (2*a*(a + b*x) - (a + b*x)^2/6 - 2*
a*ArcTanh[a + b*x] - (1 + 6*a^2)*(a + b*x)*ArcTanh[a + b*x] + 2*a*(a + b*x
)^2*ArcTanh[a + b*x] - ((a + b*x)^3*ArcTanh[a + b*x])/3 + 2*a*(1 + a^2)*Ar
cTanh[a + b*x]^2 + ((1 + 6*a^2 + a^4)*ArcTanh[a + b*x]^2)/2 - 4*a*(1 + a^2
)*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)] - Log[1 - (a + b*x)^2]/6 - ((1 + 6
*a^2)*Log[1 - (a + b*x)^2])/2 - 2*a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(
1 - a - b*x))])/2)/b^4)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.71

method	result
parts	$\frac{x^4 \operatorname{arctanh}(bx+a)^2}{4} - \frac{-6 \operatorname{arctanh}(bx+a)a^2(bx+a)+2 \operatorname{arctanh}(bx+a)(bx+a)^2 a-\frac{\operatorname{arctanh}(bx+a)(bx+a)^3}{3}-\operatorname{arctanh}(bx+}{}$
derivativedivides	$\frac{-\operatorname{arctanh}(bx+a)^2 a^3(bx+a)+\frac{3 \operatorname{arctanh}(bx+a)^2 a^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2 a(bx+a)^3+3 \operatorname{arctanh}(bx+a)a^2(bx+a)+\operatorname{arcta}}{}$
default	$\frac{-\operatorname{arctanh}(bx+a)^2 a^3(bx+a)+\frac{3 \operatorname{arctanh}(bx+a)^2 a^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2 a(bx+a)^3+3 \operatorname{arctanh}(bx+a)a^2(bx+a)+\operatorname{arcta}}{}$
risch	$-\frac{1}{12b^4}-\frac{5ax}{6b^3}-\frac{\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)a^3}{b^4}+\frac{\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)\ln(-bx-a+1)a^3}{b^4}-\frac{\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}-\right.}{b^4}$

input
```
int(x^3*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctanh(b*x+a)^2-1/2/b^4*(-6*arctanh(b*x+a)*a^2*(b*x+a)+2*arctanh(
b*x+a)*(b*x+a)^2*a-1/3*arctanh(b*x+a)*(b*x+a)^3-arctanh(b*x+a)*(b*x+a)-1/2
*arctanh(b*x+a)*ln(b*x+a-1)*a^4+2*arctanh(b*x+a)*ln(b*x+a-1)*a^3-3*arctanh
(b*x+a)*ln(b*x+a-1)*a^2+2*arctanh(b*x+a)*ln(b*x+a-1)*a-1/2*arctanh(b*x+a)*
ln(b*x+a-1)+1/2*arctanh(b*x+a)*ln(b*x+a+1)*a^4+2*arctanh(b*x+a)*ln(b*x+a+1
)*a^3+3*arctanh(b*x+a)*ln(b*x+a+1)*a^2+2*arctanh(b*x+a)*ln(b*x+a+1)*a+1/2*
arctanh(b*x+a)*ln(b*x+a+1)+2*(b*x+a)*a-1/6*(b*x+a)^2-1/6*(18*a^2-6*a+4)*ln
(b*x+a-1)+1/6*(-18*a^2-6*a-4)*ln(b*x+a+1)-1/6*(3*a^4-12*a^3+18*a^2-12*a+3)
*(1/4*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*
x+1/2*a+1/2))-1/6*(-3*a^4-12*a^3-18*a^2-12*a-3)*(1/2*(ln(b*x+a+1)-ln(1/2*b
*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(
b*x+a+1)^2))
```

3.1.5 Fricas [F]

$$\int x^3\text{arctanh}(a+bx)^2\,dx=\int x^3\,\text{artanh}\,(bx+a)^2\,dx$$

input
```
integrate(x^3*arctanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctanh(b*x + a)^2, x)
```

3.1.6 Sympy [F]

$$\int x^3\text{arctanh}(a+bx)^2\,dx=\int x^3\,\text{atanh}^2\,(a+bx)\,dx$$

input
```
integrate(x**3*atanh(b*x+a)**2,x)
```

output
```
Integral(x**3*atanh(a + b*x)**2, x)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.22

$$\int x^3\text{arctanh}(a+bx)^2\,dx=\frac{1}{4}\,x^4\,\text{artanh}\,(bx+a)^2$$
$$+\frac{1}{48}\,b^2\left(\frac{48\,(a^3+a)\big(\log\,(bx+a-1)\log\,\big(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\big)+\text{Li}_2\big(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\big)\big)}{b^6}+\frac{4\,(13\,a^3+18\,a^2+\right.$$
$$+\frac{1}{12}\,b\left(\frac{2\,(b^2x^3-3\,abx^2+3\,(3\,a^2+1)x)}{b^4}-\frac{3\,(a^4+4\,a^3+6\,a^2+4\,a+1)\log\,(bx+a+1)}{b^5}+\frac{3\,(a^4-4\,a^3\right.$$
$$+a)$$

input
```
integrate(x^3*arctanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arctanh(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 +
18*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 +
4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a
 + 1)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1
)*log(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6
) + 1/12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3
 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a +
1)*log(b*x + a - 1)/b^5)*arctanh(b*x + a)
```

3.1.8 Giac [F]

$$\int x^3 \operatorname{arctanh}(a+bx)^2\,dx=\int x^3\operatorname{artanh}\left(bx+a\right)^2\,dx$$

input
```
integrate(x^3*arctanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctanh(b*x + a)^2, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arctanh}(a+bx)^2\,dx=\int x^3\,\operatorname{atanh}(a+b\,x)^2\,dx$$

input
```
int(x^3*atanh(a + b*x)^2,x)
```

output
```
int(x^3*atanh(a + b*x)^2, x)
```

3.2 $\int x^2 \operatorname{arctanh}(a+bx)^2 \, dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 204

$$\begin{aligned}\int x^2 \operatorname{arctanh}(a+bx)^2 \, dx = {} & \frac{x}{3b^2} - \frac{\operatorname{arctanh}(a+bx)}{3b^3} - \frac{2a(a+bx)\operatorname{arctanh}(a+bx)}{b^3} \\ & + \frac{(a+bx)^2\operatorname{arctanh}(a+bx)}{3b^3} + \frac{a(3+a^2)\operatorname{arctanh}(a+bx)^2}{3b^3} \\ & + \frac{(1+3a^2)\operatorname{arctanh}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\operatorname{arctanh}(a+bx)^2 \\ & - \frac{2(1+3a^2)\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{3b^3} \\ & - \frac{a\log\left(1-(a+bx)^2\right)}{b^3} - \frac{(1+3a^2)\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{3b^3}\end{aligned}$$

output
```
1/3*x/b^2-1/3*arctanh(b*x+a)/b^3-2*a*(b*x+a)*arctanh(b*x+a)/b^3+1/3*(b*x+a
)^2*arctanh(b*x+a)/b^3+1/3*a*(a^2+3)*arctanh(b*x+a)^2/b^3+1/3*(3*a^2+1)*ar
ctanh(b*x+a)^2/b^3+1/3*x^3*arctanh(b*x+a)^2-2/3*(3*a^2+1)*arctanh(b*x+a)*l
n(2/(-b*x-a+1))/b^3-a*ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*polylog(2,(-b*x-a-
1)/(-b*x-a+1))/b^3
```

3.2.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 463 vs. 2(204) = 408.

Time = 1.64 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.27

$$\int x^2 \text{arctanh}(a+bx)^2\,dx =$$

$$-\frac{(1-(a+bx)^2)^{3/2}\left(-\frac{a+bx}{\sqrt{1-(a+bx)^2}}+\frac{6a(a+bx)\text{arctanh}(a+bx)}{\sqrt{1-(a+bx)^2}}+\frac{3(a+bx)\text{arctanh}(a+bx)^2}{\sqrt{1-(a+bx)^2}}-\frac{3a^2(a+bx)\text{arctanh}(a+b}{\sqrt{1-(a+bx)^2}}\right.}{}$$

input
```
Integrate[x^2*ArcTanh[a + b*x]^2,x]
```

output
```
-1/12*((1 - (a + b*x)^2)^(3/2)*(-((a + b*x)/Sqrt[1 - (a + b*x)^2]) + (6*a*
(a + b*x)*ArcTanh[a + b*x])/Sqrt[1 - (a + b*x)^2] + (3*(a + b*x)*ArcTanh[a
 + b*x]^2)/Sqrt[1 - (a + b*x)^2] - (3*a^2*(a + b*x)*ArcTanh[a + b*x]^2)/Sq
rt[1 - (a + b*x)^2] + ArcTanh[a + b*x]^2*Cosh[3*ArcTanh[a + b*x]] + 3*a^2*
ArcTanh[a + b*x]^2*Cosh[3*ArcTanh[a + b*x]] + 2*ArcTanh[a + b*x]*Cosh[3*Ar
cTanh[a + b*x]]*Log[1 + E^(-2*ArcTanh[a + b*x])] + 6*a^2*ArcTanh[a + b*x]*
Cosh[3*ArcTanh[a + b*x]]*Log[1 + E^(-2*ArcTanh[a + b*x])] - 6*a*Cosh[3*Arc
Tanh[a + b*x]]*Log[1/Sqrt[1 - (a + b*x)^2]] + (3*(1 - 4*a + 3*a^2)*ArcTanh
[a + b*x]^2 + 2*ArcTanh[a + b*x]*(2 + (3 + 9*a^2)*Log[1 + E^(-2*ArcTanh[a
+ b*x])]) - 18*a*Log[1/Sqrt[1 - (a + b*x)^2]])/Sqrt[1 - (a + b*x)^2] - (4*
(1 + 3*a^2)*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/(1 - (a + b*x)^2)^(3/2)
- Sinh[3*ArcTanh[a + b*x]] + 6*a*ArcTanh[a + b*x]*Sinh[3*ArcTanh[a + b*x]]
 - ArcTanh[a + b*x]^2*Sinh[3*ArcTanh[a + b*x]] - 3*a^2*ArcTanh[a + b*x]^2*
Sinh[3*ArcTanh[a + b*x]]))/b^3
```

3.2.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{arctanh}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int x^2 \text{arctanh}(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int b^2 x^2 \text{arctanh}(a+bx)^2 d(a+bx)}{b^3}$$

$$\Big\downarrow \; 6480$$

$$\frac{\frac{2}{3}\int \left(-3a\text{arctanh}(a+bx)+(a+bx)\text{arctanh}(a+bx)+\frac{(a(a^2+3)-(3a^2+1)(a+bx))\text{arctanh}(a+bx)}{1-(a+bx)^2}\right)d(a+bx)+\frac{1}{3}b^3x^3\text{a}}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}a(a^2+3)\,\text{arctanh}(a+bx)^2+\frac{1}{2}(3a^2+1)\,\text{arctanh}(a+bx)^2-(3a^2+1)\,\text{arctanh}(a+bx)\log\left(\frac{2}{-a-bx+1}\right)-\frac{1}{2}(3}{}$$

input
```
Int[x^2*ArcTanh[a + b*x]^2,x]
```

output
```
((b^3*x^3*ArcTanh[a + b*x]^2)/3 + (2*((a + b*x)/2 - ArcTanh[a + b*x]/2 - 3
*a*(a + b*x)*ArcTanh[a + b*x] + ((a + b*x)^2*ArcTanh[a + b*x])/2 + (a*(3 +
 a^2)*ArcTanh[a + b*x]^2)/2 + ((1 + 3*a^2)*ArcTanh[a + b*x]^2)/2 - (1 + 3*
a^2)*ArcTanh[a + b*x]*Log[2/(1 - a - b*x)] - (3*a*Log[1 - (a + b*x)^2])/2
- ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/2))/3)/b^3
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.72

method	result
parts	$\frac{x^3 \operatorname{arctanh}(bx+a)^2}{3} - \frac{2\left(3 \operatorname{arctanh}(bx+a)(bx+a)a - \frac{\operatorname{arctanh}(bx+a)(bx+a)^2}{2} + \frac{\operatorname{arctanh}(bx+a)\ln(bx+a-1)a^3}{2} - \frac{3 \operatorname{arctanh}(bx-}{}\right.}{}$
derivativedivides	$\frac{-\frac{\operatorname{arctanh}(bx+a)^2 a^3}{3} + \operatorname{arctanh}(bx+a)^2 a^2 (bx+a) - \operatorname{arctanh}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arctanh}(bx+a)^2 (bx+a)^3}{3} - 2 \operatorname{arctanh}(bx+a)(}{}$
default	$\frac{-\frac{\operatorname{arctanh}(bx+a)^2 a^3}{3} + \operatorname{arctanh}(bx+a)^2 a^2 (bx+a) - \operatorname{arctanh}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arctanh}(bx+a)^2 (bx+a)^3}{3} - 2 \operatorname{arctanh}(bx+a)(}{}$
risch	$-\frac{1}{3b^3} + \frac{x}{3b^2} + \frac{a}{3b^3} - \frac{\ln(-bx-a+1)^2}{12b^3} + \frac{\ln(-bx-a+1)}{6b^3} + \frac{x^3 \ln(-bx-a+1)^2}{12} + \frac{\ln\left(-\frac{bx}{2}-\frac{a}{2}+\frac{1}{2}\right)\ln\left(\frac{bx}{2}+\frac{a}{2}+\frac{1}{2}\right)}{b^3}$

input
```
int(x^2*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arctanh(b*x+a)^2-2/3/b^3*(3*arctanh(b*x+a)*(b*x+a)*a-1/2*arctanh(b
*x+a)*(b*x+a)^2+1/2*arctanh(b*x+a)*ln(b*x+a-1)*a^3-3/2*arctanh(b*x+a)*ln(b
*x+a-1)*a^2+3/2*arctanh(b*x+a)*ln(b*x+a-1)*a-1/2*arctanh(b*x+a)*ln(b*x+a-1
)-1/2*arctanh(b*x+a)*ln(b*x+a+1)*a^3-3/2*arctanh(b*x+a)*ln(b*x+a+1)*a^2-3/
2*arctanh(b*x+a)*ln(b*x+a+1)*a-1/2*arctanh(b*x+a)*ln(b*x+a+1)-1/2*(a^3+3*a
^2+3*a+1)*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-
1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2)-1/2*b*x-1/2*a+1/4*(6*a-1)*
ln(b*x+a-1)-1/4*(-6*a-1)*ln(b*x+a+1)-1/2*(-a^3+3*a^2-3*a+1)*(1/4*ln(b*x+a-
1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)))
```

3.2.5 Fricas [F]

$$\int x^2\text{arctanh}(a+bx)^2\,dx=\int x^2\,\text{artanh}\,(bx+a)^2\,dx$$

input
```
integrate(x^2*arctanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arctanh(b*x + a)^2, x)
```

3.2.6 Sympy [F]

$$\int x^2\text{arctanh}(a+bx)^2\,dx=\int x^2\,\text{atanh}^2\,(a+bx)\,dx$$

input
```
integrate(x**2*atanh(b*x+a)**2,x)
```

output
```
Integral(x**2*atanh(a + b*x)**2, x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.27

$$\int x^2\text{arctanh}(a+bx)^2\,dx = \frac{1}{3}x^3\,\text{artanh}\,(bx+a)^2 - \frac{1}{12}b^2\left(\frac{4\,(3\,a^2+1)\left(\log\,(bx+a-1)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{b^5}+\frac{2\,(5\,a^2+6\,a+1)\,\text{l}}{b^5}\right. + \frac{1}{3}\,b\left(\frac{bx^2-4\,ax}{b^3}+\frac{(a^3+3\,a^2+3\,a+1)\log\,(bx+a+1)}{b^4}-\frac{(a^3-3\,a^2+3\,a-1)\log\,(bx+a-1)}{b^4}\right)\text{arta}$$
$$+\,a)$$

input
```
integrate(x^2*arctanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arctanh(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 +
6*a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^
2 - 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3
*a^2 + 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x +
 a - 1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b
*x + a + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arctanh(b*
x + a)
```

3.2.8 Giac [F]

$$\int x^2\text{arctanh}(a+bx)^2\,dx = \int x^2\,\text{artanh}\,(bx+a)^2\,dx$$

input
```
integrate(x^2*arctanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctanh(b*x + a)^2, x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arctanh}(a+bx)^2\,dx = \int x^2\,\text{atanh}(a+b\,x)^2\,dx$$

input
```
int(x^2*atanh(a + b*x)^2,x)
```

output
```
int(x^2*atanh(a + b*x)^2, x)
```

3.3 $\int x\text{arctanh}(a+bx)^2\,dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 136

$$\begin{aligned}\int x\text{arctanh}(a+bx)^2\,dx = {} & \frac{(a+bx)\text{arctanh}(a+bx)}{b^2}-\frac{a\text{arctanh}(a+bx)^2}{b^2}\\ & -\frac{(1+a^2)\,\text{arctanh}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\text{arctanh}(a+bx)^2\\ & +\frac{2a\text{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b^2}\\ & +\frac{\log\left(1-(a+bx)^2\right)}{2b^2}+\frac{a\,\text{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b^2}\end{aligned}$$

output
```
(b*x+a)*arctanh(b*x+a)/b^2-a*arctanh(b*x+a)^2/b^2-1/2*(a^2+1)*arctanh(b*x+
a)^2/b^2+1/2*x^2*arctanh(b*x+a)^2+2*a*arctanh(b*x+a)*ln(2/(-b*x-a+1))/b^2+
1/2*ln(1-(b*x+a)^2)/b^2+a*polylog(2,(-b*x-a-1)/(-b*x-a+1))/b^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int x\text{arctanh}(a+bx)^2\,dx$$

$$=\frac{(-1+2a-a^2+b^2x^2)\,\text{arctanh}(a+bx)^2+2\text{arctanh}(a+bx)\left(a+bx+2a\log\left(1+e^{-2\text{arctanh}(a+bx)}\right)\right)-2\,\text{lo}}{2b^2}$$

input
```
Integrate[x*ArcTanh[a + b*x]^2,x]
```

output
```
((-1 + 2*a - a^2 + b^2*x^2)*ArcTanh[a + b*x]^2 + 2*ArcTanh[a + b*x]*(a + b
*x + 2*a*Log[1 + E^(-2*ArcTanh[a + b*x])]) - 2*Log[1/Sqrt[1 - (a + b*x)^2]
] - 2*a*PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/(2*b^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6661, 25, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{arctanh}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int x\text{arctanh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x\text{arctanh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx\text{arctanh}(a+bx)^2d(a+bx)}{b^2}$$

$$\Big\downarrow \text{6480}$$

$$-\frac{\int \left(\frac{(a^2-2(a+bx)a+1)\text{arctanh}(a+bx)}{1-(a+bx)^2}-\text{arctanh}(a+bx)\right)d(a+bx)-\frac{1}{2}b^2x^2\text{arctanh}(a+bx)^2}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\left(a^2+1\right)\text{arctanh}(a+bx)^2-\frac{1}{2}b^2x^2\text{arctanh}(a+bx)^2+a\text{arctanh}(a+bx)^2-(a+bx)\text{arctanh}(a+bx)-2a\text{arcta}}{b^2}$$

input
```
Int[x*ArcTanh[a + b*x]^2,x]
```

output
```
-((-((a + b*x)*ArcTanh[a + b*x]) + a*ArcTanh[a + b*x]^2 + ((1 + a^2)*ArcTa
nh[a + b*x]^2)/2 - (b^2*x^2*ArcTanh[a + b*x]^2)/2 - 2*a*ArcTanh[a + b*x]*L
og[2/(1 - a - b*x)] - Log[1 - (a + b*x)^2]/2 - a*PolyLog[2, -((1 + a + b*x
)/(1 - a - b*x))])/b^2)
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{\frac{\operatorname{arctanh}(bx+a)^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2(bx+a)a+\operatorname{arctanh}(bx+a)(bx+a)-\operatorname{arctanh}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arctanh}(bx+a)}{2}}{}$
default	$\frac{\frac{\operatorname{arctanh}(bx+a)^2(bx+a)^2}{2}-\operatorname{arctanh}(bx+a)^2(bx+a)a+\operatorname{arctanh}(bx+a)(bx+a)-\operatorname{arctanh}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arctanh}(bx+a)}{2}}{}$
parts	$\frac{x^2\operatorname{arctanh}(bx+a)^2}{2}-\frac{-\operatorname{arctanh}(bx+a)(bx+a)-\frac{\operatorname{arctanh}(bx+a)\ln(bx+a-1)a^2}{2}+\operatorname{arctanh}(bx+a)\ln(bx+a-1)a-\frac{\operatorname{arctanh}(b}{}}{}$
risch	$-\frac{\ln(-bx-a+1)^2}{8b^2}+\frac{\ln(-bx-a+1)}{2b^2}+\left(-\frac{x^2\ln(-bx-a+1)}{4}-\frac{-\ln(-bx-a+1)a^2+2\ln(-bx-a+1)a-2bx-\ln(-bx}{4b^2}\right.$

input
```
int(x*arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arctanh(b*x+a)^2*(b*x+a)^2-arctanh(b*x+a)^2*(b*x+a)*a+arctanh(b
*x+a)*(b*x+a)-arctanh(b*x+a)*ln(b*x+a-1)*a+1/2*arctanh(b*x+a)*ln(b*x+a-1)-
arctanh(b*x+a)*ln(b*x+a+1)*a-1/2*arctanh(b*x+a)*ln(b*x+a+1)+1/2*ln(b*x+a-1
)+1/2*ln(b*x+a+1)+1/2*(-2*a+1)*(1/4*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+
1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2))+1/2*(-2*a-1)*(1/2*(ln(b*x+a+1)
-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2
)-1/4*ln(b*x+a+1)^2))
```

3.3.5 Fricas [F]

$$\int x\operatorname{arctanh}(a+bx)^2\,dx=\int x\operatorname{artanh}\left(bx+a\right)^2\,dx$$

input
```
integrate(x*arctanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x*arctanh(b*x + a)^2, x)
```

3.3.6 Sympy [F]

$$\int x\text{arctanh}(a+bx)^2\,dx=\int x\,\text{atanh}^2\left(a+bx\right)dx$$

input
```
integrate(x*atanh(b*x+a)**2,x)
```

output
```
Integral(x*atanh(a + b*x)**2, x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int x\text{arctanh}(a+bx)^2\,dx=\frac{1}{2}\,x^2\,\text{artanh}\left(bx+a\right)^2 +\frac{1}{8}\,b^2\left(\frac{8\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)a}{b^4}+\frac{4\left(a+1\right)\log\left(bx+a+1\right)}{b^4}+\right. +\frac{1}{2}\,b\left(\frac{2\,x}{b^2}-\frac{\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{b^3}+\frac{\left(a^2-2\,a+1\right)\log\left(bx+a-1\right)}{b^3}\right)\text{artanh}\left(bx+a\right)$$

input
```
integrate(x*arctanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arctanh(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/
2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a
+ 1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x
 + a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1
)*log(b*x + a - 1))/b^4) + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a +
1)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arctanh(b*x + a)
```

3.3.8 Giac [F]

$$\int x\text{arctanh}(a+bx)^2\,dx=\int x\,\text{artanh}\,(bx+a)^2\,\,dx$$

input
```
integrate(x*arctanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*arctanh(b*x + a)^2, x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x\text{arctanh}(a+bx)^2\,dx=\int x\,\text{atanh}(a+b\,x)^2\,dx$$

input
```
int(x*atanh(a + b*x)^2,x)
```

output
```
int(x*atanh(a + b*x)^2, x)
```

3.4 $\int \operatorname{arctanh}(a+bx)^2\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \operatorname{arctanh}(a+bx)^2\,dx=\frac{\operatorname{arctanh}(a+bx)^2}{b}+\frac{(a+bx)\operatorname{arctanh}(a+bx)^2}{b}-\frac{2\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b}$$

output
```
arctanh(b*x+a)^2/b+(b*x+a)*arctanh(b*x+a)^2/b-2*arctanh(b*x+a)*ln(2/(-b*x-
a+1))/b-polylog(2,(-b*x-a-1)/(-b*x-a+1))/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \operatorname{arctanh}(a+bx)^2\,dx=\frac{\operatorname{arctanh}(a+bx)\left((-1+a+bx)\operatorname{arctanh}(a+bx)-2\log\left(1+e^{-2\operatorname{arctanh}(a+bx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(a+bx)}\right)}{b}$$

input
```
Integrate[ArcTanh[a + b*x]^2,x]
```

output
```
(ArcTanh[a + b*x]*((-1 + a + b*x)*ArcTanh[a + b*x] - 2*Log[1 + E^(-2*ArcTa
nh[a + b*x])]) + PolyLog[2, -E^(-2*ArcTanh[a + b*x])])/b
```

3.4.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6653, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6653}$$

$$\frac{\int \operatorname{arctanh}(a+bx)^2d(a+bx)}{b}$$

$$\Big\downarrow \text{6436}$$

$$\frac{(a+bx)\operatorname{arctanh}(a+bx)^2-2\int \frac{(a+bx)\operatorname{arctanh}(a+bx)}{1-(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{6546}$$

$$\frac{(a+bx)\operatorname{arctanh}(a+bx)^2-2\left(\int \frac{\operatorname{arctanh}(a+bx)}{-a-bx+1}d(a+bx)-\frac{1}{2}\operatorname{arctanh}(a+bx)^2\right)}{b}$$

$$\Big\downarrow \text{6470}$$

$$\frac{(a+bx)\operatorname{arctanh}(a+bx)^2-2\left(-\int \frac{\log\left(\frac{2}{-a-bx+1}\right)}{1-(a+bx)^2}d(a+bx)-\frac{1}{2}\operatorname{arctanh}(a+bx)^2+\operatorname{arctanh}(a+bx)\log\left(\frac{2}{-a-bx+1}\right)\right.}{b}$$

$$\Big\downarrow \text{2849}$$

$$\frac{(a+bx)\operatorname{arctanh}(a+bx)^2-2\left(\int \frac{\log\left(\frac{2}{-a-bx+1}\right)}{1-\frac{2}{-a-bx+1}}d\frac{1}{-a-bx+1}-\frac{1}{2}\operatorname{arctanh}(a+bx)^2+\operatorname{arctanh}(a+bx)\log\left(\frac{2}{-a-bx+1}\right)\right)}{b}$$

$$\Big\downarrow \text{2752}$$

$$\frac{(a+bx)\operatorname{arctanh}(a+bx)^2-2\left(-\frac{1}{2}\operatorname{arctanh}(a+bx)^2+\operatorname{arctanh}(a+bx)\log\left(\frac{2}{-a-bx+1}\right)+\frac{1}{2}\operatorname{PolyLog}\left(2,1-\frac{2}{-a-bx+\cdots}\right.\right.}{b}$$

input
```
Int[ArcTanh[a + b*x]^2,x]
```

output
```
((a + b*x)*ArcTanh[a + b*x]^2 - 2*(-1/2*ArcTanh[a + b*x]^2 + ArcTanh[a + b
*x]*Log[2/(1 - a - b*x)] + PolyLog[2, 1 - 2/(1 - a - b*x)]/2))/b
```

3.4.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6653
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\operatorname{arctanh}(bx+a)^2(bx+a-1)+2\operatorname{arctanh}(bx+a)^2-2\operatorname{arctanh}(bx+a)\ln\left(1+\frac{(bx+a+1)^2}{1-(bx+a)^2}\right)-\operatorname{polylog}\left(2,-\frac{(bx+a+1)^2}{1-(bx+a)^2}\right)}{b}$
default	$\frac{\operatorname{arctanh}(bx+a)^2(bx+a-1)+2\operatorname{arctanh}(bx+a)^2-2\operatorname{arctanh}(bx+a)\ln\left(1+\frac{(bx+a+1)^2}{1-(bx+a)^2}\right)-\operatorname{polylog}\left(2,-\frac{(bx+a+1)^2}{1-(bx+a)^2}\right)}{b}$
risch	$\frac{(bx+a+1)\ln(bx+a+1)^2}{4b}+\left(-\frac{x\ln(-bx-a+1)}{2}+\frac{-\ln(-bx-a+1)a+\ln(-bx-a+1)}{2b}\right)\ln(bx+a+1)+\frac{x\ln(}{}$

input
```
int(arctanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctanh(b*x+a)^2*(b*x+a-1)+2*arctanh(b*x+a)^2-2*arctanh(b*x+a)*ln(1+(
b*x+a+1)^2/(1-(b*x+a)^2))-polylog(2,-(b*x+a+1)^2/(1-(b*x+a)^2)))
```

3.4.5 Fricas [F]

$$\int \operatorname{arctanh}(a+bx)^2\,dx=\int \operatorname{artanh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arctanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)^2, x)
```

3.4.6 Sympy [F]

$$\int \operatorname{arctanh}(a+bx)^2\,dx=\int \operatorname{atanh}^2\left(a+bx\right)dx$$

input
```
integrate(atanh(b*x+a)**2,x)
```

output
```
Integral(atanh(a + b*x)**2, x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.72

$$\int \operatorname{arctanh}(a+bx)^2\,dx =$$

$$-\frac{1}{4}\,b^2\left(\frac{(a+1)\log\left(bx+a+1\right)^2-2\,(a+1)\log\left(bx+a+1\right)\log\left(bx+a-1\right)+(a-1)\log\left(bx+a-1\right)^2}{b^3}\right.$$

$$+\,b\left(\frac{(a+1)\log\left(bx+a+1\right)}{b^2}-\frac{(a-1)\log\left(bx+a-1\right)}{b^2}\right)\operatorname{artanh}\left(bx+a\right)$$

$$+\,x\operatorname{artanh}\left(bx+a\right)^2$$

input
```
integrate(arctanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x
 + a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*
b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(
b*x + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arctanh(b*x + a) + x*arct
anh(b*x + a)^2
```

3.4.8 Giac [F]

$$\int \operatorname{arctanh}(a+bx)^2\,dx = \int \operatorname{artanh}\left(bx+a\right)^2\,dx$$

input
```
integrate(arctanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)^2, x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{arctanh}(a+bx)^2\,dx=\int \operatorname{atanh}(a+b\,x)^2\,dx$$

input
```
int(atanh(a + b*x)^2,x)
```

output
```
int(atanh(a + b*x)^2, x)
```

3.5 $\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 148

$$\begin{aligned}\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = {} & -\operatorname{arctanh}(a+bx)^2\log\left(\frac{2}{1+a+bx}\right)\\ & +\operatorname{arctanh}(a+bx)^2\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)\\ & +\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)\\ & -\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)\\ & +\frac{1}{2}\operatorname{PolyLog}\left(3,1-\frac{2}{1+a+bx}\right)\\ & -\frac{1}{2}\operatorname{PolyLog}\left(3,1-\frac{2bx}{(1-a)(1+a+bx)}\right)\end{aligned}$$

output
```
-arctanh(b*x+a)^2*ln(2/(b*x+a+1))+arctanh(b*x+a)^2*ln(2*b*x/(1-a)/(b*x+a+1
))+arctanh(b*x+a)*polylog(2,1-2/(b*x+a+1))-arctanh(b*x+a)*polylog(2,1-2*b*
x/(1-a)/(b*x+a+1))+1/2*polylog(3,1-2/(b*x+a+1))-1/2*polylog(3,1-2*b*x/(1-a
)/(b*x+a+1))
```

3.5.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 753, normalized size of antiderivative = 5.09

$$
\begin{aligned}
\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = & -\frac{4}{3}\operatorname{arctanh}(a+bx)^3-\frac{2\operatorname{arctanh}(a+bx)^3}{3a} \\
& +\frac{2\sqrt{1-a^2}e^{\operatorname{arctanh}(a)}\operatorname{arctanh}(a+bx)^3}{3a} \\
& -\operatorname{arctanh}(a+bx)^2\log\left(1+e^{-2\operatorname{arctanh}(a+bx)}\right) \\
& -i\pi\operatorname{arctanh}(a+bx)\log\left(\frac{1}{2}\left(e^{-\operatorname{arctanh}(a+bx)}+e^{\operatorname{arctanh}(a+bx)}\right)\right) \\
& -\operatorname{arctanh}(a+bx)^2\log\left(1-\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right) \\
& -\operatorname{arctanh}(a+bx)^2\log\left(1+\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right) \\
& +\operatorname{arctanh}(a+bx)^2\log\left(\frac{1}{2}e^{-\operatorname{arctanh}(a+bx)}\left(1+a-e^{2\operatorname{arctanh}(a+bx)}\right.\right. \\
& \qquad \left.\left.+ae^{2\operatorname{arctanh}(a+bx)}\right)\right) \\
& +\operatorname{arctanh}(a+bx)^2\log\left(1-e^{-\operatorname{arctanh}(a)+\operatorname{arctanh}(a+bx)}\right) \\
& +\operatorname{arctanh}(a+bx)^2\log\left(1+e^{-\operatorname{arctanh}(a)+\operatorname{arctanh}(a+bx)}\right) \\
& -2\operatorname{arctanh}(a)\operatorname{arctanh}(a \\
& +bx)\log\left(\frac{1}{2}i\left(-e^{\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx)}+e^{-\operatorname{arctanh}(a)+\operatorname{arctanh}(a+bx)}\right)\right) \\
& +\operatorname{arctanh}(a+bx)^2\log\left(1-e^{-2\operatorname{arctanh}(a)+2\operatorname{arctanh}(a+bx)}\right) \\
& +i\pi\operatorname{arctanh}(a+bx)\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right) \\
& -\operatorname{arctanh}(a+bx)^2\log\left(-\frac{bx}{\sqrt{1-(a+bx)^2}}\right) \\
& +2\operatorname{arctanh}(a)\operatorname{arctanh}(a \\
& \qquad +bx)\log(-i\sinh(\operatorname{arctanh}(a)-\operatorname{arctanh}(a+bx))) \\
& +\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(a+bx)}\right) \\
& -2\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,-\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right) \\
& -2\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right) \\
& +2\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arctanh}(a)+\operatorname{arctanh}(a+bx)}\right) \\
& +2\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,e^{-\operatorname{arctanh}(a)+\operatorname{arctanh}(a+bx)}\right) \\
& +\operatorname{arctanh}(a+bx)\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}(a)+2\operatorname{arctanh}(a+bx)}\right) \\
& +\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(a+bx)}\right) \\
& +2\operatorname{PolyLog}\left(3,-\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right) \\
& +2\operatorname{PolyLog}\left(3,\frac{\sqrt{-1+a}e^{\operatorname{arctanh}(a+bx)}}{\sqrt{-1-a}}\right)
\end{aligned}
$$

input
```
Integrate[ArcTanh[a + b*x]^2/x,x]
```

output
```
(-4*ArcTanh[a + b*x]^3)/3 - (2*ArcTanh[a + b*x]^3)/(3*a) + (2*Sqrt[1 - a^2
]*E^ArcTanh[a]*ArcTanh[a + b*x]^3)/(3*a) - ArcTanh[a + b*x]^2*Log[1 + E^(-
2*ArcTanh[a + b*x])] - I*Pi*ArcTanh[a + b*x]*Log[(E^(-ArcTanh[a + b*x]) +
E^ArcTanh[a + b*x])/2] - ArcTanh[a + b*x]^2*Log[1 - (Sqrt[-1 + a]*E^ArcTan
h[a + b*x])/Sqrt[-1 - a]] - ArcTanh[a + b*x]^2*Log[1 + (Sqrt[-1 + a]*E^Arc
Tanh[a + b*x])/Sqrt[-1 - a]] + ArcTanh[a + b*x]^2*Log[(1 + a - E^(2*ArcTan
h[a + b*x]) + a*E^(2*ArcTanh[a + b*x]))/(2*E^ArcTanh[a + b*x])] + ArcTanh[
a + b*x]^2*Log[1 - E^(-ArcTanh[a] + ArcTanh[a + b*x])] + ArcTanh[a + b*x]^
2*Log[1 + E^(-ArcTanh[a] + ArcTanh[a + b*x])] - 2*ArcTanh[a]*ArcTanh[a + b
*x]*Log[(I/2)*(-E^(ArcTanh[a] - ArcTanh[a + b*x]) + E^(-ArcTanh[a] + ArcTa
nh[a + b*x]))] + ArcTanh[a + b*x]^2*Log[1 - E^(-2*ArcTanh[a] + 2*ArcTanh[a
 + b*x])] + I*Pi*ArcTanh[a + b*x]*Log[1/Sqrt[1 - (a + b*x)^2]] - ArcTanh[a
 + b*x]^2*Log[-((b*x)/Sqrt[1 - (a + b*x)^2])] + 2*ArcTanh[a]*ArcTanh[a + b
*x]*Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]] + ArcTanh[a + b*x]*PolyL
og[2, -E^(-2*ArcTanh[a + b*x])] - 2*ArcTanh[a + b*x]*PolyLog[2, -((Sqrt[-1
 + a]*E^ArcTanh[a + b*x])/Sqrt[-1 - a])] - 2*ArcTanh[a + b*x]*PolyLog[2, (
Sqrt[-1 + a]*E^ArcTanh[a + b*x])/Sqrt[-1 - a]] + 2*ArcTanh[a + b*x]*PolyLo
g[2, -E^(-ArcTanh[a] + ArcTanh[a + b*x])] + 2*ArcTanh[a + b*x]*PolyLog[2,
E^(-ArcTanh[a] + ArcTanh[a + b*x])] + ArcTanh[a + b*x]*PolyLog[2, E^(-2*Ar
cTanh[a] + 2*ArcTanh[a + b*x])] + PolyLog[3, -E^(-2*ArcTanh[a + b*x])]/...
```

3.5.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6661, 25, 27, 6474}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int \frac{\operatorname{arctanh}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{\text{arctanh}(a+bx)^2}{x}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\int -\frac{\text{arctanh}(a+bx)^2}{bx}d(a+bx)$$

$$\Big\downarrow \; 6474$$

$$\text{arctanh}(a+bx)\,\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-\text{arctanh}(a+bx)\,\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\text{arctanh}(a+bx)^2\left(-\log\left(\frac{2}{a+bx+1}\right)\right)+\text{arctanh}(a+bx)^2\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)+\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2bx}{(1-a)(a+bx+1)}\right)$$

input
```
Int[ArcTanh[a + b*x]^2/x,x]
```

output
```
-(ArcTanh[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcTanh[a + b*x]^2*Log[(2*b*x
)/((1 - a)*(1 + a + b*x))] + ArcTanh[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*
x)] - ArcTanh[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + P
olyLog[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a
+ b*x))]/2
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6474
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.5.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 901, normalized size of antiderivative = 6.09

method	result	size
derivativedivides	Expression too large to display	901
default	Expression too large to display	901
parts	Expression too large to display	1544

input
```
int(arctanh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(-b*x)*arctanh(b*x+a)^2-arctanh(b*x+a)^2*ln(-(b*x+a+1)^2/(1-(b*x+a)^2)+1
+a*(1+(b*x+a+1)^2/(1-(b*x+a)^2)))+1/2*I*Pi*csgn(I*((b*x+a+1)^2/((b*x+a)^2-
1)+1+a*(1-(b*x+a+1)^2/((b*x+a)^2-1)))/(1-(b*x+a+1)^2/((b*x+a)^2-1)))*(csgn
(I*((b*x+a+1)^2/((b*x+a)^2-1)+1+a*(1-(b*x+a+1)^2/((b*x+a)^2-1))))*csgn(I/(
1-(b*x+a+1)^2/((b*x+a)^2-1)))-csgn(I*((b*x+a+1)^2/((b*x+a)^2-1)+1+a*(1-(b*
x+a+1)^2/((b*x+a)^2-1)))/(1-(b*x+a+1)^2/((b*x+a)^2-1)))*csgn(I/(1-(b*x+a+1
)^2/((b*x+a)^2-1)))-csgn(I*((b*x+a+1)^2/((b*x+a)^2-1)+1+a*(1-(b*x+a+1)^2/(
(b*x+a)^2-1))))*csgn(I*((b*x+a+1)^2/((b*x+a)^2-1)+1+a*(1-(b*x+a+1)^2/((b*x
+a)^2-1)))/(1-(b*x+a+1)^2/((b*x+a)^2-1)))+csgn(I*((b*x+a+1)^2/((b*x+a)^2-1
)+1+a*(1-(b*x+a+1)^2/((b*x+a)^2-1)))/(1-(b*x+a+1)^2/((b*x+a)^2-1)))^2)*arc
tanh(b*x+a)^2-arctanh(b*x+a)*polylog(2,-(b*x+a+1)^2/(1-(b*x+a)^2))+1/2*pol
ylog(3,-(b*x+a+1)^2/(1-(b*x+a)^2))+1/(-1+a)*a*arctanh(b*x+a)^2*ln(1-(-1+a)
*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))+1/(-1+a)*a*arctanh(b*x+a)*polylog(2,(-1
+a)*(b*x+a+1)^2/(1-(b*x+a)^2)/(-1-a))-1/2/(-1+a)*a*polylog(3,(-1+a)*(b*x+a
+1)^2/(1-(b*x+a)^2)/(-1-a))-1/(-1+a)*arctanh(b*x+a)^2*ln(1-(-1+a)*(b*x+a+1
)^2/(1-(b*x+a)^2)/(-1-a))-1/(-1+a)*arctanh(b*x+a)*polylog(2,(-1+a)*(b*x+a+
1)^2/(1-(b*x+a)^2)/(-1-a))+1/2/(-1+a)*polylog(3,(-1+a)*(b*x+a+1)^2/(1-(b*x
+a)^2)/(-1-a))
```

3.5.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)^2/x, x)
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{atanh}^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(atanh(b*x+a)**2/x,x)
```

output
```
Integral(atanh(a + b*x)**2/x, x)
```

3.5.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
integrate(arctanh(b*x + a)^2/x, x)
```

3.5.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)^2/x, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)^2}{x}\,dx$$

input
```
int(atanh(a + b*x)^2/x,x)
```

output
```
int(atanh(a + b*x)^2/x, x)
```

3.6 $\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx$

3.6.1 Optimal result

Integrand size = 12, antiderivative size = 251

$$
\begin{aligned}
\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx = & -\frac{\operatorname{arctanh}(a+bx)^2}{x}+\frac{b\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} \\
& +\frac{b\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1-a^2} \\
& +\frac{2b\operatorname{arctanh}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)}{1-a^2} \\
& +\frac{b\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{2(1-a)}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2(1+a)} \\
& +\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{1-a^2}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)}{1-a^2}
\end{aligned}
$$

output
```
-arctanh(b*x+a)^2/x+b*arctanh(b*x+a)*ln(2/(-b*x-a+1))/(1-a)+b*arctanh(b*x+
a)*ln(2/(b*x+a+1))/(1+a)-2*b*arctanh(b*x+a)*ln(2/(b*x+a+1))/(-a^2+1)+2*b*a
rctanh(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)+1/2*b*polylog(2,(-b*x-a-1
)/(-b*x-a+1))/(1-a)-1/2*b*polylog(2,1-2/(b*x+a+1))/(1+a)+b*polylog(2,1-2/(
b*x+a+1))/(-a^2+1)-b*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)
```

3.6.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx$$
$$= \frac{-\left(\left(-a+a^3+a^2bx+b\left(-1+\sqrt{1-a^2}e^{\operatorname{arctanh}(a)}\right)x\right)\operatorname{arctanh}(a+bx)^2\right)+abx\operatorname{arctanh}(a+bx)\left(-i\pi+2\mathrm{ar}\right.}{}$$

input
```
Integrate[ArcTanh[a + b*x]^2/x^2,x]
```

output
```
(-((-a + a^3 + a^2*b*x + b*(-1 + Sqrt[1 - a^2]*E^ArcTanh[a])*x)*ArcTanh[a
+ b*x]^2) + a*b*x*ArcTanh[a + b*x]*((-I)*Pi + 2*ArcTanh[a] - 2*Log[1 - E^(
2*ArcTanh[a] - 2*ArcTanh[a + b*x])]) + a*b*x*(I*Pi*(Log[1 + E^(2*ArcTanh[a
 + b*x])] - Log[1/Sqrt[1 - (a + b*x)^2]]) + 2*ArcTanh[a]*(Log[1 - E^(2*Arc
Tanh[a] - 2*ArcTanh[a + b*x])] - Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*
x]]])) + a*b*x*PolyLog[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])])/(a*(-1 +
 a^2)*x)
```

3.6.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6659, 7292, 6671, 25, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx$$

$$\Big\downarrow \; 6659$$

$$2b\int \frac{\operatorname{arctanh}(a+bx)}{x\left(1-(a+bx)^2\right)}dx-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \; 7292$$

$$2b\int \frac{\operatorname{arctanh}(a+bx)}{x\left(-a^2-2bxa-b^2x^2+1\right)}dx-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \text{6671}$$

$$2\int \frac{\operatorname{arctanh}(a+bx)}{x\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{\operatorname{arctanh}(a+bx)}{x\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \text{27}$$

$$-2b\int -\frac{\operatorname{arctanh}(a+bx)}{bx\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \text{7276}$$

$$-2b\int \left(\frac{\operatorname{arctanh}(a+bx)}{(a^2-1)\,bx}-\frac{\operatorname{arctanh}(a+bx)}{2(a-1)(a+bx-1)}+\frac{\operatorname{arctanh}(a+bx)}{2(a+1)(a+bx+1)}\right)d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

$$\Big\downarrow \text{2009}$$

$$-2b\left(\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{1-a^2}-\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2\left(1-a^2\right)}+\operatorname{PolyL}\right.$$

$$\frac{\operatorname{arctanh}(a+bx)^2}{x}$$

input
```
Int[ArcTanh[a + b*x]^2/x^2,x]
```

output
```
-(ArcTanh[a + b*x]^2/x) - 2*b*(-1/2*(ArcTanh[a + b*x]*Log[2/(1 - a - b*x)]
)/(1 - a) - (ArcTanh[a + b*x]*Log[2/(1 + a + b*x)])/(2*(1 + a)) + (ArcTanh
[a + b*x]*Log[2/(1 + a + b*x)])/(1 - a^2) - (ArcTanh[a + b*x]*Log[(2*b*x)/
((1 - a)*(1 + a + b*x))])/(1 - a^2) - PolyLog[2, -((1 + a + b*x)/(1 - a -
b*x))]/(4*(1 - a)) + PolyLog[2, 1 - 2/(1 + a + b*x)]/(4*(1 + a)) - PolyLog
[2, 1 - 2/(1 + a + b*x)]/(2*(1 - a^2)) + PolyLog[2, 1 - (2*b*x)/((1 - a)*(
1 + a + b*x))]/(2*(1 - a^2)))
```

3.6. $\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.6.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{\operatorname{arctanh}(bx+a)^2}{x}+2b\left(-\frac{\operatorname{arctanh}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2a+2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a-1}{-2+2a}\right.$
derivativedivides	$b\left(-\frac{\operatorname{arctanh}(bx+a)^2}{bx}-\frac{2\operatorname{arctanh}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2a+2}+\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a-}{-2+2a}\right.$
default	$b\left(-\frac{\operatorname{arctanh}(bx+a)^2}{bx}-\frac{2\operatorname{arctanh}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2a+2}+\frac{2\operatorname{arctanh}(bx+a)\ln(bx+a-}{-2+2a}\right.$

input
```
int(arctanh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(b*x+a)^2/x+2*b*(-arctanh(b*x+a)/(-1+a)/(1+a)*ln(-b*x)-arctanh(b*x
+a)/(2*a+2)*ln(b*x+a+1)+arctanh(b*x+a)/(-2+2*a)*ln(b*x+a-1)-1/(-1+a)/(1+a)
*(1/2*dilog(1/(1-a)*(-b*x-a+1))+1/2*ln(-b*x)*ln(1/(1-a)*(-b*x-a+1))-1/2*di
log((-b*x-a-1)/(-1-a))-1/2*ln(-b*x)*ln((-b*x-a-1)/(-1-a)))+1/2/(-1+a)*(1/4
*ln(b*x+a-1)^2-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2
*a+1/2))-1/2/(1+a)*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/
2*a+1/2)-1/2*dilog(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2))
```

3.6.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)^2/x^2, x)
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{atanh}^2(a+bx)}{x^2}\,dx$$

input
```
integrate(atanh(b*x+a)**2/x**2,x)
```

output
```
Integral(atanh(a + b*x)**2/x**2, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx$$
$$= \frac{1}{4}b^2\left(\frac{(a-1)\log(bx+a+1)^2 - 2(a-1)\log(bx+a+1)\log(bx+a-1) + (a+1)\log(bx+a-1)^2}{a^2b-b} - \right.$$
$$-b\left(\frac{\log(bx+a+1)}{a+1} - \frac{\log(bx+a-1)}{a-1} + \frac{2\log(x)}{a^2-1}\right)\operatorname{artanh}(bx+a) - \frac{\operatorname{artanh}(bx+a)^2}{x}$$

input
```
integrate(arctanh(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/4*b^2*(((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x
+ a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*l
og(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4
*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(
b*x/(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b)) - b*(log(b*x +
 a + 1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arctanh(b
*x + a) - arctanh(b*x + a)^2/x
```

3.6.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)^2/x^2, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^2}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)^2}{x^2}\,dx$$

input
```
int(atanh(a + b*x)^2/x^2,x)
```

output
```
int(atanh(a + b*x)^2/x^2, x)
```

3.7 $\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3} dx$

3.7.1 Optimal result

Integrand size = 12, antiderivative size = 370

$$\begin{aligned}\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3} dx = & -\frac{b\operatorname{arctanh}(a+bx)}{(1-a^2)\,x} - \frac{\operatorname{arctanh}(a+bx)^2}{2x^2} \\ & + \frac{b^2\log(x)}{(1-a^2)^2} + \frac{b^2\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} \\ & - \frac{b^2\log(1-a-bx)}{2(1-a)^2(1+a)} - \frac{b^2\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{2(1+a)^2} \\ & - \frac{2ab^2\operatorname{arctanh}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{(1-a^2)^2} \\ & + \frac{2ab^2\operatorname{arctanh}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)}{(1-a^2)^2} \\ & - \frac{b^2\log(1+a+bx)}{2(1-a)(1+a)^2} + \frac{b^2\operatorname{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{4(1-a)^2} \\ & + \frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{4(1+a)^2} + \frac{ab^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{(1-a^2)^2} \\ & - \frac{ab^2\operatorname{PolyLog}\left(2,1-\frac{2bx}{(1-a)(1+a+bx)}\right)}{(1-a^2)^2}\end{aligned}$$

output
```
-b*arctanh(b*x+a)/(-a^2+1)/x-1/2*arctanh(b*x+a)^2/x^2+b^2*ln(x)/(-a^2+1)^2
+1/2*b^2*arctanh(b*x+a)*ln(2/(-b*x-a+1))/(1-a)^2-1/2*b^2*ln(-b*x-a+1)/(1-a
)^2/(1+a)-1/2*b^2*arctanh(b*x+a)*ln(2/(b*x+a+1))/(1+a)^2-2*a*b^2*arctanh(b
*x+a)*ln(2/(b*x+a+1))/(-a^2+1)^2+2*a*b^2*arctanh(b*x+a)*ln(2*b*x/(1-a)/(b*
x+a+1))/(-a^2+1)^2-1/2*b^2*ln(b*x+a+1)/(1-a)/(1+a)^2+1/4*b^2*polylog(2,(-b
*x-a-1)/(-b*x-a+1))/(1-a)^2+1/4*b^2*polylog(2,1-2/(b*x+a+1))/(1+a)^2+a*b^2
*polylog(2,1-2/(b*x+a+1))/(-a^2+1)^2-a*b^2*polylog(2,1-2*b*x/(1-a)/(b*x+a+
1))/(-a^2+1)^2
```

3.7.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx$$
$$= \frac{-\left(\left(1+a^4-b^2\left(-1+2\sqrt{1-a^2}e^{\operatorname{arctanh}(a)}\right)x^2-a^2(2+b^2x^2)\right)\operatorname{arctanh}(a+bx)^2\right)+2bx\operatorname{arctanh}(a+bx)\left(-\right.}{}$$

input
```
Integrate[ArcTanh[a + b*x]^2/x^3,x]
```

output
```
(-((1 + a^4 - b^2*(-1 + 2*Sqrt[1 - a^2]*E^ArcTanh[a])*x^2 - a^2*(2 + b^2*x
^2))*ArcTanh[a + b*x]^2) + 2*b*x*ArcTanh[a + b*x]*(-1 + a^2 + a*b*x + I*a*
b*Pi*x - 2*a*b*x*ArcTanh[a] + 2*a*b*x*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[
a + b*x])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcTanh[a + b*x])] + I*a*P
i*Log[1/Sqrt[1 - (a + b*x)^2]] + Log[-((b*x)/Sqrt[1 - (a + b*x)^2])] - 2*a
*ArcTanh[a]*(Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - Log[(-I)*Sin
h[ArcTanh[a] - ArcTanh[a + b*x]]])) - 2*a*b^2*x^2*PolyLog[2, E^(2*ArcTanh[
a] - 2*ArcTanh[a + b*x])])/(2*(-1 + a^2)^2*x^2)
```

3.7.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6659, 7292, 6671, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}dx$$

$$\Big\downarrow \text{6659}$$

$$b\int \frac{\operatorname{arctanh}(a+bx)}{x^2\left(1-(a+bx)^2\right)}dx-\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{7292}$$

$$b\int \frac{\operatorname{arctanh}(a+bx)}{x^2\left(-a^2-2bxa-b^2x^2+1\right)}dx-\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{6671}$$

$$\int \frac{\operatorname{arctanh}(a+bx)}{x^2\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$b^2\int \frac{\operatorname{arctanh}(a+bx)}{b^2x^2\left(1-(a+bx)^2\right)}d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{7276}$$

$$b^2\int \left(\frac{2a\operatorname{arctanh}(a+bx)}{\left(a^2-1\right)^2bx}-\frac{\operatorname{arctanh}(a+bx)}{2(a-1)^2(a+bx-1)}+\frac{\operatorname{arctanh}(a+bx)}{2(a+1)^2(a+bx+1)}-\frac{\operatorname{arctanh}(a+bx)}{\left(a^2-1\right)b^2x^2}\right)d(a+bx)-\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$b^2\left(-\frac{\operatorname{arctanh}(a+bx)}{\left(1-a^2\right)bx}-\frac{2a\operatorname{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{\left(1-a^2\right)^2}+\frac{2a\operatorname{arctanh}(a+bx)\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{\left(1-a^2\right)^2}+\frac{a\operatorname{PolyLog}}{(1}\right.$$
$$\frac{\operatorname{arctanh}(a+bx)^2}{2x^2}$$

input
```
Int[ArcTanh[a + b*x]^2/x^3,x]
```

output
```
-1/2*ArcTanh[a + b*x]^2/x^2 + b^2*(-(ArcTanh[a + b*x]/((1 - a^2)*b*x)) + L
og[-(b*x)]/(1 - a^2)^2 + (ArcTanh[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a
)^2) - Log[1 - a - b*x]/(2*(1 - a)^2*(1 + a)) - (ArcTanh[a + b*x]*Log[2/(1
 + a + b*x)])/(2*(1 + a)^2) - (2*a*ArcTanh[a + b*x]*Log[2/(1 + a + b*x)])/
(1 - a^2)^2 + (2*a*ArcTanh[a + b*x]*Log[(2*b*x)/((1 - a)*(1 + a + b*x))])/
(1 - a^2)^2 - Log[1 + a + b*x]/(2*(1 - a)*(1 + a)^2) + PolyLog[2, -((1 + a
 + b*x)/(1 - a - b*x))]/(4*(1 - a)^2) + PolyLog[2, 1 - 2/(1 + a + b*x)]/(4
*(1 + a)^2) + (a*PolyLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2)^2 - (a*PolyLog
[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[(((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.7.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{\operatorname{arctanh}(bx+a)^2}{2x^2}+b^2\left(\frac{\operatorname{arctanh}(bx+a)}{(-1+a)(1+a)bx}+\frac{2\operatorname{arctanh}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2(1+a)^2}-\text{arcta}\right.$
derivativedivides	$b^2\left(-\frac{\operatorname{arctanh}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arctanh}(bx+a)}{(-1+a)(1+a)bx}+\frac{2\operatorname{arctanh}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2(1+a)^2}-\text{arcta}\right.$
default	$b^2\left(-\frac{\operatorname{arctanh}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arctanh}(bx+a)}{(-1+a)(1+a)bx}+\frac{2\operatorname{arctanh}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\operatorname{arctanh}(bx+a)\ln(bx+a+1)}{2(1+a)^2}-\text{arcta}\right.$

input
```
int(arctanh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(b*x+a)^2/x^2+b^2*(arctanh(b*x+a)/(-1+a)/(1+a)/b/x+2*arctanh(b
*x+a)*a/(-1+a)^2/(1+a)^2*ln(-b*x)+1/2*arctanh(b*x+a)/(1+a)^2*ln(b*x+a+1)-1
/2*arctanh(b*x+a)/(-1+a)^2*ln(b*x+a-1)-1/2/(-1+a)^2*(1/4*ln(b*x+a-1)^2-1/2
*dilog(1/2*b*x+1/2*a+1/2)-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2))+1/2/(1+a)
^2*(1/2*(ln(b*x+a+1)-ln(1/2*b*x+1/2*a+1/2))*ln(-1/2*b*x-1/2*a+1/2)-1/2*dil
og(1/2*b*x+1/2*a+1/2)-1/4*ln(b*x+a+1)^2)-1/(-1+a)/(1+a)*(-1/(-1+a)/(1+a)*l
n(-b*x)-1/(2*a+2)*ln(b*x+a+1)+1/(-2+2*a)*ln(b*x+a-1))+2*a/(-1+a)^2/(1+a)^2
*(1/2*dilog(1/(1-a)*(-b*x-a+1))+1/2*ln(-b*x)*ln(1/(1-a)*(-b*x-a+1))-1/2*di
log((-b*x-a-1)/(-1-a))-1/2*ln(-b*x)*ln((-b*x-a-1)/(-1-a))))
```

3.7.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)^2/x^3, x)
```

3.7.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{atanh}^2(a+bx)}{x^3}\,dx$$

input
```
integrate(atanh(b*x+a)**2/x**3,x)
```

output
```
Integral(atanh(a + b*x)**2/x**3, x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx = \frac{1}{8}\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)a}{a^4-2a^2+1} - \frac{8\left(\log\left(\frac{bx}{a+1}+1\right)\log(x)+\operatorname{Li}_2\left(-\right.\right.}{a^4-2a^2+1}\right. + \frac{1}{2}\left(\frac{4ab\log(x)}{a^4-2a^2+1}+\frac{b\log(bx+a+1)}{a^2+2a+1}-\frac{b\log(bx+a-1)}{a^2-2a+1}+\frac{2}{(a^2-1)x}\right)b\operatorname{artanh}(bx+a) - \frac{\operatorname{artanh}(bx+a)^2}{2x^2}$$

input
```
integrate(arctanh(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2
*a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b
*x/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(
-b*x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 -
 2*(a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log
(b*x + a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1
) - 4*log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b
^2 + 1/2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a +
 1) - b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arctanh(b*x
+ a) - 1/2*arctanh(b*x + a)^2/x^2
```

3.7.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)^2}{x^3}\,dx$$

input
```
integrate(arctanh(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)^2/x^3, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)^2}{x^3}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)^2}{x^3}\,dx$$

input
```
int(atanh(a + b*x)^2/x^3,x)
```

output
```
int(atanh(a + b*x)^2/x^3, x)
```

3.8 $\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx$

3.8.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = -\operatorname{arctanh}(1+bx)^2\log\left(-\frac{2}{bx}\right) - \operatorname{arctanh}(1+bx)\operatorname{PolyLog}\left(2,1+\frac{2}{bx}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,1+\frac{2}{bx}\right)$$

output
```
-arctanh(b*x+1)^2*ln(-2/b/x)-arctanh(b*x+1)*polylog(2,1+2/b/x)+1/2*polylog
(3,1+2/b/x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = -\frac{2}{3}\operatorname{arctanh}(1+bx)^3 - \operatorname{arctanh}(1+bx)^2\log\left(1+e^{-2\operatorname{arctanh}(1+bx)}\right) + \operatorname{arctanh}(1+bx)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(1+bx)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(1+bx)}\right)$$

input
```
Integrate[ArcTanh[1 + b*x]^2/x,x]
```

output
```
(-2*ArcTanh[1 + b*x]^3)/3 - ArcTanh[1 + b*x]^2*Log[1 + E^(-2*ArcTanh[1 + b
*x])] + ArcTanh[1 + b*x]*PolyLog[2, -E^(-2*ArcTanh[1 + b*x])] + PolyLog[3,
 -E^(-2*ArcTanh[1 + b*x])]/2
```

3.8.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6661, 25, 27, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(bx+1)^2}{x} \, dx$$

$$\Big\downarrow \text{ 6661}$$

$$\frac{\int \frac{\text{arctanh}(bx+1)^2}{x}d(bx+1)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{\text{arctanh}(bx+1)^2}{x}d(bx+1)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\int -\frac{\text{arctanh}(bx+1)^2}{bx}d(bx+1)$$

$$\Big\downarrow \text{ 6470}$$

$$2\int \frac{\text{arctanh}(bx+1)\log\left(-\frac{2}{bx}\right)}{1-(bx+1)^2}d(bx+1)-\text{arctanh}(bx+1)^2\log\left(-\frac{2}{bx}\right)$$

$$\Big\downarrow \text{ 6620}$$

$$2\left(\frac{1}{2}\int \frac{\text{PolyLog}\left(2,1+\frac{2}{bx}\right)}{1-(bx+1)^2}d(bx+1)-\frac{1}{2}\text{arctanh}(bx+1)\text{PolyLog}\left(2,1+\frac{2}{bx}\right)\right)-\text{arctanh}(bx+1)^2\log\left(-\frac{2}{bx}\right)$$

$$\Big\downarrow \text{ 7164}$$

$$2\left(\frac{1}{4}\text{PolyLog}\left(3,1+\frac{2}{bx}\right)-\frac{1}{2}\text{arctanh}(bx+1)\text{PolyLog}\left(2,1+\frac{2}{bx}\right)\right)-\text{arctanh}(bx+1)^2\log\left(-\frac{2}{bx}\right)$$

input
```
Int[ArcTanh[1 + b*x]^2/x,x]
```

output
```
-(ArcTanh[1 + b*x]^2*Log[-2/(b*x)]) + 2*(-1/2*(ArcTanh[1 + b*x]*PolyLog[2,
 1 + 2/(b*x)]) + PolyLog[3, 1 + 2/(b*x)]/4)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.8.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

method	result
derivativedivides	$\ln(bx)\operatorname{arctanh}(bx+1)^{2}-\operatorname{arctanh}(bx+1)\operatorname{polylog}\left(2,-\frac{(bx+2)^{2}}{-(bx+1)^{2}+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(bx+}{-(bx+}\right.}{2}$
default	$\ln(bx)\operatorname{arctanh}(bx+1)^{2}-\operatorname{arctanh}(bx+1)\operatorname{polylog}\left(2,-\frac{(bx+2)^{2}}{-(bx+1)^{2}+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(bx+}{-(bx+}\right.}{2}$
parts	$\ln(x)\operatorname{arctanh}(bx+1)^{2}-2b\left(\frac{\operatorname{arctanh}(bx+1)\operatorname{polylog}\left(2,-\frac{(bx+2)^{2}}{-(bx+1)^{2}+1}\right)}{2b}-\frac{\operatorname{polylog}\left(3,-\frac{(bx+2)^{2}}{-(bx+1)^{2}+1}\right)}{4b}-\right.$
risch	Expression too large to display

input
```
int(arctanh(b*x+1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(b*x)*arctanh(b*x+1)^2-arctanh(b*x+1)*polylog(2,-(b*x+2)^2/(-(b*x+1)^2+1
))+1/2*polylog(3,-(b*x+2)^2/(-(b*x+1)^2+1))-(-I*Pi*csgn(I/(1-(b*x+2)^2/((b
*x+1)^2-1)))^2+I*Pi*csgn(I/(1-(b*x+2)^2/((b*x+1)^2-1)))^3+I*Pi+ln(2))*arct
anh(b*x+1)^2
```

3.8.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}(bx+1)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+1)^2/x,x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + 1)^2/x, x)
```

3.8.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = \int \frac{\operatorname{atanh}^2(bx+1)}{x}\,dx$$

input
```
integrate(atanh(b*x+1)**2/x,x)
```

output
```
Integral(atanh(b*x + 1)**2/x, x)
```

3.8.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}(bx+1)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+1)^2/x,x, algorithm="maxima")
```

output
```
1/12*log(-b*x)^3 + 1/4*log(b*x + 2)^2*log(-x) - 1/4*integrate(2*(b*x*log(b
) + 2*(b*x + 1)*log(-x) + 2*log(b))*log(b*x + 2)/(b*x^2 + 2*x), x)
```

3.8.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = \int \frac{\operatorname{artanh}(bx+1)^2}{x}\,dx$$

input
```
integrate(arctanh(b*x+1)^2/x,x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + 1)^2/x, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(1+bx)^2}{x}\,dx = \int \frac{\operatorname{atanh}(b\,x+1)^2}{x}\,dx$$

input
```
int(atanh(b*x + 1)^2/x,x)
```

output
```
int(atanh(b*x + 1)^2/x, x)
```

3.9 $\int (ce+dex)^3(a+b\,\mathrm{arctanh}(c+dx))\,dx$

3.9.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (ce+dex)^3(a+b\,\mathrm{arctanh}(c+dx))\,dx = \frac{1}{4}be^3x+\frac{be^3(c+dx)^3}{12d}-\frac{be^3\mathrm{arctanh}(c+dx)}{4d}+\frac{e^3(c+dx)^4(a+b\,\mathrm{arctanh}(c+dx))}{4d}$$

output
```
1/4*b*e^3*x+1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctanh(d*x+c)/d+1/4*e^3*(d*x
+c)^4*(a+b*arctanh(d*x+c))/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int (ce+dex)^3(a+b\,\mathrm{arctanh}(c+dx))\,dx = \frac{e^3(6b(c+dx)+2b(c+dx)^3+6a(c+dx)^4+6b(c+dx)^4\mathrm{arctanh}(c+dx)+3b\log(1-c-dx)-3b\log(1}{24d}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e^3*(6*b*(c + d*x) + 2*b*(c + d*x)^3 + 6*a*(c + d*x)^4 + 6*b*(c + d*x)^4*
ArcTanh[c + d*x] + 3*b*Log[1 - c - d*x] - 3*b*Log[1 + c + d*x]))/(24*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6657, 27, 6452, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{ 6657}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))-\frac{1}{4}b\int \frac{(c+dx)^4}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))-\frac{1}{4}b\int \left(-(c+dx)^2+\frac{1}{1-(c+dx)^2}-1\right)d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))-\frac{1}{4}b\left(\text{arctanh}(c+dx)-\frac{1}{3}(c+dx)^3-c-dx\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e^3*(-1/4*(b*(-c - d*x - (c + d*x)^3/3 + ArcTanh[c + d*x])) + ((c + d*x)^
4*(a + b*ArcTanh[c + d*x]))/4))/d
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{e^3 a (dx+c)^4}{4}+e^3 b \left(\frac{(dx+c)^4 \operatorname{arctanh}(dx+c)}{4}+\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}+\frac{\ln(dx+c-1)}{8}-\frac{\ln(dx+c+1)}{8}\right)}{d}$
default	$\frac{\frac{e^3 a (dx+c)^4}{4}+e^3 b \left(\frac{(dx+c)^4 \operatorname{arctanh}(dx+c)}{4}+\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}+\frac{\ln(dx+c-1)}{8}-\frac{\ln(dx+c+1)}{8}\right)}{d}$
parts	$\frac{e^3 a (dx+c)^4}{4d}+\frac{e^3 b \left(\frac{(dx+c)^4 \operatorname{arctanh}(dx+c)}{4}+\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}+\frac{\ln(dx+c-1)}{8}-\frac{\ln(dx+c+1)}{8}\right)}{d}$
parallelrisch	$-\frac{-3b d^5 e^3 \operatorname{arctanh}(dx+c) x^4-3x^4 a d^5 e^3-12bc d^4 e^3 x^3 \operatorname{arctanh}(dx+c)-12x^3 ac d^4 e^3-18x^2 \operatorname{arctanh}(dx+c) b c^2 d^3 e^3-x^3}{}$
risch	$\frac{e^3 (dx+c)^4 b \ln(dx+c+1)}{8d}-\frac{e^3 d^3 b x^4 \ln(-dx-c+1)}{8}-\frac{e^3 d^2 bc x^3 \ln(-dx-c+1)}{2}+\frac{e^3 d^3 a x^4}{4}-\frac{3e^3 db c^2 x^2 \ln(-dx-}{4}$

input
```
int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arctanh(d*x+c)+1/12*(d*x+c)^
3+1/4*d*x+1/4*c+1/8*ln(d*x+c-1)-1/8*ln(d*x+c+1)))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.21

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))\,dx$$
$$=\frac{6\,ad^4e^3x^4+2\,(12\,ac+b)d^3e^3x^3+6\,(6\,ac^2+bc)d^2e^3x^2+6\,(4\,ac^3+bc^2+b)de^3x+3\,(bd^4e^3x^4+4\,bcd^3e^3x^3}{24\,d}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(6*a*d^4*e^3*x^4 + 2*(12*a*c + b)*d^3*e^3*x^3 + 6*(6*a*c^2 + b*c)*d^2
*e^3*x^2 + 6*(4*a*c^3 + b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*
e^3*x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*log(-(d
*x + c + 1)/(d*x + c - 1)))/d
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(61) = 122$.

Time = 0.46 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))\,dx$$
$$=\begin{cases} ac^3e^3x+\frac{3ac^2de^3x^2}{2}+acd^2e^3x^3+\frac{ad^3e^3x^4}{4}+\frac{bc^4e^3\,\text{atanh}\,(c+dx)}{4d}+bc^3e^3x\,\text{atanh}\,(c+dx)+\frac{3bc^2de^3x^2\,\text{atanh}\,(c+dx)}{2}+ \\ c^3e^3x(a+b\,\text{atanh}\,(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a
*d**3*e**3*x**4/4 + b*c**4*e**3*atanh(c + d*x)/(4*d) + b*c**3*e**3*x*atanh
(c + d*x) + 3*b*c**2*d*e**3*x**2*atanh(c + d*x)/2 + b*c**2*e**3*x/4 + b*c*
d**2*e**3*x**3*atanh(c + d*x) + b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atanh
(c + d*x)/4 + b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atanh(c + d*x)/(4*
d), Ne(d, 0)), (c**3*e**3*x*(a + b*atanh(c)), True))
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(64) = 128$.

Time = 0.19 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.96

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))\,dx = \frac{1}{4}\,ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}\,ac^2de^3x^2$$
$$+\frac{3}{4}\left(2\,x^2\,\text{artanh}\,(dx+c)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3}+\frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right)$$
$$+\frac{1}{2}\left(2\,x^3\,\text{artanh}\,(dx+c)+d\left(\frac{dx^2-4\,cx}{d^3}+\frac{(c^3+3\,c^2+3\,c+1)\log{(dx+c+1)}}{d^4}-\frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+\frac{1}{24}\left(6\,x^4\,\text{artanh}\,(dx+c)+d\left(\frac{2\,(d^2x^3-3\,cdx^2+3\,(3\,c^2+1)x)}{d^4}-\frac{3\,(c^4+4\,c^3+6\,c^2+4\,c+1)\log{(dx}}{d^5}\right.\right.$$
$$+ac^3e^3x+\frac{\left(2\,(dx+c)\,\text{artanh}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)bc^3e^3}{2\,d}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arc
tanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 -
 2*c + 1)*log(d*x + c - 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctanh(d*x + c)
 + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 -
 (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*b*c*d^2*e^3 + 1/24*(6*x^4*
arctanh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c
^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^
2 - 4*c + 1)*log(d*x + c - 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x
+ c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*c^3*e^3/d
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 5.04

$$\int (ce + dex)^3(a + b\text{arctanh}(c + dx))\,dx$$

$$= \frac{1}{6}\left((c+1)d - (c-1)d\right)\left(\frac{3\left(\frac{(dx+c+1)^3be^3}{(dx+c-1)^3} + \frac{(dx+c+1)be^3}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4} - \frac{4(dx+c+1)^3d^2}{(dx+c-1)^3} + \frac{6(dx+c+1)^2d^2}{(dx+c-1)^2} - \frac{4(dx+c+1)d^2}{dx+c-1} + d^2} + \frac{\frac{6(dx+c+1)^3ae^3}{(dx+c-1)^3} + 6(d}{\frac{(dx+c+1)}{(dx+c-1}}\right.$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*e^3/(d*x + c - 1)^3 + (d
*x + c + 1)*b*e^3/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x +
 c + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*
(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) +
d^2) + (6*(d*x + c + 1)^3*a*e^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a*e^3/(d
*x + c - 1) + 3*(d*x + c + 1)^3*b*e^3/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*
b*e^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*e^3/(d*x + c - 1) - 2*b*e^3)/((d
*x + c + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3
+ 6*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1
) + d^2))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 414, normalized size of antiderivative = 5.75

$$\int (ce+dex)^3(a+b\operatorname{arctanh}(c+dx))\,dx$$
$$= x^3\left(\frac{d^2\,e^3\,(b+20\,a\,c)}{12}-\frac{2\,a\,c\,d^2\,e^3}{3}\right)$$
$$+\ln(c+d\,x+1)\,\left(\frac{b\,c^3\,e^3\,x}{2}+\frac{3\,b\,c^2\,d\,e^3\,x^2}{4}+\frac{b\,c\,d^2\,e^3\,x^3}{2}+\frac{b\,d^3\,e^3\,x^4}{8}\right)$$
$$-\ln(1-d\,x-c)\,\left(\frac{b\,c^3\,e^3\,x}{2}+\frac{3\,b\,c^2\,d\,e^3\,x^2}{4}+\frac{b\,c\,d^2\,e^3\,x^3}{2}+\frac{b\,d^3\,e^3\,x^4}{8}\right)$$
$$-x^2\left(\frac{c\left(\frac{d^2\,e^3\,(b+20\,a\,c)}{4}-2\,a\,c\,d^2\,e^3\right)}{d}-\frac{d\,e^3\,(10\,a\,c^2+b\,c-a)}{2}+\frac{a\,d\,e^3\,(4\,c^2-4)}{8}\right)$$
$$+x\left(\frac{c\,e^3\,(20\,a\,c^2+3\,b\,c-6\,a)}{2}-\frac{(4\,c^2-4)\,\left(\frac{d^2\,e^3\,(b+20\,a\,c)}{4}-2\,a\,c\,d^2\,e^3\right)}{4\,d^2}\right.$$
$$\left.+\frac{2\,c\left(\frac{2\,c\left(\frac{d^2\,e^3\,(b+20\,a\,c)}{4}-2\,a\,c\,d^2\,e^3\right)}{d}-d\,e^3\,(10\,a\,c^2+b\,c-a)+\frac{a\,d\,e^3\,(4\,c^2-4)}{4}\right)}{d}\right)$$
$$+\frac{\ln(c+d\,x-1)\,\left(b\,e^3-b\,c^4\,e^3\right)}{8\,d}+\frac{a\,d^3\,e^3\,x^4}{4}$$
$$+\frac{b\,e^3\,\ln(c+d\,x+1)\,\left(c^2+1\right)\,(c-1)\,(c+1)}{8\,d}$$

input
```
int((c*e + d*e*x)^3*(a + b*atanh(c + d*x)),x)
```

output
```
x^3*((d^2*e^3*(b + 20*a*c))/12 - (2*a*c*d^2*e^3)/3) + log(c + d*x + 1)*((b
*d^3*e^3*x^4)/8 + (b*c^3*e^3*x)/2 + (3*b*c^2*d*e^3*x^2)/4 + (b*c*d^2*e^3*x
^3)/2) - log(1 - d*x - c)*((b*d^3*e^3*x^4)/8 + (b*c^3*e^3*x)/2 + (3*b*c^2*
d*e^3*x^2)/4 + (b*c*d^2*e^3*x^3)/2) - x^2*((c*((d^2*e^3*(b + 20*a*c))/4 -
2*a*c*d^2*e^3))/d - (d*e^3*(b*c - a + 10*a*c^2))/2 + (a*d*e^3*(4*c^2 - 4))
/8) + x*((c*e^3*(3*b*c - 6*a + 20*a*c^2))/2 - ((4*c^2 - 4)*((d^2*e^3*(b +
20*a*c))/4 - 2*a*c*d^2*e^3))/(4*d^2) + (2*c*((2*c*((d^2*e^3*(b + 20*a*c))/
4 - 2*a*c*d^2*e^3))/d - d*e^3*(b*c - a + 10*a*c^2) + (a*d*e^3*(4*c^2 - 4))
/4))/d) + (log(c + d*x - 1)*(b*e^3 - b*c^4*e^3))/(8*d) + (a*d^3*e^3*x^4)/4
 + (b*e^3*log(c + d*x + 1)*(c^2 + 1)*(c - 1)*(c + 1))/(8*d)
```

3.9. $\int (ce+dex)^3(a+b\text{arctanh}(c+dx))\,dx$

3.10 $\int (ce+dex)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx$

3.10.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (ce+dex)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx=\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3(a+b\,\mathrm{arctanh}(c+dx))}{3d}+\frac{be^2\log\left(1-(c+dx)^2\right)}{6d}$$

output
```
1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctanh(d*x+c))/d+1/6*b*e^2*l
n(1-(d*x+c)^2)/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int (ce+dex)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx=\frac{e^2\left((c+dx)^2(b+2a(c+dx))+2b(c+dx)^3\mathrm{arctanh}(c+dx)+b\log\left(1-(c+dx)^2\right)\right)}{6d}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e^2*((c + d*x)^2*(b + 2*a*(c + d*x)) + 2*b*(c + d*x)^3*ArcTanh[c + d*x] +
 b*Log[1 - (c + d*x)^2]))/(6*d)
```

3.10.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6657, 27, 6452, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))-\frac{1}{3}b\int \frac{(c+dx)^3}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))-\frac{1}{6}b\int \frac{(c+dx)^2}{-c-dx+1}d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))-\frac{1}{6}b\int \left(\frac{1}{-c-dx+1}-1\right)d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))-\frac{1}{6}b(-\log(-c-dx+1)-c-dx))}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTanh[c + d*x]))/3 - (b*(-c - d*x - Log[1 - c
- d*x]))/6))/d
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.10.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{3}+\frac{(dx+c)^2}{6}+\frac{\ln(dx+c-1)}{6}+\frac{\ln(dx+c+1)}{6}\right)}{d}$
default	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{3}+\frac{(dx+c)^2}{6}+\frac{\ln(dx+c-1)}{6}+\frac{\ln(dx+c+1)}{6}\right)}{d}$
parts	$\frac{e^2a(dx+c)^3}{3d}+\frac{e^2b\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{3}+\frac{(dx+c)^2}{6}+\frac{\ln(dx+c-1)}{6}+\frac{\ln(dx+c+1)}{6}\right)}{d}$
risch	$\frac{e^2(dx+c)^3b\ln(dx+c+1)}{6d}-\frac{e^2d^2b\,x^3\ln(-dx-c+1)}{6}-\frac{e^2dbc\,x^2\ln(-dx-c+1)}{2}+\frac{e^2d^2a\,x^3}{3}-\frac{e^2bc^2x\ln(-dx-c+1)}{2}$
parallelrisch	$2b\,d^4e^2\operatorname{arctanh}(dx+c)x^3+2x^3a\,d^4e^2+6x^2\operatorname{arctanh}(dx+c)bc\,d^3e^2+6x^2ac\,d^3e^2+6x\operatorname{arctanh}(dx+c)b\,c^2d^2e^2+x^2b\,d^3e^2+6$

input
```
int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arctanh(d*x+c)+1/6*(d*x+c)^2
+1/6*ln(d*x+c-1)+1/6*ln(d*x+c+1)))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(63) = 126$.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.09

$$\int (ce+dex)^2(a+b\operatorname{arctanh}(c+dx))\,dx$$
$$=\frac{2\,ad^3e^2x^3+(6\,ac+b)d^2e^2x^2+2\,(3\,ac^2+bc)de^2x+(bc^3+b)e^2\log{(dx+c+1)}-(bc^3-b)e^2\log{(dx+c}}{6\,d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*e^2*x^3 + (6*a*c + b)*d^2*e^2*x^2 + 2*(3*a*c^2 + b*c)*d*e^2*x
 + (b*c^3 + b)*e^2*log(d*x + c + 1) - (b*c^3 - b)*e^2*log(d*x + c - 1) + (
b*d^3*e^2*x^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x)*log(-(d*x + c + 1)/(d
*x + c - 1)))/d
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(56) = 112$.

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.61

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))\,dx$$
$$= \begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\,\text{atanh}\,(c+dx)}{3d} + bc^2e^2x\,\text{atanh}\,(c+dx) + bcde^2x^2\,\text{atanh}\,(c+dx) + \frac{bce^2x}{3} + \\ c^2e^2x(a+b\,\text{atanh}\,(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e
**2*atanh(c + d*x)/(3*d) + b*c**2*e**2*x*atanh(c + d*x) + b*c*d*e**2*x**2*
atanh(c + d*x) + b*c*e**2*x/3 + b*d**2*e**2*x**3*atanh(c + d*x)/3 + b*d*e*
*2*x**2/6 + b*e**2*log(c/d + x + 1/d)/(3*d) - b*e**2*atanh(c + d*x)/(3*d),
 Ne(d, 0)), (c**2*e**2*x*(a + b*atanh(c)), True))
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(63) = 126$.

Time = 0.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.26

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))\,dx = \frac{1}{3}ad^2e^2x^3 + acde^2x^2$$
$$+ \frac{1}{2}\left(2x^2\,\text{artanh}\,(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$
$$+ \frac{1}{6}\left(2x^3\,\text{artanh}\,(dx+c) + d\left(\frac{dx^2-4cx}{d^3} + \frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3c^2+3c-1)}{d^4}\right.\right.$$
$$+ ac^2e^2x + \frac{\left(2(dx+c)\,\text{artanh}\,(dx+c) + \log\left(-(dx+c)^2+1\right)\right)bc^2e^2}{2d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d
^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c -
1)/d^3))*b*c*d*e^2 + 1/6*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3
+ (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*l
og(d*x + c - 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctanh(d
*x + c) + log(-(d*x + c)^2 + 1))*b*c^2*e^2/d
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 4.67

$$\int (ce+dex)^2(a+b\operatorname{arctanh}(c+dx))\,dx =$$

$$-\frac{1}{6}((c+1)d-(c-1)d)\left(\frac{be^2\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2}-\frac{be^2\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2}-\frac{\left(\frac{3(dx+c+1)^2be^2}{(dx+c-1)^2}+be^2\right)\log\left(-\right.}{\frac{(dx+c+1)^3d^2}{(dx+c-1)^3}-\frac{3(dx+c+1)^2d^2}{(dx+c-1)^2}+\frac{3(d}{d}}\right.$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*((c + 1)*d - (c - 1)*d)*(b*e^2*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/
d^2 - b*e^2*log(-(d*x + c + 1)/(d*x + c - 1))/d^2 - (3*(d*x + c + 1)^2*b*e
^2/(d*x + c - 1)^2 + b*e^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c +
1)^3*d^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 + 3*(d*x
+ c + 1)*d^2/(d*x + c - 1) - d^2) - 2*(3*(d*x + c + 1)^2*a*e^2/(d*x + c -
1)^2 + a*e^2 + (d*x + c + 1)^2*b*e^2/(d*x + c - 1)^2 - (d*x + c + 1)*b*e^2
/(d*x + c - 1))/((d*x + c + 1)^3*d^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d
^2/(d*x + c - 1)^2 + 3*(d*x + c + 1)*d^2/(d*x + c - 1) - d^2))
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.43

$$\int (ce+dex)^2(a+b\operatorname{arctanh}(c+dx))\,dx = \frac{a\,d^2\,e^2\,x^3}{3}+\frac{b\,c\,e^2\,x}{3}+\frac{b\,e^2\,\ln\left(c+d\,x-1\right)}{6\,d}+\frac{b\,e^2\,\ln\left(c+d\,x+1\right)}{6\,d}+a\,c^2\,e^2\,x+\frac{b\,d\,e^2\,x^2}{6}+a\,c\,d\,e^2\,x^2+\frac{b\,c^2\,e^2\,x\,\ln\left(c+d\,x+1\right)}{2}-\frac{b\,c^3\,e^2\,\ln\left(c+d\,x-1\right)}{6\,d}+\frac{b\,c^3\,e^2\,\ln\left(c+d\,x+1\right)}{6\,d}-\frac{b\,c^2\,e^2\,x\,\ln\left(1-d\,x-c\right)}{2}+\frac{b\,d^2\,e^2\,x^3\,\ln\left(c+d\,x+1\right)}{6}-\frac{b\,d^2\,e^2\,x^3\,\ln\left(1-d\,x-c\right)}{6}+\frac{b\,c\,d\,e^2\,x^2\,\ln\left(c+d\,x+1\right)}{2}-\frac{b\,c\,d\,e^2\,x^2\,\ln\left(1-d\,x-c\right)}{2}$$

input
```
int((c*e + d*e*x)^2*(a + b*atanh(c + d*x)),x)
```

output
```
(a*d^2*e^2*x^3)/3 + (b*c*e^2*x)/3 + (b*e^2*log(c + d*x - 1))/(6*d) + (b*e^
2*log(c + d*x + 1))/(6*d) + a*c^2*e^2*x + (b*d*e^2*x^2)/6 + a*c*d*e^2*x^2
+ (b*c^2*e^2*x*log(c + d*x + 1))/2 - (b*c^3*e^2*log(c + d*x - 1))/(6*d) +
(b*c^3*e^2*log(c + d*x + 1))/(6*d) - (b*c^2*e^2*x*log(1 - d*x - c))/2 + (b
*d^2*e^2*x^3*log(c + d*x + 1))/6 - (b*d^2*e^2*x^3*log(1 - d*x - c))/6 + (b
*c*d*e^2*x^2*log(c + d*x + 1))/2 - (b*c*d*e^2*x^2*log(1 - d*x - c))/2
```

3.11 $\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx=\frac{bex}{2}-\frac{b\text{earctanh}(c+dx)}{2d}+\frac{e(c+dx)^2(a+b\text{arctanh}(c+dx))}{2d}$$

output
```
1/2*b*e*x-1/2*b*e*arctanh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx=\frac{e(2bc+2ac^2+2bdx+4acdx+2ad^2x^2+2b(c+dx)^2\text{arctanh}(c+dx)+b\log(1-c-dx)-b\log(1+c+$$

$$4d$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e*(2*b*c + 2*a*c^2 + 2*b*d*x + 4*a*c*d*x + 2*a*d^2*x^2 + 2*b*(c + d*x)^2*
ArcTanh[c + d*x] + b*Log[1 - c - d*x] - b*Log[1 + c + d*x]))/(4*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6657, 27, 6452, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int e(c+dx)(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\frac{1}{2}b\int \frac{(c+dx)^2}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\frac{1}{2}b\left(\int \frac{1}{1-(c+dx)^2}d(c+dx)-c-dx\right)\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{e(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\frac{1}{2}b(\text{arctanh}(c+dx)-c-dx))}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(e*(-1/2*(b*(-c - d*x + ArcTanh[c + d*x])) + ((c + d*x)^2*(a + b*ArcTanh[c
 + d*x]))/2))/d
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}+\frac{\ln(dx+c-1)}{4}-\frac{\ln(dx+c+1)}{4}\right)}{d}$
default	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}+\frac{\ln(dx+c-1)}{4}-\frac{\ln(dx+c+1)}{4}\right)}{d}$
parts	$ea\left(\frac{1}{2}d\,x^2+cx\right)+\frac{be\left(\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}+\frac{\ln(dx+c-1)}{4}-\frac{\ln(dx+c+1)}{4}\right)}{d}$
parallelrisch	$-\frac{-x^2\operatorname{arctanh}(dx+c)b\,d^3e-x^2a\,d^3e-2x\,\operatorname{arctanh}(dx+c)bc\,d^2e-2xac\,d^2e-\operatorname{arctanh}(dx+c)b\,c^2de-xb\,d^2e+5a\,c^2de+\operatorname{arct}}{2d^2}$
risch	$\frac{ebx(dx+2c)\ln(dx+c+1)}{4}-\frac{edb\,x^2\ln(-dx-c+1)}{4}-\frac{ebcx\ln(-dx-c+1)}{2}+\frac{eda\,x^2}{2}+\frac{e\ln(-dx-c-1)b\,c^2}{4d}-\frac{e\ln(d}{}$

input
```
int((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a*(d*x+c)^2+b*e*(1/2*(d*x+c)^2*arctanh(d*x+c)+1/2*d*x+1/2*c+1/4
*ln(d*x+c-1)-1/4*ln(d*x+c+1)))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int(ce+dex)(a+b\operatorname{arctanh}(c+dx))\,dx = \frac{2\,ad^2ex^2+2\,(2\,ac+b)dex+(bd^2ex^2+2\,bcdex+(bc^2-b)e)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{4\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d^2*e*x^2 + 2*(2*a*c + b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b
*c^2 - b)*e)*log(-(d*x + c + 1)/(d*x + c - 1)))/d
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx$$
$$=\begin{cases} acex+\frac{adex^2}{2}+\frac{bc^2e\,\text{atanh}\,(c+dx)}{2d}+bcex\,\text{atanh}\,(c+dx)+\frac{bdex^2\,\text{atanh}\,(c+dx)}{2}+\frac{bex}{2}-\frac{be\,\text{atanh}\,(c+dx)}{2d} & \text{for } d\neq 0 \\ cex(a+b\,\text{atanh}\,(c)) & \text{otherwise} \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atanh(c + d*x)/(2*d) + b*c*e*
x*atanh(c + d*x) + b*d*e*x**2*atanh(c + d*x)/2 + b*e*x/2 - b*e*atanh(c + d
*x)/(2*d), Ne(d, 0)), (c*e*x*(a + b*atanh(c)), True))
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.35

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))\,dx=\frac{1}{2}\,adex^2$$
$$+\frac{1}{4}\left(2\,x^2\,\text{artanh}\,(dx+c)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3}+\frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right)$$
$$+acex+\frac{(2\,(dx+c)\,\text{artanh}\,(dx+c)+\log{(-(dx+c)^2+1)})bce}{2\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*d*e*x^2 + 1/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)
*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*d*e + a*c
*e*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*c*e/d
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.75

$$\int (ce+dex)(a+b\operatorname{arctanh}(c+dx))\,dx = \frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{(dx+c+1)be\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\left(\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2(dx+c+1)d^2}{dx+c-1}+d^2\right)(dx+c-1)}+\frac{\frac{2(dx+c+1)ae}{dx+c-1}+\frac{(dx+c+1)be}{dx+c-1}-be}{\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2(dx+c+1)d^2}{dx+c-1}+d^2}\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*((d*x + c + 1)*b*e*log(-(d*x + c + 1)/(d*x + c
 - 1))/(((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x +
c - 1) + d^2)*(d*x + c - 1)) + (2*(d*x + c + 1)*a*e/(d*x + c - 1) + (d*x +
 c + 1)*b*e/(d*x + c - 1) - b*e)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*
(d*x + c + 1)*d^2/(d*x + c - 1) + d^2))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int (ce+dex)(a+b\operatorname{arctanh}(c+dx))\,dx = \frac{b\,e\,x}{2}+a\,c\,e\,x-\frac{b\,e\,\mathrm{atanh}(c+d\,x)}{2\,d}+\frac{a\,d\,e\,x^2}{2}+\frac{b\,c^2\,e\,\mathrm{atanh}(c+d\,x)}{2\,d}+b\,c\,e\,x\,\mathrm{atanh}(c+d\,x)+\frac{b\,d\,e\,x^2\,\mathrm{atanh}(c+d\,x)}{2}$$

input
```
int((c*e + d*e*x)*(a + b*atanh(c + d*x)),x)
```

output
```
(b*e*x)/2 + a*c*e*x - (b*e*atanh(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*
e*atanh(c + d*x))/(2*d) + b*c*e*x*atanh(c + d*x) + (b*d*e*x^2*atanh(c + d*
x))/2
```

3.12 $\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx$

3.12.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx=\frac{a\log(c+dx)}{de}-\frac{b\,\text{PolyLog}(2,-c-dx)}{2de}+\frac{b\,\text{PolyLog}(2,c+dx)}{2de}$$

output
```
a*ln(d*x+c)/d/e-1/2*b*polylog(2,-d*x-c)/d/e+1/2*b*polylog(2,d*x+c)/d/e
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx=\frac{a\log(c+dx)}{de}-\frac{b\,\text{PolyLog}(2,-c-dx)}{2de}+\frac{b\,\text{PolyLog}(2,c+dx)}{2de}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(a*Log[c + d*x])/(d*e) - (b*PolyLog[2, -c - d*x])/(2*d*e) + (b*PolyLog[2,
c + d*x])/(2*d*e)
```

3.12.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6657, 27, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6446}$$

$$\frac{a\log(c+dx)-\frac{1}{2}b\,\text{PolyLog}(2,-c-dx)+\frac{1}{2}b\,\text{PolyLog}(2,c+dx)}{de}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x),x]
```

output
```
(a*Log[c + d*x] - (b*PolyLog[2, -c - d*x])/2 + (b*PolyLog[2, c + d*x])/2)/
(d*e)
```

3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{a\ln(-dx-c)}{de}+\frac{b\,\mathrm{dilog}(-dx-c+1)}{2de}-\frac{b\,\mathrm{dilog}(dx+c+1)}{2ed}$	54
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(\ln(dx+c)\,\mathrm{arctanh}(dx+c)-\frac{\mathrm{dilog}(dx+c)}{2}-\frac{\mathrm{dilog}(dx+c+1)}{2}-\frac{\ln(dx+c)\ln(dx+c+1)}{2}\right)}{e}}{d}$	68
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(\ln(dx+c)\,\mathrm{arctanh}(dx+c)-\frac{\mathrm{dilog}(dx+c)}{2}-\frac{\mathrm{dilog}(dx+c+1)}{2}-\frac{\ln(dx+c)\ln(dx+c+1)}{2}\right)}{e}}{d}$	68
parts	$\frac{a\ln(dx+c)}{de}+\frac{b\left(\ln(dx+c)\,\mathrm{arctanh}(dx+c)-\frac{\mathrm{dilog}(dx+c)}{2}-\frac{\mathrm{dilog}(dx+c+1)}{2}-\frac{\ln(dx+c)\ln(dx+c+1)}{2}\right)}{ed}$	70

input
```
int((a+b*arctanh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d/e*a*ln(-d*x-c)+1/2/d/e*b*dilog(-d*x-c+1)-1/2*b/e/d*dilog(d*x+c+1)
```

3.12.5 Fricas [F]

$$\int \frac{a+b\mathrm{arctanh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\mathrm{artanh}\,(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b*arctanh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.12.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx=\frac{\int \frac{a}{c+dx}\,dx+\int \frac{b\,\text{atanh}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*atanh(d*x+c))/(d*e*x+c*e),x)
```

output
```
(Integral(a/(c + d*x), x) + Integral(b*atanh(c + d*x)/(c + d*x), x))/e
```

3.12.7 Maxima [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{artanh}\,(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(d*x + c + 1) - log(-d*x - c + 1))/(d*e*x + c*e), x) +
 a*log(d*e*x + c*e)/(d*e)
```

3.12.8 Giac [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{ce+dex}\,dx=\int \frac{b\,\text{artanh}\,(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)/(d*e*x + c*e), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(c + dx)}{ce + dex}\, dx = \int \frac{a + b\,\text{atanh}(c + d\,x)}{c\,e + d\,e\,x}\, dx$$

input
```
int((a + b*atanh(c + d*x))/(c*e + d*e*x),x)
```

output
```
int((a + b*atanh(c + d*x))/(c*e + d*e*x), x)
```

3.13 $\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^2} dx$

3.13.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^2} dx = -\frac{a+b\text{arctanh}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1-(c+dx)^2\right)}{2de^2}$$

output
```
(-a-b*arctanh(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1-(d*x+c)^2
)/d/e^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^2} dx = -\frac{\frac{2a}{c+dx}+\frac{2b\text{arctanh}(c+dx)}{c+dx}-2b\log(c+dx)+b\log\left(1-c^2-2cdx-d^2x^2\right)}{2de^2}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
-1/2*((2*a)/(c + d*x) + (2*b*ArcTanh[c + d*x])/(c + d*x) - 2*b*Log[c + d*x
] + b*Log[1 - c^2 - 2*c*d*x - d^2*x^2])/(d*e^2)
```

3.13.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6657, 27, 6452, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{ 6657}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{b\int \frac{1}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{2}b\int \frac{1}{(-c-dx+1)(c+dx)^2}d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\int \frac{1}{(c+dx)^2}d(c+dx)^2\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\log\left((c+dx)^2\right)\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log(-c-dx+1)\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}}{de^2}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTanh[c + d*x])/(c + d*x)) + (b*(-Log[1 - c - d*x] + Log[(c +
d*x)^2]))/2)/(d*e^2)
```

3.13.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{dx+c}-\frac{\ln(dx+c-1)}{2}+\ln(dx+c)-\frac{\ln(dx+c+1)}{2}\right)}{e^2}}{d}$
default	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{dx+c}-\frac{\ln(dx+c-1)}{2}+\ln(dx+c)-\frac{\ln(dx+c+1)}{2}\right)}{e^2}}{d}$
parts	$-\frac{a}{d\,e^2(dx+c)}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{dx+c}-\frac{\ln(dx+c-1)}{2}+\ln(dx+c)-\frac{\ln(dx+c+1)}{2}\right)}{e^2 d}$
parallelrisch	$\frac{3\ln(dx+c)xbc\,d^2-3\ln(dx+c-1)xbc\,d^2-3x\ \operatorname{arctanh}(dx+c)bc\,d^2+3\ln(dx+c)b\,c^2d-3\ln(dx+c-1)b\,c^2d-3\ \operatorname{arctanh}(dx+c}{3(dx+c)c\,d^2e^2}$
risch	$-\frac{b\ln(dx+c+1)}{2d\,e^2(dx+c)}+\frac{2\ln(-dx-c)bdx-\ln\left(d^2x^2+2cdx+c^2-1\right)bdx+2\ln(-dx-c)bc-\ln\left(d^2x^2+2cdx+c^2-1\right)bc+b\ln(-dx-}{2e^2(dx+c)d}$

input

```
int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arctanh(d*x+c)-1/2*ln(d*x+c-1)+ln(d*
x+c)-1/2*ln(d*x+c+1)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(ce+dex)^2}\,dx = -\frac{(bdx+bc)\log\left(d^2x^2+2\,cdx+c^2-1\right)-2\,(bdx+bc)\log\left(dx+c\right)+b\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,a}{2\left(d^2e^2x+cde^2\right)}$$

input

```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output

```
-1/2*((b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 - 1) - 2*(b*d*x + b*c)*log
(d*x + c) + b*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a)/(d^2*e^2*x + c*d*e^
2)
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(53) = 106$.

Time = 0.61 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.48

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^2} dx$$
$$= \begin{cases} -\frac{a}{cde^2+d^2e^2x} + \frac{bc\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x} - \frac{bc\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{cde^2+d^2e^2x} + \frac{bc\,\text{atanh}\,(c+dx)}{cde^2+d^2e^2x} + \frac{bdx\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x} - \frac{bdx\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{cde^2+d^2e^2x} + \frac{bdx\,\text{atanh}\,(c+dx)}{cde^2+d^2e^2x} - \\ \frac{x(a+b\,\text{atanh}\,(c))}{c^2e^2} \end{cases}$$

input
```
integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**2,x)
```

output
```
Piecewise((-a/(c*d*e**2 + d**2*e**2*x) + b*c*log(c/d + x)/(c*d*e**2 + d**2
*e**2*x) - b*c*log(c/d + x + 1/d)/(c*d*e**2 + d**2*e**2*x) + b*c*atanh(c +
 d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/d + x)/(c*d*e**2 + d**2*e**2*
x) - b*d*x*log(c/d + x + 1/d)/(c*d*e**2 + d**2*e**2*x) + b*d*x*atanh(c + d
*x)/(c*d*e**2 + d**2*e**2*x) - b*atanh(c + d*x)/(c*d*e**2 + d**2*e**2*x),
Ne(d, 0)), (x*(a + b*atanh(c))/(c**2*e**2), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^2} dx$$
$$= -\frac{1}{2}\left(d\left(\frac{\log(dx + c + 1)}{d^2e^2} - \frac{2\log(dx + c)}{d^2e^2} + \frac{\log(dx + c - 1)}{d^2e^2}\right) + \frac{2\,\text{artanh}\,(dx + c)}{d^2e^2x + cde^2}\right)b - \frac{a}{d^2e^2x + cde^2}$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-1/2*(d*(log(d*x + c + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2) + log(d*x +
 c - 1)/(d^2*e^2)) + 2*arctanh(d*x + c)/(d^2*e^2*x + c*d*e^2))*b - a/(d^2*
e^2*x + c*d*e^2)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^2}\, dx$$
$$= \frac{1}{2}\left((c+1)d - (c-1)d\right)\left(\frac{b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)d^2e^2}{dx+c-1} + d^2e^2} + \frac{2\,a}{\frac{(dx+c+1)d^2e^2}{dx+c-1} + d^2e^2} + \frac{b\log\left(-\frac{dx+c+1}{dx+c-1} - 1\right)}{d^2e^2} - \frac{b\log\left(-\frac{dx+c+}{dx+c-}\right.}{d^2e^2}\right.$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c
 + 1)*d^2*e^2/(d*x + c - 1) + d^2*e^2) + 2*a/((d*x + c + 1)*d^2*e^2/(d*x +
 c - 1) + d^2*e^2) + b*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d^2*e^2) - b
*log(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^2))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^2}\, dx = \frac{b\,\ln\left(1 - d\,x - c\right)}{2\,x\,d^2\,e^2 + 2\,c\,d\,e^2} - \frac{b\,\ln\left(c + d\,x + 1\right)}{2\,\left(x\,d^2\,e^2 + c\,d\,e^2\right)} - \frac{a}{x\,d^2\,e^2 + c\,d\,e^2} - \frac{b\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 - 1\right)}{2\,d\,e^2} + \frac{b\,\ln\left(c + d\,x\right)}{d\,e^2}$$

input
```
int((a + b*atanh(c + d*x))/(c*e + d*e*x)^2,x)
```

output
```
(b*log(1 - d*x - c))/(2*d^2*e^2*x + 2*c*d*e^2) - (b*log(c + d*x + 1))/(2*(
d^2*e^2*x + c*d*e^2)) - a/(d^2*e^2*x + c*d*e^2) - (b*log(c^2 + d^2*x^2 + 2
*c*d*x - 1))/(2*d*e^2) + (b*log(c + d*x))/(d*e^2)
```

3.14 $\int \frac{a+b\mathbf{arctanh}(c+dx)}{(ce+dex)^3}\,dx$

3.14.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^3}\,dx=-\frac{b}{2de^3(c+dx)}+\frac{b\text{arctanh}(c+dx)}{2de^3}-\frac{a+b\text{arctanh}(c+dx)}{2de^3(c+dx)^2}$$

output
```
-1/2*b/d/e^3/(d*x+c)+1/2*b*arctanh(d*x+c)/d/e^3+1/2*(-a-b*arctanh(d*x+c))/
d/e^3/(d*x+c)^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.59

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^3}\,dx=-\frac{a}{2de^3(c+dx)^2}-\frac{b}{2de^3(c+dx)}-\frac{b\text{arctanh}(c+dx)}{2de^3(c+dx)^2}-\frac{b\log(1-c-dx)}{4de^3}+\frac{b\log(1+c+dx)}{4de^3}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
-1/2*a/(d*e^3*(c + d*x)^2) - b/(2*d*e^3*(c + d*x)) - (b*ArcTanh[c + d*x])/
(2*d*e^3*(c + d*x)^2) - (b*Log[1 - c - d*x])/(4*d*e^3) + (b*Log[1 + c + d*
x])/(4*d*e^3)
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6657, 27, 6452, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^3}\,dx$$

$$\Big\downarrow \text{ 6657}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{\frac{1}{2}b\int \frac{1}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{1}{2}b\left(\int \frac{1}{1-(c+dx)^2}d(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}b\left(\text{arctanh}(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
((b*(-(c + d*x)^(-1) + ArcTanh[c + d*x]))/2 - (a + b*ArcTanh[c + d*x])/(2*
(c + d*x)^2))/(d*e^3)
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)^2}-\frac{\ln(dx+c-1)}{4}+\frac{\ln(dx+c+1)}{4}-\frac{1}{2(dx+c)}\right)}{e^3}}{d}$
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)^2}-\frac{\ln(dx+c-1)}{4}+\frac{\ln(dx+c+1)}{4}-\frac{1}{2(dx+c)}\right)}{e^3}}{d}$
parts	$-\frac{a}{2e^3(dx+c)^2 d}+\frac{b\left(-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)^2}-\frac{\ln(dx+c-1)}{4}+\frac{\ln(dx+c+1)}{4}-\frac{1}{2(dx+c)}\right)}{e^3 d}$
parallelrisch	$\frac{4x^2\operatorname{arctanh}(dx+c)bc\,d^4+8x\operatorname{arctanh}(dx+c)b\,c^2d^3+b\,d^4x^2+4\operatorname{arctanh}(dx+c)b\,c^3d^2-2xbc\,d^3-4\operatorname{arctanh}(dx+c)bc\,d^2-3}{8(dx+c)^2e^3c\,d^3}$
risch	$-\frac{b\ln(dx+c+1)}{4d\,e^3(dx+c)^2}+\frac{\ln(-dx-c-1)b\,d^2x^2-b\,d^2x^2\ln(-dx-c+1)+2\ln(-dx-c-1)bcdx-2bcdx\ln(-dx-c+1)+\ln(-dx-}{4e^3(dx+c)^2d}$

input
```
int((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arctanh(d*x+c)-1/4*ln(d*x+
c-1)+1/4*ln(d*x+c+1)-1/2/(d*x+c)))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(ce+dex)^3}\,dx = -\frac{2\,bdx+2\,bc-(bd^2x^2+2\,bcdx+bc^2-b)\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,a}{4\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*d*x + 2*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - b)*log(-(d*x + c
+ 1)/(d*x + c - 1)) + 2*a)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(54) = 108$.

Time = 0.91 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.97

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(ce+dex)^3}\,dx$$
$$= \begin{cases} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}+\frac{bc^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}+\frac{2bcdx\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}+\frac{bd^2x^2\operatorname{ata}}{2c^2de^3+4cd^2} \\ \frac{x(a+b\operatorname{atanh}(c))}{c^3e^3} \end{cases}$$

input
```
integrate((a+b*atanh(d*x+c))/(d*e*x+c*e)**3,x)
```

output
```
Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*c**
2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*
b*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2
) - b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b*d**2*x**2
*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d
*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atanh(c + d*x)
/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a +
b*atanh(c))/(c**3*e**3), True))
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(57) = 114$.

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.08

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^3}\,dx = -\frac{1}{4}\left(d\left(\frac{2}{d^3e^3x + cd^2e^3} - \frac{\log(dx + c + 1)}{d^2e^3} + \frac{\log(dx + c - 1)}{d^2e^3}\right) + \frac{2\,\text{artanh}(dx + c)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right)b - \frac{a}{2\,(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3)}$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/4*(d*(2/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c + 1)/(d^2*e^3) + log(d*x
+ c - 1)/(d^2*e^3)) + 2*arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^
2*d*e^3))*b - 1/2*a/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.08

$$\int \frac{a + b\text{arctanh}(c + dx)}{(ce + dex)^3}\,dx = \frac{1}{2}\left((c+1)d - (c-1)d\right)\left(\frac{(dx + c + 1)b\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\left(\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2} + \frac{2\,(dx+c+1)d^2e^3}{dx+c-1} + d^2e^3\right)(dx + c - 1)} + \frac{\frac{2\,(dx+c+1)a}{dx+c-1} + \frac{(dx+c+1)b}{dx+c-1}}{\frac{(dx+c+1)^2d^2e^3}{(dx+c-1)^2} + \frac{2\,(dx+c+1)d^2e^{}}{dx+c-1}}\right.$$

input
```
integrate((a+b*arctanh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*((d*x + c + 1)*b*log(-(d*x + c + 1)/(d*x + c -
 1))/((((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(
d*x + c - 1) + d^2*e^3)*(d*x + c - 1)) + (2*(d*x + c + 1)*a/(d*x + c - 1)
+ (d*x + c + 1)*b/(d*x + c - 1) + b)/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1
)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^3))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+b\text{arctanh}(c+dx)}{(ce+dex)^3}\,dx = \frac{b\,\text{atanh}(c+d\,x)}{2\,d\,e^3} - \frac{\frac{a}{2}+\frac{b\,c}{2}+\frac{b\,\ln(c+d\,x+1)}{4}-\frac{b\,\ln(1-d\,x-c)}{4}+\frac{b\,d\,x}{2}}{d\,e^3\,(c+d\,x)^2}$$

input
```
int((a + b*atanh(c + d*x))/(c*e + d*e*x)^3,x)
```

output
```
(b*atanh(c + d*x))/(2*d*e^3) - (a/2 + (b*c)/2 + (b*log(c + d*x + 1))/4 - (
b*log(1 - d*x - c))/4 + (b*d*x)/2)/(d*e^3*(c + d*x)^2)
```

3.15 $\int (ce+dex)^3(a+b\,\mathrm{arctanh}(c+dx))^2\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\int (ce+dex)^3(a+b\,\mathrm{arctanh}(c+dx))^2\,dx = \frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\mathrm{arctanh}(c+dx)}{2d} + \frac{be^3(c+dx)^3(a+b\,\mathrm{arctanh}(c+dx))}{6d} - \frac{e^3(a+b\,\mathrm{arctanh}(c+dx))^2}{4d} + \frac{e^3(c+dx)^4(a+b\,\mathrm{arctanh}(c+dx))^2}{4d} + \frac{b^2e^3\log\left(1-(c+dx)^2\right)}{3d}$$

output
```
1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctanh(d*x+c)/
d+1/6*b*e^3*(d*x+c)^3*(a+b*arctanh(d*x+c))/d-1/4*e^3*(a+b*arctanh(d*x+c))^
2/d+1/4*e^3*(d*x+c)^4*(a+b*arctanh(d*x+c))^2/d+1/3*b^2*e^3*ln(1-(d*x+c)^2)
/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))^2\,dx$$
$$=\frac{e^3(6ab(c+dx)+b^2(c+dx)^2+2ab(c+dx)^3+3a^2(c+dx)^4+2b(c+dx)\left(3b+b(c+dx)^2+3a(c+dx)^3\right)$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(e^3*(6*a*b*(c + d*x) + b^2*(c + d*x)^2 + 2*a*b*(c + d*x)^3 + 3*a^2*(c + d
*x)^4 + 2*b*(c + d*x)*(3*b + b*(c + d*x)^2 + 3*a*(c + d*x)^3)*ArcTanh[c +
d*x] + 3*b^2*(-1 + (c + d*x)^4)*ArcTanh[c + d*x]^2 + b*(3*a + 4*b)*Log[1 -
 c - d*x] + b*(-3*a + 4*b)*Log[1 + c + d*x]))/(12*d)
```

3.15.3 Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6657, 27, 6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int e^3(c+dx)^3(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^3\int (c+dx)^3(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\int \frac{(c+dx)^4(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6542}$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)-\int (c+dx)^2(a+b\text{arctanh}(c+\right.\right.}{d}$$

$\downarrow$ 6452

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{1}{3}b\int \frac{(c+dx)^3}{1-(c+dx)^2}d(c+dx)-\frac{1}{3}(\right.\right.}{d}$$

$\downarrow$ 243

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{1}{6}b\int \frac{(c+dx)^2}{-c-dx+1}d(c+dx)^2-\frac{1}{3}(\right.\right.}{d}$$

$\downarrow$ 49

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{1}{6}b\int \left(\frac{1}{-c-dx+1}-1\right)d(c+dx\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)-\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+\right.\right.}{d}$$

$\downarrow$ 6542

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(-\int (a+b\text{arctanh}(c+dx))d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))-\right.\right.}{d}$$

$\downarrow$ 6510

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\text{arctanh}(c+dx))^2-\frac{1}{2}b\left(-\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-a(c+d\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

3.15. $\int (ce+dex)^3(a+b\text{arctanh}(c+dx))^2\,dx$

output
```
(e^3*(((c + d*x)^4*(a + b*ArcTanh[c + d*x])^2)/4 - (b*(-(a*(c + d*x)) - b*
(c + d*x)*ArcTanh[c + d*x] - ((c + d*x)^3*(a + b*ArcTanh[c + d*x]))/3 + (a
 + b*ArcTanh[c + d*x])^2/(2*b) + (b*(-c - d*x - Log[1 - c - d*x]))/6 - (b*
Log[1 - (c + d*x)^2])/2))/2))/d
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.15.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.64

method	result
derivativedivides	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arctanh}(dx+c)^2}{4}+\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{6}+\frac{(dx+c)\operatorname{arctanh}(dx+c)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}-\ldots\right.}{\ldots}$
default	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\operatorname{arctanh}(dx+c)^2}{4}+\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{6}+\frac{(dx+c)\operatorname{arctanh}(dx+c)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}-\ldots\right.}{\ldots}$
parts	$\frac{e^3a^2(dx+c)^4}{4d}+\frac{e^3b^2\left(\frac{(dx+c)^4\operatorname{arctanh}(dx+c)^2}{4}+\frac{(dx+c)^3\operatorname{arctanh}(dx+c)}{6}+\frac{(dx+c)\operatorname{arctanh}(dx+c)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}\ldots\right.}{\ldots}$
parallelrisch	$18d\,e^3c^2a^2-5d\,e^3c^2b^2+e^3b^2d-18ab\,c^3d\,e^3-6abcd\,e^3+12x^3a^2c\,d^4e^3+2x^3ab\,d^4e^3+6x\operatorname{arctanh}(dx+c)b^2d^2e^3+12x\,a^2c^3d\ldots$
risch	$-e^3d^2abc\,x^3\ln(-dx-c+1)-\frac{3e^3dab\,c^2x^2\ln(-dx-c+1)}{2}+\frac{ab\,e^3x}{2}+e^3d^2a^2c\,x^3+\frac{3e^3d\,a^2c^2x^2}{2}+\ldots$

input
```
int((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arctanh(d*x+c)^2+1/6*(d*
x+c)^3*arctanh(d*x+c)+1/2*(d*x+c)*arctanh(d*x+c)+1/4*arctanh(d*x+c)*ln(d*x
+c-1)-1/4*arctanh(d*x+c)*ln(d*x+c+1)-1/8*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)
+1/16*ln(d*x+c-1)^2+1/16*ln(d*x+c+1)^2-1/8*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1
/2))*ln(-1/2*d*x-1/2*c+1/2)+1/12*(d*x+c)^2+1/3*ln(d*x+c-1)+1/3*ln(d*x+c+1)
)+2*e^3*a*b*(1/4*(d*x+c)^4*arctanh(d*x+c)+1/12*(d*x+c)^3+1/4*d*x+1/4*c+1/8
*ln(d*x+c-1)-1/8*ln(d*x+c+1)))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.41

$$\int (ce + dex)^3(a + b\text{arctanh}(c + dx))^2\, dx$$
$$= \frac{12\,a^2d^4e^3x^4 + 8\,(6\,a^2c + ab)d^3e^3x^3 + 4\,(18\,a^2c^2 + 6\,abc + b^2)d^2e^3x^2 + 8\,(6\,a^2c^3 + 3\,abc^2 + b^2c + 3\,ab)de^3x}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(12*a^2*d^4*e^3*x^4 + 8*(6*a^2*c + a*b)*d^3*e^3*x^3 + 4*(18*a^2*c^2 +
 6*a*b*c + b^2)*d^2*e^3*x^2 + 8*(6*a^2*c^3 + 3*a*b*c^2 + b^2*c + 3*a*b)*d*
e^3*x + 4*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b + 4*b^2)*e^3*log(d*x + c
+ 1) - 4*(3*a*b*c^4 + b^2*c^3 + 3*b^2*c - 3*a*b - 4*b^2)*e^3*log(d*x + c -
 1) + 3*(b^2*d^4*e^3*x^4 + 4*b^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4
*b^2*c^3*d*e^3*x + (b^2*c^4 - b^2)*e^3)*log(-(d*x + c + 1)/(d*x + c - 1))^
2 + 4*(3*a*b*d^4*e^3*x^4 + (12*a*b*c + b^2)*d^3*e^3*x^3 + 3*(6*a*b*c^2 + b
^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 + b^2*c^2 + b^2)*d*e^3*x)*log(-(d*x + c +
 1)/(d*x + c - 1)))/d
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(138) = 276$.

Time = 0.68 (sec) , antiderivative size = 581, normalized size of antiderivative = 3.65

$$\int (ce + dex)^3(a + b\text{arctanh}(c + dx))^2\, dx$$
$$= \begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{atanh}(c+dx)}{2d} + 2abc^3e^3x\operatorname{atanh}(c+dx) + 3abc^2de^3x^2\operatorname{a} \\ c^3e^3x(a + b\operatorname{atanh}(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*atanh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3
*x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atanh(c + d*x)/(2*d) + 2*a*b
*c**3*e**3*x*atanh(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atanh(c + d*x) + a*b*
c**2*e**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atanh(c + d*x) + a*b*c*d*e**3*x**2/
2 + a*b*d**3*e**3*x**4*atanh(c + d*x)/2 + a*b*d**2*e**3*x**3/6 + a*b*e**3*
x/2 - a*b*e**3*atanh(c + d*x)/(2*d) + b**2*c**4*e**3*atanh(c + d*x)**2/(4*
d) + b**2*c**3*e**3*x*atanh(c + d*x)**2 + b**2*c**3*e**3*atanh(c + d*x)/(6
*d) + 3*b**2*c**2*d*e**3*x**2*atanh(c + d*x)**2/2 + b**2*c**2*e**3*x*atanh
(c + d*x)/2 + b**2*c*d**2*e**3*x**3*atanh(c + d*x)**2 + b**2*c*d*e**3*x**2
*atanh(c + d*x)/2 + b**2*c*e**3*x/6 + b**2*c*e**3*atanh(c + d*x)/(2*d) + b
**2*d**3*e**3*x**4*atanh(c + d*x)**2/4 + b**2*d**2*e**3*x**3*atanh(c + d*x
)/6 + b**2*d*e**3*x**2/12 + b**2*e**3*x*atanh(c + d*x)/2 + 2*b**2*e**3*log
(c/d + x + 1/d)/(3*d) - b**2*e**3*atanh(c + d*x)**2/(4*d) - 2*b**2*e**3*at
anh(c + d*x)/(3*d), Ne(d, 0)), (c**3*e**3*x*(a + b*atanh(c))**2, True))
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(145) = 290$.

Time = 0.39 (sec) , antiderivative size = 827, normalized size of antiderivative = 5.20

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))^2\,dx = \frac{1}{4}a^2d^3e^3x^4 + a^2cd^2e^3x^3 + \frac{3}{2}a^2c^2de^3x^2$$
$$+\frac{3}{2}\left(2x^2\,\text{artanh}\,(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$
$$+\left(2x^3\,\text{artanh}\,(dx+c) + d\left(\frac{dx^2-4cx}{d^3} + \frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3c^2+3c-1)\,\text{lo}}{d^4}\right.\right.$$
$$+\frac{1}{12}\left(6x^4\,\text{artanh}\,(dx+c) + d\left(\frac{2\left(d^2x^3-3cdx^2+3\left(3c^2+1\right)x\right)}{d^4} - \frac{3\left(c^4+4c^3+6c^2+4c+1\right)\log(dx}{d^5}\right.\right.$$
$$+a^2c^3e^3x + \frac{\left(2\,(dx+c)\,\text{artanh}\,(dx+c) + \log\left(-(dx+c)^2+1\right)\right)abc^3e^3}{d}$$
$$+\frac{4b^2d^2e^3x^2 + 8b^2cde^3x + 3\left(b^2d^4e^3x^4 + 4b^2cd^3e^3x^3 + 6b^2c^2d^2e^3x^2 + 4b^2c^3de^3x + (c^4e^3-e^3)b^2\right)\log(dx}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x
^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 +
(c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctanh(d*x
+ c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d
^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*a*b*c*d^2*e^3 + 1/12*(
6*x^4*arctanh(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4
- 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3
+ 6*c^2 - 4*c + 1)*log(d*x + c - 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*e^3*x + (2
*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/48*
(4*b^2*d^2*e^3*x^2 + 8*b^2*c*d*e^3*x + 3*(b^2*d^4*e^3*x^4 + 4*b^2*c*d^3*e^
3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (c^4*e^3 - e^3)*b^2)*l
og(d*x + c + 1)^2 + 3*(b^2*d^4*e^3*x^4 + 4*b^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d
^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (c^4*e^3 - e^3)*b^2)*log(-d*x - c + 1)^2
+ 4*(b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*(c^2*d*e^3 + d*e^3)*b^2*x +
 (c^3*e^3 + 3*c*e^3 + 4*e^3)*b^2)*log(d*x + c + 1) - 2*(2*b^2*d^3*e^3*x^3
+ 6*b^2*c*d^2*e^3*x^2 + 6*(c^2*d*e^3 + d*e^3)*b^2*x + 2*(c^3*e^3 + 3*c*e^3
 - 4*e^3)*b^2 + 3*(b^2*d^4*e^3*x^4 + 4*b^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e
^3*x^2 + 4*b^2*c^3*d*e^3*x + (c^4*e^3 - e^3)*b^2)*log(d*x + c + 1))*log(-d
*x - c + 1))/d
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(145) = 290$.

Time = 0.32 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.61

$$\int (ce+dex)^3(a+b\operatorname{arctanh}(c+dx))^2\,dx =$$

$$-\frac{1}{12}\left(\frac{4\,b^2e^3\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2}-\frac{4\,b^2e^3\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2}-\frac{3\left(\frac{(dx+c+1)^3b^2e^3}{(dx+c-1)^3}+\frac{(dx+c+1)b^2e^3}{dx+c-1}\right)\log\left(-\frac{dx-}{dx-}\right.}{\frac{(dx+c+1)^4d^2}{(dx+c-1)^4}-\frac{4\,(dx+c+1)^3d^2}{(dx+c-1)^3}+\frac{6\,(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{4\,(dx}{dx}}\right.$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/12*(4*b^2*e^3*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^2 - 4*b^2*e^3*log
(-(d*x + c + 1)/(d*x + c - 1))/d^2 - 3*((d*x + c + 1)^3*b^2*e^3/(d*x + c -
 1)^3 + (d*x + c + 1)*b^2*e^3/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c -
 1))^2/((d*x + c + 1)^4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x +
 c - 1)^3 + 6*(d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d
*x + c - 1) + d^2) - 2*(6*(d*x + c + 1)^3*a*b*e^3/(d*x + c - 1)^3 + 6*(d*x
 + c + 1)*a*b*e^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b^2*e^3/(d*x + c - 1)^
3 - 6*(d*x + c + 1)^2*b^2*e^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b^2*e^3/(d
*x + c - 1) - 2*b^2*e^3)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^
4*d^2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x + c
 + 1)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2) - 2
*(6*(d*x + c + 1)^3*a^2*e^3/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a^2*e^3/(d*x
 + c - 1) + 6*(d*x + c + 1)^3*a*b*e^3/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2
*a*b*e^3/(d*x + c - 1)^2 + 10*(d*x + c + 1)*a*b*e^3/(d*x + c - 1) - 4*a*b*
e^3 + (d*x + c + 1)^3*b^2*e^3/(d*x + c - 1)^3 - 2*(d*x + c + 1)^2*b^2*e^3/
(d*x + c - 1)^2 + (d*x + c + 1)*b^2*e^3/(d*x + c - 1))/((d*x + c + 1)^4*d^
2/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^2/(d*x + c - 1)^3 + 6*(d*x + c + 1
)^2*d^2/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2))*((c +
1)*d - (c - 1)*d)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 1730, normalized size of antiderivative = 10.88

$$\int (ce+dex)^3(a+b\text{arctanh}(c+dx))^2\,dx = \text{Too large to display}$$

input
```
int((c*e + d*e*x)^3*(a + b*atanh(c + d*x))^2,x)
```

output
```
log(1 - d*x - c)^2*((b^2*c^3*e^3*x)/4 - (b^2*e^3 - b^2*c^4*e^3)/(16*d) + (
b^2*d^3*e^3*x^4)/16 + (3*b^2*c^2*d*e^3*x^2)/8 + (b^2*c*d^2*e^3*x^3)/4) + x
*((c*e^3*(b^2 - 6*a^2 + 20*a^2*c^2 + 6*a*b*c))/2 + ((6*c^2 - 6)*(2*a^2*c*d
^2*e^3 - (a*d^2*e^3*(b + 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e^
3 - (a*d^2*e^3*(b + 10*a*c))/2))/d + (d*e^3*(b^2 - 6*a^2 + 60*a^2*c^2 + 12
*a*b*c))/6 - (a^2*d*e^3*(6*c^2 - 6))/6))/d) - log(1 - d*x - c)*(log(c + d*
x + 1)*((b^2*c^3*e^3*x)/2 - ((b^2*e^3)/2 - (b^2*c^4*e^3)/2)/(4*d) + (b^2*d
^3*e^3*x^4)/8 + (3*b^2*c^2*d*e^3*x^2)/4 + (b^2*c*d^2*e^3*x^3)/2) + (x^2*((
(d*(c - 1) + d*(c + 1))*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*(c - 1) + d*(
c + 1)) + 8*b^2*d^4*e^3*(c - 1)))/d^2 - 48*b^2*c^2*d^3*e^3 + 8*b^2*d^3*e^3
*(c - 1)*(c + 1) - 32*b^2*c*d^3*e^3*(c - 1)))/(128*d^2) - (x^2*(((d*(c - 1
) + d*(c + 1))*(32*b*d^4*e^3*(8*a*c - 2*a + b*c) - 8*b*d^3*e^3*(d*(c - 1)
+ d*(c + 1))*(8*a + b) + 8*b*d^4*e^3*(8*a + b)*(c + 1)))/d^2 - 48*b*c*d^3*
e^3*(8*a*c - 4*a + b*c) - 32*b*d^3*e^3*(c + 1)*(8*a*c - 2*a + b*c) + 8*b*d
^3*e^3*(8*a + b)*(c - 1)*(c + 1)))/(128*d^2) + (x^3*(32*b*d^4*e^3*(8*a*c -
 2*a + b*c) - 8*b*d^3*e^3*(d*(c - 1) + d*(c + 1))*(8*a + b) + 8*b*d^4*e^3*
(8*a + b)*(c + 1)))/(192*d^2) - (x^3*(32*b^2*c*d^4*e^3 - 8*b^2*d^3*e^3*(d*
(c - 1) + d*(c + 1)) + 8*b^2*d^4*e^3*(c - 1)))/(192*d^2) + (x*(((d*(c - 1)
 + d*(c + 1))*(((d*(c - 1) + d*(c + 1))*(32*b*d^4*e^3*(8*a*c - 2*a + b*c)
- 8*b*d^3*e^3*(d*(c - 1) + d*(c + 1))*(8*a + b) + 8*b*d^4*e^3*(8*a + b)...
```

3.15. $\int (ce + dex)^3 (a + b\text{arctanh}(c + dx))^2 \, dx$

3.16 $\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx$

3.16.1 Optimal result

Integrand size = 23, antiderivative size = 179

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx=\frac{1}{3}b^2e^2x-\frac{b^2e^2\text{arctanh}(c+dx)}{3d}+\frac{be^2(c+dx)^2(a+b\text{arctanh}(c+dx))}{3d}+\frac{e^2(a+b\text{arctanh}(c+dx))^2}{3d}+\frac{e^2(c+dx)^3(a+b\text{arctanh}(c+dx))^2}{3d}-\frac{2be^2(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{3d}-\frac{b^2e^2\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{3d}$$

output
```
1/3*b^2*e^2*x-1/3*b^2*e^2*arctanh(d*x+c)/d+1/3*b*e^2*(d*x+c)^2*(a+b*arctan
h(d*x+c))/d+1/3*e^2*(a+b*arctanh(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan
h(d*x+c))^2/d-2/3*b*e^2*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-1/3*b^2*e^
2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx$$
$$=\frac{e^2\left(a^2(c+dx)^3+ab((c+dx)^2+2(c+dx)^3\text{arctanh}(c+dx)+\log\left(-1+(c+dx)^2\right))+b^2\left(c+dx-\text{arctan}\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(e^2*(a^2*(c + d*x)^3 + a*b*((c + d*x)^2 + 2*(c + d*x)^3*ArcTanh[c + d*x]
+ Log[-1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTanh[c + d*x] + (c + d*x)^2*A
rcTanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*ArcTanh[c + d*x]^2 - 2*
ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])] + PolyLog[2, -E^(-2*ArcT
anh[c + d*x])])))/(3*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.80, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6657, 27, 6452, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6657}$$

$$\frac{\int e^2(c+dx)^2(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\int (c+dx)^2(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\int \frac{(c+dx)^3(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)-\int(c+dx)(a+b\text{arctanh}(c+d\right.\right.}{d}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\int\frac{(c+dx)^2}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}(c\right.\right.}{d}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int\frac{1}{1-(c+dx)^2}d(c+dx)-c\right.\right.\right.}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+\right.\right.}{d}$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(\int\frac{a+b\text{arctanh}(c+dx)}{-c-dx+1}d(c+dx)-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\right.\right.}{d}$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(-b\int\frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\right.\right.}{d}$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(b\int\frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-\frac{2}{-c-dx+1}}d\frac{1}{-c-dx+1}-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\frac{(a\right.\right.}{d}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^2-\frac{2}{3}b\left(-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{}{-c}\right.\right.\right.}{d}$$

3.16. $\int(ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTanh[c + d*x])^2)/3 - (2*b*((b*(-c - d*x + Ar
cTanh[c + d*x]))/2 - ((c + d*x)^2*(a + b*ArcTanh[c + d*x]))/2 - (a + b*Arc
Tanh[c + d*x])^2/(2*b) + (a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)] + (
b*PolyLog[2, 1 - 2/(1 - c - d*x)])/2))/3))/d
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)^2}{3}+\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1}{3}\right.}{}$
default	$\frac{\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)^2}{3}+\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1}{3}\right.}{}$
parts	$\frac{e^2a^2(dx+c)^3}{3d}+\frac{b^2e^2\left(\frac{(dx+c)^3\operatorname{arctanh}(dx+c)^2}{3}+\frac{(dx+c)^2\operatorname{arctanh}(dx+c)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+}{3}\right.}{}$
risch	$-\frac{e^2b^2cd\ln(dx+c+1)\ln(-dx-c+1)x^2}{2}+e^2badc\ln\left(dx+c+1\right)x^2-e^2dab\ln\left(-dx-c+1\right)cx^2$

input
```
int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^2*(d*x+c)^3+b^2*e^2*(1/3*(d*x+c)^3*arctanh(d*x+c)^2+1/3*(d*
x+c)^2*arctanh(d*x+c)+1/3*arctanh(d*x+c)*ln(d*x+c-1)+1/3*arctanh(d*x+c)*ln
(d*x+c+1)+1/3*d*x+1/3*c+1/6*ln(d*x+c-1)-1/6*ln(d*x+c+1)-1/3*dilog(1/2*d*x+
1/2*c+1/2)-1/6*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/12*ln(d*x+c-1)^2-1/12*l
n(d*x+c+1)^2+1/6*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2
))+2*e^2*a*b*(1/3*(d*x+c)^3*arctanh(d*x+c)+1/6*(d*x+c)^2+1/6*ln(d*x+c-1)+1
/6*ln(d*x+c+1)))
```

3.16.5 Fricas [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx=\int (dex+ce)^2(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctanh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
 + 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctanh(d*x + c), x)
```

3.16.6 Sympy [F]

$$\begin{aligned}&\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx\\&=e^2\left(\int a^2c^2\,dx+\int a^2d^2x^2\,dx+\int b^2c^2\,\text{atanh}^2\,(c+dx)\,dx+\int 2abc^2\,\text{atanh}\,(c+dx)\,dx\right.\\&\quad+\int 2a^2cdx\,dx+\int b^2d^2x^2\,\text{atanh}^2\,(c+dx)\,dx+\int 2abd^2x^2\,\text{atanh}\,(c+dx)\,dx\\&\quad\left.+\int 2b^2cdx\,\text{atanh}^2\,(c+dx)\,dx+\int 4abcdx\,\text{atanh}\,(c+dx)\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**2,x)
```

output
```
e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2
*c**2*atanh(c + d*x)**2, x) + Integral(2*a*b*c**2*atanh(c + d*x), x) + Int
egral(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atanh(c + d*x)**2, x) + I
ntegral(2*a*b*d**2*x**2*atanh(c + d*x), x) + Integral(2*b**2*c*d*x*atanh(c
 + d*x)**2, x) + Integral(4*a*b*c*d*x*atanh(c + d*x), x))
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(157) = 314$.

Time = 0.38 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.46

$$\int (ce+dex)^2(a+b\operatorname{arctanh}(c+dx))^2\,dx = \frac{1}{3}a^2d^2e^2x^3 + a^2cde^2x^2$$
$$+ \left(2x^2\operatorname{artanh}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)ab$$
$$+ \frac{1}{3}\left(2x^3\operatorname{artanh}(dx+c) + d\left(\frac{dx^2-4cx}{d^3} + \frac{(c^3+3c^2+3c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3c^2+3c-1)}{d^4}\right.\right.$$
$$+ a^2c^2e^2x + \frac{\left(2(dx+c)\operatorname{artanh}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)abc^2e^2}{d}$$
$$+ \frac{\left(\log(dx+c+1)\log\left(-\frac{1}{2}dx - \frac{1}{2}c + \frac{1}{2}\right) + \operatorname{Li}_2\left(\frac{1}{2}dx + \frac{1}{2}c + \frac{1}{2}\right)\right)b^2e^2}{3d}$$
$$+ \frac{(c^2e^2-e^2)b^2\log(dx+c+1)}{6d} - \frac{(c^2e^2-e^2)b^2\log(dx+c-1)}{6d}$$
$$+ \frac{4b^2de^2x + (b^2d^3e^2x^3 + 3b^2cd^2e^2x^2 + 3b^2c^2de^2x + (c^3e^2+e^2)b^2)\log(dx+c+1)^2 + (b^2d^3e^2x^3 + 3b^2cd^2}{}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arctanh(d*x + c) + d*(2*x/d
^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c -
1)/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^
3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)
*log(d*x + c - 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + (2*(d*x + c)*arctanh
(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/3*(log(d*x + c + 1)*l
og(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2))*b^2*e^2/d + 1/6
*(c^2*e^2 - e^2)*b^2*log(d*x + c + 1)/d - 1/6*(c^2*e^2 - e^2)*b^2*log(d*x
+ c - 1)/d + 1/12*(4*b^2*d*e^2*x + (b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x^2
+ 3*b^2*c^2*d*e^2*x + (c^3*e^2 + e^2)*b^2)*log(d*x + c + 1)^2 + (b^2*d^3*e
^2*x^3 + 3*b^2*c*d^2*e^2*x^2 + 3*b^2*c^2*d*e^2*x + (c^3*e^2 - e^2)*b^2)*lo
g(-d*x - c + 1)^2 + 2*(b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x)*log(d*x + c + 1)
 - 2*(b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e
^2*x^2 + 3*b^2*c^2*d*e^2*x + (c^3*e^2 + e^2)*b^2)*log(d*x + c + 1))*log(-d
*x - c + 1))/d
```

3.16.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx=\int (dex+ce)^2(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arctanh(d*x + c) + a)^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^2\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{atanh}(c+d\,x))^2\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^2, x)
```

3.17 $\int (ce + dex)(a + b\,\mathbf{arctanh}(c + dx))^2\,dx$

3.17.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (ce + dex)(a + b\,\mathrm{arctanh}(c + dx))^2\,dx = abex + \frac{b^2e(c+dx)\mathrm{arctanh}(c+dx)}{d} - \frac{e(a+b\,\mathrm{arctanh}(c+dx))^2}{2d} + \frac{e(c+dx)^2(a+b\,\mathrm{arctanh}(c+dx))^2}{2d} + \frac{b^2e\log\left(1-(c+dx)^2\right)}{2d}$$

output
```
a*b*e*x+b^2*e*(d*x+c)*arctanh(d*x+c)/d-1/2*e*(a+b*arctanh(d*x+c))^2/d+1/2*
e*(d*x+c)^2*(a+b*arctanh(d*x+c))^2/d+1/2*b^2*e*ln(1-(d*x+c)^2)/d
```

3.17.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int (ce + dex)(a + b\text{arctanh}(c + dx))^2\, dx = e\left(\frac{ab(c + dx)}{d} + \frac{a^2(c + dx)^2}{2d} + \frac{b(c + dx)(b + a(c + dx))\text{arctanh}(c + dx)}{d} + \frac{(-b^2 + b^2(c + dx)^2)\,\text{arctanh}(c + dx)^2}{2d} + \frac{(ab + b^2)\log(1 - c - dx)}{2d} + \frac{(-ab + b^2)\log(1 + c + dx)}{2d}\right)$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
e*((a*b*(c + d*x))/d + (a^2*(c + d*x)^2)/(2*d) + (b*(c + d*x)*(b + a*(c +
d*x))*ArcTanh[c + d*x])/d + ((-b^2 + b^2*(c + d*x)^2)*ArcTanh[c + d*x]^2)/
(2*d) + ((a*b + b^2)*Log[1 - c - d*x])/(2*d) + ((-(a*b) + b^2)*Log[1 + c +
 d*x])/(2*d))
```

3.17.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6657, 27, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce + dex)(a + b\text{arctanh}(c + dx))^2\, dx$$

$$\Big\downarrow \; 6657$$

$$\frac{\int e(c + dx)(a + b\text{arctanh}(c + dx))^2 d(c + dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int (c + dx)(a + b\text{arctanh}(c + dx))^2 d(c + dx)}{d}$$

$$\Big\downarrow \text{6452}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^2-b\int\frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{6542}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^2-b\left(\int\frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\int(a+b\text{arctanh}(c+dx))d(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^2-b\left(\int\frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-a(c+dx)-b(c+dx)\text{arctanh}(c+dx)-\right.\right.}{d}$$

$$\Big\downarrow \text{6510}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^2-b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-a(c+dx)-b(c+dx)\text{arctanh}(c+dx)-\frac{1}{2}b\log\left(\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcTanh[c + d*x])^2)/2 - b*(-(a*(c + d*x)) - b*(c
+ d*x)*ArcTanh[c + d*x] + (a + b*ArcTanh[c + d*x])^2/(2*b) - (b*Log[1 - (c
 + d*x)^2])/2)))/d
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(89) = 178$.

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

method	result
derivativedivides	$\frac{e a^2 (dx+c)^2}{2} + e b^2 \Big(\frac{(dx+c)^2 \operatorname{arctanh}(dx+c)^2}{2} + (dx+c) \operatorname{arctanh}(dx+c) + \frac{\operatorname{arctanh}(dx+c) \ln(dx+c-1)}{2} - \frac{\operatorname{arctanh}(dx+c) \ln(dx+c+1)}{2}$
default	$\frac{e a^2 (dx+c)^2}{2} + e b^2 \Big(\frac{(dx+c)^2 \operatorname{arctanh}(dx+c)^2}{2} + (dx+c) \operatorname{arctanh}(dx+c) + \frac{\operatorname{arctanh}(dx+c) \ln(dx+c-1)}{2} - \frac{\operatorname{arctanh}(dx+c) \ln(dx+c+1)}{2}$
parts	$e a^2 \left(\frac{1}{2} d x^2 + cx\right) + e b^2 \Big(\frac{(dx+c)^2 \operatorname{arctanh}(dx+c)^2}{2} + (dx+c) \operatorname{arctanh}(dx+c) + \frac{\operatorname{arctanh}(dx+c) \ln(dx+c-1)}{2} - \operatorname{arctanh}(dx+$
parallelrisch	$d^3 e b^2 x^2 \operatorname{arctanh}(dx+c)^2 + 2x^2 \operatorname{arctanh}(dx+c) ab d^3 e + 2ce b^2 x \operatorname{arctanh}(dx+c)^2 d^2 + x^2 a^2 d^3 e + 4x \operatorname{arctanh}(dx+c) abc d^2 e+$
risch	$\frac{e b^2 (d^2 x^2 + 2cdx + c^2 - 1) \ln(dx+c+1)^2}{8d} + \frac{be(-b d^2 x^2 \ln(-dx-c+1) + 2a d^2 x^2 - 2bcdx \ln(-dx-c+1) + 4acdx - \ln(-dx-c}{4d}$

input
```
int((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/2*(d*x+c)^2*arctanh(d*x+c)^2+(d*x+c)*arc
tanh(d*x+c)+1/2*arctanh(d*x+c)*ln(d*x+c-1)-1/2*arctanh(d*x+c)*ln(d*x+c+1)-
1/4*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)+1/8*ln(d*x+c-1)^2+1/2*ln(d*x+c-1)+1/
2*ln(d*x+c+1)+1/8*ln(d*x+c+1)^2-1/4*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln
(-1/2*d*x-1/2*c+1/2))+2*e*a*b*(1/2*(d*x+c)^2*arctanh(d*x+c)+1/2*d*x+1/2*c+
1/4*ln(d*x+c-1)-1/4*ln(d*x+c+1)))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(89) = 178$.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^2\,dx$$
$$= \frac{4\,a^2d^2ex^2 + 8\,(a^2c+ab)dex + 4\,(abc^2+b^2c-ab+b^2)e\log{(dx+c+1)} - 4\,(abc^2+b^2c-ab-b^2)e\log{(d}}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(4*a^2*d^2*e*x^2 + 8*(a^2*c + a*b)*d*e*x + 4*(a*b*c^2 + b^2*c - a*b +
b^2)*e*log(d*x + c + 1) - 4*(a*b*c^2 + b^2*c - a*b - b^2)*e*log(d*x + c -
1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 - b^2)*e)*log(-(d*x + c + 1
)/(d*x + c - 1))^2 + 4*(a*b*d^2*e*x^2 + (2*a*b*c + b^2)*d*e*x)*log(-(d*x +
 c + 1)/(d*x + c - 1)))/d
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(83) = 166$.

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.51

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^2\,dx$$
$$= \begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\,\text{atanh}\,(c+dx)}{d} + 2abcex\,\text{atanh}\,(c+dx) + abdex^2\,\text{atanh}\,(c+dx) + abex - \frac{abe\,\text{atanh}\,(c+d}{d} \\ cex(a+b\,\text{atanh}\,(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atanh(c + d*x)/d + 2*
a*b*c*e*x*atanh(c + d*x) + a*b*d*e*x**2*atanh(c + d*x) + a*b*e*x - a*b*e*a
tanh(c + d*x)/d + b**2*c**2*e*atanh(c + d*x)**2/(2*d) + b**2*c*e*x*atanh(c
 + d*x)**2 + b**2*c*e*atanh(c + d*x)/d + b**2*d*e*x**2*atanh(c + d*x)**2/2
 + b**2*e*x*atanh(c + d*x) + b**2*e*log(c/d + x + 1/d)/d - b**2*e*atanh(c
+ d*x)**2/(2*d) - b**2*e*atanh(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atanh(
c))**2, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(89) = 178$.

Time = 0.40 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.33

$$\int (ce + dex)(a + b\operatorname{arctanh}(c + dx))^2\, dx = \frac{1}{2}\, a^2 dex^2$$
$$+ \frac{1}{2}\left(2\,x^2 \operatorname{artanh}\left(dx + c\right) + d\left(\frac{2\,x}{d^2} - \frac{(c^2 + 2\,c + 1)\log\left(dx + c + 1\right)}{d^3} + \frac{(c^2 - 2\,c + 1)\log\left(dx + c - 1\right)}{d^3}\right)\right)$$
$$+ a^2 cex + \frac{\left(2\left(dx + c\right)\operatorname{artanh}\left(dx + c\right) + \log\left(-(dx + c)^2 + 1\right)\right)abce}{d}$$
$$+ \frac{(b^2 d^2 ex^2 + 2\,b^2 cdex + (c^2 e - e)b^2)\log\left(dx + c + 1\right)^2 + (b^2 d^2 ex^2 + 2\,b^2 cdex + (c^2 e - e)b^2)\log\left(-dx - c\right.}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c +
1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*d*e +
 a^2*c*e*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*c*
e/d + 1/8*((b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*log(d*x + c +
 1)^2 + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*log(-d*x - c + 1
)^2 + 4*(b^2*d*e*x + (c*e + e)*b^2)*log(d*x + c + 1) - 2*(2*b^2*d*e*x + 2*
(c*e - e)*b^2 + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (c^2*e - e)*b^2)*log(d*x
+ c + 1))*log(-d*x - c + 1))/d
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.69

$$\int (ce + dex)(a + b\text{arctanh}(c + dx))^2\, dx$$

$$= \frac{1}{4}\left(\frac{(dx+c+1)b^2e\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\left(\frac{(dx+c+1)^2d^2}{(dx+c-1)^2}-\frac{2\,(dx+c+1)d^2}{dx+c-1}+d^2\right)(dx+c-1)} - \frac{2\,b^2e\log\left(-\frac{dx+c+1}{dx+c-1}+1\right)}{d^2} + \frac{2\,b^2e\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{d^2} + \frac{2\left(\right.}{}\right.$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/4*((d*x + c + 1)*b^2*e*log(-(d*x + c + 1)/(d*x + c - 1))^2/(((d*x + c +
1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c - 1) + d^2)*(d*x +
 c - 1)) - 2*b^2*e*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^2 + 2*b^2*e*log
(-(d*x + c + 1)/(d*x + c - 1))/d^2 + 2*(2*(d*x + c + 1)*a*b*e/(d*x + c - 1
) + (d*x + c + 1)*b^2*e/(d*x + c - 1) - b^2*e)*log(-(d*x + c + 1)/(d*x + c
 - 1))/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^2/(d*x + c
 - 1) + d^2) + 4*((d*x + c + 1)*a^2*e/(d*x + c - 1) + (d*x + c + 1)*a*b*e/
(d*x + c - 1) - a*b*e)/((d*x + c + 1)^2*d^2/(d*x + c - 1)^2 - 2*(d*x + c +
 1)*d^2/(d*x + c - 1) + d^2))*((c + 1)*d - (c - 1)*d)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.55

$$\int (ce+dex)(a+b\operatorname{arctanh}(c+dx))^2\,dx$$
$$= x\left(a\,e\,(b+3\,a\,c)-2\,a^2\,c\,e\right)+\ln\left(1-d\,x-c\right)^2\left(\frac{b^2\,c\,e\,x}{4}-\frac{b^2\,e-b^2\,c^2\,e}{8\,d}+\frac{b^2\,d\,e\,x^2}{8}\right)$$
$$-\ln\left(1-d\,x-c\right)\left(\ln\left(c+d\,x+1\right)\left(\frac{b^2\,c\,e\,x}{2}-\frac{\frac{b^2\,e}{2}-\frac{b^2\,c^2\,e}{2}}{2\,d}+\frac{b^2\,d\,e\,x^2}{4}\right)\right.$$
$$-\frac{x\left(4\,b^2\,d^2\,e\,(c-1)-4\,b^2\,d\,e\,(d\,(c-1)+d\,(c+1))+8\,b^2\,c\,d^2\,e\right)}{16\,d^2}$$
$$+\frac{x\left(8\,b\,d^2\,e\,(4\,a\,c-2\,a+b\,c)+4\,b\,d^2\,e\,(4\,a+b)\,(c+1)-4\,b\,d\,e\,(d\,(c-1)+d\,(c+1))\,(4\,a+b)\right)}{16\,d^2}$$
$$\left.-\frac{b^2\,d\,e\,x^2}{8}+\frac{b\,d\,e\,x^2\,(4\,a+b)}{8}\right)+\ln\left(c+d\,x+1\right)^2\left(\frac{b^2\,c\,e\,x}{4}-\frac{b^2\,e-b^2\,c^2\,e}{8\,d}+\frac{b^2\,d\,e\,x^2}{8}\right)$$
$$+\frac{\ln\left(c+d\,x+1\right)\,\left(e\,b^2\,c+e\,b^2+a\,e\,b\,c^2-a\,e\,b\right)}{2\,d}$$
$$+\frac{\ln\left(c+d\,x-1\right)\,\left(-e\,b^2\,c+e\,b^2-a\,e\,b\,c^2+a\,e\,b\right)}{2\,d}$$
$$+d\,\ln\left(c+d\,x+1\right)\left(\frac{x\,(e\,b^2+2\,a\,c\,e\,b)}{2\,d}+\frac{a\,b\,e\,x^2}{2}\right)+\frac{a^2\,d\,e\,x^2}{2}$$

input
```
int((c*e + d*e*x)*(a + b*atanh(c + d*x))^2,x)
```

output
```
x*(a*e*(b + 3*a*c) - 2*a^2*c*e) + log(1 - d*x - c)^2*((b^2*c*e*x)/4 - (b^2
*e - b^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) - log(1 - d*x - c)*(log(c + d*x +
 1)*((b^2*c*e*x)/2 - ((b^2*e)/2 - (b^2*c^2*e)/2)/(2*d) + (b^2*d*e*x^2)/4)
- (x*(4*b^2*d^2*e*(c - 1) - 4*b^2*d*e*(d*(c - 1) + d*(c + 1)) + 8*b^2*c*d^
2*e))/(16*d^2) + (x*(8*b*d^2*e*(4*a*c - 2*a + b*c) + 4*b*d^2*e*(4*a + b)*(
c + 1) - 4*b*d*e*(d*(c - 1) + d*(c + 1))*(4*a + b)))/(16*d^2) - (b^2*d*e*x
^2)/8 + (b*d*e*x^2*(4*a + b))/8) + log(c + d*x + 1)^2*((b^2*c*e*x)/4 - (b^
2*e - b^2*c^2*e)/(8*d) + (b^2*d*e*x^2)/8) + (log(c + d*x + 1)*(b^2*e - a*b
*e + b^2*c*e + a*b*c^2*e))/(2*d) + (log(c + d*x - 1)*(b^2*e + a*b*e - b^2*
c*e - a*b*c^2*e))/(2*d) + d*log(c + d*x + 1)*((x*(b^2*e + 2*a*b*c*e))/(2*d
) + (a*b*e*x^2)/2) + (a^2*d*e*x^2)/2
```

3.18 $\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{ce+dex}\,dx$

3.18.1 Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{ce+dex}\,dx = \frac{2(a+b\,\mathrm{arctanh}(c+dx))^2\mathrm{arctanh}\left(1-\frac{2}{1-c-dx}\right)}{de} - \frac{b(a+b\,\mathrm{arctanh}(c+dx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{de} + \frac{b(a+b\,\mathrm{arctanh}(c+dx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-c-dx}\right)}{de} + \frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2de} - \frac{b^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-c-dx}\right)}{2de}$$

output
```
-2*(a+b*arctanh(d*x+c))^2*arctanh(-1+2/(-d*x-c+1))/d/e-b*(a+b*arctanh(d*x+
c))*polylog(2,1-2/(-d*x-c+1))/d/e+b*(a+b*arctanh(d*x+c))*polylog(2,-1+2/(-
d*x-c+1))/d/e+1/2*b^2*polylog(3,1-2/(-d*x-c+1))/d/e-1/2*b^2*polylog(3,-1+2
/(-d*x-c+1))/d/e
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.52

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{ce+dex}\,dx = \frac{a^2\log(c+dx)}{de}$$

$$-\frac{2iab\Big(i\text{arctanh}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+\log\left(\frac{i(c+dx)}{\sqrt{1-(c+dx)^2}}\right)\right)+\frac{1}{2}\Big(-\frac{1}{4}i(\pi-2i\text{arctanh}(c+dx))^2+$$

$$+\frac{b^2\left(\text{arctanh}(c+dx)^2\log\left(1-e^{-2\text{arctanh}(c+dx)}\right)-\text{arctanh}(c+dx)^2\log\left(1+e^{-2\text{arctanh}(c+dx)}\right)+\text{arctanh}(c$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(a^2*Log[c + d*x])/(d*e) - ((2*I)*a*b*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 -
 (c + d*x)^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) + ((-1/4*I)*(Pi
 - (2*I)*ArcTanh[c + d*x])^2 + I*ArcTanh[c + d*x]^2 + (Pi - (2*I)*ArcTanh[
c + d*x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] + (2*I)*ArcTanh[c +
 d*x]*Log[1 - E^(-2*ArcTanh[c + d*x])] - (2*I)*ArcTanh[c + d*x]*Log[((2*I)
*(c + d*x))/Sqrt[1 - (c + d*x)^2]] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2*S
in[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTa
nh[c + d*x]))] - I*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/2))/(d*e) + (b^2*(
ArcTanh[c + d*x]^2*Log[1 - E^(-2*ArcTanh[c + d*x])] - ArcTanh[c + d*x]^2*L
og[1 + E^(-2*ArcTanh[c + d*x])] + ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTa
nh[c + d*x])] - ArcTanh[c + d*x]*PolyLog[2, E^(-2*ArcTanh[c + d*x])] + Pol
yLog[3, -E^(-2*ArcTanh[c + d*x])]/2 - PolyLog[3, E^(-2*ArcTanh[c + d*x])]/
2))/(d*e)
```

3.18.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6657, 27, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{ce+dex}\,dx$$

$$\Big\downarrow \; 6657$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \; 6448$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-4b\int \frac{(a+b\text{arctanh}(c+dx))\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)}{de}$$

$$\Big\downarrow \; 6614$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-4b\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(c+dx))\log\left(2-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}\int \frac{(a+b}{}\right.}{de}$$

$$\Big\downarrow \; 6620$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-4b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))-\right.\right.}{}$$

$$\Big\downarrow \; 7164$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-4b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))-\right.\right.}{}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(2*(a + b*ArcTanh[c + d*x])^2*ArcTanh[1 - 2/(1 - c - d*x)] - 4*b*((((a + b
*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/2 - (b*PolyLog[3, 1 -
2/(1 - c - d*x)])/4)/2 + (-1/2*((a + b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2
/(1 - c - d*x)]) + (b*PolyLog[3, -1 + 2/(1 - c - d*x)])/4)/2))/(d*e)
```

3.18. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{ce+dex}\,dx$

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.18.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 705, normalized size of antiderivative = 4.20

method	result
derivativedivides	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(\ln(dx+c)\operatorname{arctanh}(dx+c)^2-\operatorname{arctanh}(dx+c)\operatorname{polylog}\left(2,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)}{2}-\operatorname{arctanh}(dx+c)}{}}{}$
default	$\frac{\frac{a^2\ln(dx+c)}{e}+\frac{b^2\left(\ln(dx+c)\operatorname{arctanh}(dx+c)^2-\operatorname{arctanh}(dx+c)\operatorname{polylog}\left(2,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)}{2}-\operatorname{arctanh}(dx+c)}{}}{}$
parts	$\frac{a^2\ln(dx+c)}{ed}+\frac{b^2\left(\ln(dx+c)\operatorname{arctanh}(dx+c)^2-\operatorname{arctanh}(dx+c)\operatorname{polylog}\left(2,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)}{2}-\text{ar}}{}$

input
```
int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2/e*ln(d*x+c)+b^2/e*(ln(d*x+c)*arctanh(d*x+c)^2-arctanh(d*x+c)*poly
log(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/2*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2)
)-arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)+arctanh(d*x+c)^2*ln(1-(
d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*arctanh(d*x+c)*polylog(2,(d*x+c+1)/(1-(d*x
+c)^2)^(1/2))-2*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+arctanh(d*x+c)^2*
ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*arctanh(d*x+c)*polylog(2,-(d*x+c+1)/
(1-(d*x+c)^2)^(1/2))-2*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+1/2*I*Pi*
csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*(csgn
(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))-c
sgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1
)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))-csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-
(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))+csgn(I*(
-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2)*arctanh(d*
x+c)^2)+2*a*b/e*(ln(d*x+c)*arctanh(d*x+c)-1/2*dilog(d*x+c)-1/2*dilog(d*x+c
+1)-1/2*ln(d*x+c)*ln(d*x+c+1)))
```

3.18.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{ce+dex}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d*e*x +
c*e), x)
```

3.18.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{ce+dex}\,dx=\frac{\int \frac{a^2}{c+dx}\,dx+\int \frac{b^2\operatorname{atanh}^2(c+dx)}{c+dx}\,dx+\int \frac{2ab\operatorname{atanh}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e),x)
```

output
```
(Integral(a**2/(c + d*x), x) + Integral(b**2*atanh(c + d*x)**2/(c + d*x),
x) + Integral(2*a*b*atanh(c + d*x)/(c + d*x), x))/e
```

3.18.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{ce + dex}\,dx = \int \frac{(b\,\text{artanh}\,(dx + c) + a)^2}{dex + ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^2*log(d*e*x + c*e)/(d*e) + integrate(1/4*b^2*(log(d*x + c + 1) - log(-d*
x - c + 1))^2/(d*e*x + c*e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(
d*e*x + c*e), x)
```

3.18.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{ce + dex}\,dx = \int \frac{(b\,\text{artanh}\,(dx + c) + a)^2}{dex + ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{ce + dex}\,dx = \int \frac{(a + b\,\text{atanh}(c + d\,x))^2}{c\,e + d\,e\,x}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x),x)
```

output
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x), x)
```

3.19 $\int \frac{(a+b\,\text{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{(a+b\,\text{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{(a+b\,\text{arctanh}(c+dx))^2}{de^2} - \frac{(a+b\,\text{arctanh}(c+dx))^2}{de^2(c+dx)} + \frac{2b(a+b\,\text{arctanh}(c+dx))\log\left(2-\frac{2}{1+c+dx}\right)}{de^2} - \frac{b^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+c+dx}\right)}{de^2}$$

output
```
(a+b*arctanh(d*x+c))^2/d/e^2-(a+b*arctanh(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b
*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^2-b^2*polylog(2,-1+2/(d*x+c+1))/d/e
^2
```

3.19.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\,\text{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{b^2(-1+c+dx)\text{arctanh}(c+dx)^2 + 2b\,\text{arctanh}(c+dx)\left(-a+b(c+dx)\log\left(1-e^{-2\text{arctanh}(c+dx)}\right)\right) + a\Big(-\cdots}{de^2(c+dx)}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(-a + b*(c +
 d*x)*Log[1 - E^(-2*ArcTanh[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c + d
*x)/Sqrt[1 - (c + d*x)^2]]) - b^2*(c + d*x)*PolyLog[2, E^(-2*ArcTanh[c + d
*x])])/(d*e^2*(c + d*x))
```

3.19.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6657, 27, 6452, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6550}$$

$$\frac{2b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(c+dx+1)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6494}$$

$$\frac{2b\left(-b\int \frac{\log\left(2-\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))\right)-\frac{(a+b\text{arctan}}{c+c}}{de^2}$$

$\downarrow$ 2897

$$\frac{2b\left(\frac{(a+b\operatorname{arctanh}(c+dx))^2}{2b}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\operatorname{arctanh}(c+dx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)\right)-\frac{(a+b\operatorname{arctanh}}{c+dx}}{de^2}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTanh[c + d*x])^2/(c + d*x)) + 2*b*((a + b*ArcTanh[c + d*x])^2
/(2*b) + (a + b*ArcTanh[c + d*x])*Log[2 - 2/(1 + c + d*x)] - (b*PolyLog[2,
 -1 + 2/(1 + c + d*x)])/2))/(d*e^2)
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(104) = 208$.

Time = 0.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.42

method	result
derivativedivides	$-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{dx+c}-\operatorname{arctanh}(dx+c)\ln(dx+c-1)+2\ln(dx+c)\operatorname{arctanh}(dx+c)-\operatorname{arctanh}(dx+c)\ln(dx+c+1)+\operatorname{dilog}\left(\frac{dx}{2}\right.\right.}{}$
default	$-\frac{a^2}{e^2(dx+c)}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{dx+c}-\operatorname{arctanh}(dx+c)\ln(dx+c-1)+2\ln(dx+c)\operatorname{arctanh}(dx+c)-\operatorname{arctanh}(dx+c)\ln(dx+c+1)+\operatorname{dilog}\left(\frac{dx}{2}\right.\right.}{}$
parts	$-\frac{a^2}{e^2(dx+c)d}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{dx+c}-\operatorname{arctanh}(dx+c)\ln(dx+c-1)+2\ln(dx+c)\operatorname{arctanh}(dx+c)-\operatorname{arctanh}(dx+c)\ln(dx\right.}{}$

input
```
int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arctanh(d*x+c)^2-arctanh(d*x+c)*
ln(d*x+c-1)+2*ln(d*x+c)*arctanh(d*x+c)-arctanh(d*x+c)*ln(d*x+c+1)+dilog(1/
2*d*x+1/2*c+1/2)+1/2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/4*ln(d*x+c-1)^2+1
/4*ln(d*x+c+1)^2-1/2*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c
+1/2)-dilog(d*x+c)-dilog(d*x+c+1)-ln(d*x+c)*ln(d*x+c+1))+2*a*b/e^2*(-1/(d*
x+c)*arctanh(d*x+c)-1/2*ln(d*x+c-1)+ln(d*x+c)-1/2*ln(d*x+c+1)))
```

3.19.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d^2*e^2*
x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.19.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx=\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^2\operatorname{atanh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2ab\operatorname{atanh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atanh(c +
d*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atanh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.19.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-(d*(log(d*x + c + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2) + log(d*x + c -
 1)/(d^2*e^2)) + 2*arctanh(d*x + c)/(d^2*e^2*x + c*d*e^2))*a*b - 1/4*b^2*(
log(-d*x - c + 1)^2/(d^2*e^2*x + c*d*e^2) + integrate(-((d*x + c - 1)*log(
d*x + c + 1)^2 + 2*(d*x - (d*x + c - 1)*log(d*x + c + 1) + c)*log(-d*x - c
 + 1))/(d^3*e^2*x^3 + c^3*e^2 - c^2*e^2 + (3*c*d^2*e^2 - d^2*e^2)*x^2 + (3
*c^2*d*e^2 - 2*c*d*e^2)*x), x)) - a^2/(d^2*e^2*x + c*d*e^2)
```

3.19.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\text{atanh}(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.20 $\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b(a+b\,\mathrm{arctanh}(c+dx))}{de^3(c+dx)} + \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{2de^3} - \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3} - \frac{b^2\log\left(1-(c+dx)^2\right)}{2de^3}$$

output
```
-b*(a+b*arctanh(d*x+c))/d/e^3/(d*x+c)+1/2*(a+b*arctanh(d*x+c))^2/d/e^3-1/2
*(a+b*arctanh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1-(
d*x+c)^2)/d/e^3
```

3.20.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{-\frac{a^2}{(c+dx)^2} - \frac{2ab}{c+dx} - \frac{2b(a+b(c+dx))\mathrm{arctanh}(c+dx)}{(c+dx)^2} + \frac{b^2(-1+c^2+2cdx+d^2x^2)\mathrm{arctanh}(c+dx)^2}{(c+dx)^2} - b(a+b)\log(1-c-dx)}{2de^3}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-(a^2/(c + d*x)^2) - (2*a*b)/(c + d*x) - (2*b*(a + b*(c + d*x))*ArcTanh[c
 + d*x])/(c + d*x)^2 + (b^2*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x
]^2)/(c + d*x)^2 - b*(a + b)*Log[1 - c - d*x] + 2*b^2*Log[c + d*x] + (a -
b)*b*Log[1 + c + d*x])/(2*d*e^3)
```

3.20.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6657, 27, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^3}dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{6452}$$

$$\frac{b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6544}$$

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2}d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6452}$$

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+b\int \frac{1}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\int \frac{1}{(-c-dx+1)(c+dx)^2}d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 47

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\int \frac{1}{(c+dx)^2}d(c+dx)^2\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)-\frac{(a+b\text{arc}}{2}}{de^3}$$

$\Big\downarrow$ 14

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\log\left((c+dx)^2\right)\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)-\frac{(a+b\text{arctanh}}{2(c+d}}{de^3}$$

$\Big\downarrow$ 16

$$\frac{b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log(-c-dx+1)\right)\right)-\frac{(a+b\text{arctanh}(c+d}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 6510

$$\frac{b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{c+dx}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log(-c-dx+1)\right)\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcTanh[c + d*x])^2/(c + d*x)^2 + b*(-((a + b*ArcTanh[c + d*x
])/(c + d*x)) + (a + b*ArcTanh[c + d*x])^2/(2*b) + (b*(-Log[1 - c - d*x] +
 Log[(c + d*x)^2]))/2))/(d*e^3)
```

3.20.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.20. $\int \frac{(a+b\,\text{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx$

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

method	result
derivativedivides	$-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{2(dx+c)^2}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}-\frac{\operatorname{arctanh}(dx+c)}{dx+c}+\frac{\ln(dx+c-1)\ln(}{4}\right.}{}$
default	$-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{2(dx+c)^2}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}-\frac{\operatorname{arctanh}(dx+c)}{dx+c}+\frac{\ln(dx+c-1)\ln(}{4}\right.}{}$
parts	$-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{2(dx+c)^2}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}-\frac{\operatorname{arctanh}(dx+c)}{dx+c}+\frac{\ln(dx+c}{}\right.}{}$
parallelrisch	$\frac{-4\operatorname{arctanh}(dx+c)b^2c^3d^2-4\operatorname{arctanh}(dx+c)b^2c^2d^2+2\operatorname{arctanh}(dx+c)^2b^2c^3d^2+4\ln(dx+c)b^2c^3d^2-4\ln(dx+c-1)b^2c^3d^2}{}$
risch	$\frac{b^2(d^2x^2+2cdx+c^2-1)\ln(dx+c+1)^2}{8e^3(dx+c)^2d}-\frac{b(bd^2x^2\ln(-dx-c+1)+2bcdx\ln(-dx-c+1)+\ln(-dx-c+1)bc^2+2bdx+2bc-}{4e^3(dx+c)^2d}$

input
```
int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(-1/2/(d*x+c)^2*arctanh(d*x+c)^2-1/2*a
rctanh(d*x+c)*ln(d*x+c-1)+1/2*arctanh(d*x+c)*ln(d*x+c+1)-1/(d*x+c)*arctanh
(d*x+c)+1/4*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/8*ln(d*x+c-1)^2-1/2*ln(d*x
+c-1)+ln(d*x+c)-1/2*ln(d*x+c+1)-1/8*ln(d*x+c+1)^2+1/4*(ln(d*x+c+1)-ln(1/2*
d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2))+2*a*b/e^3*(-1/2/(d*x+c)^2*arctanh(
d*x+c)-1/4*ln(d*x+c-1)+1/4*ln(d*x+c+1)-1/2/(d*x+c)))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(113) = 226$.

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.27

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx =$$
$$-\frac{8\,abdx+8\,abc-(b^2d^2x^2+2\,b^2cdx+b^2c^2-b^2)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2+4\,a^2-4\left((ab-b^2)d^2x^2+2\,(ab-b^2)c\right.}{}$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
-1/8*(8*a*b*d*x + 8*a*b*c - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - b^2)*lo
g(-(d*x + c + 1)/(d*x + c - 1))^2 + 4*a^2 - 4*((a*b - b^2)*d^2*x^2 + 2*(a*
b - b^2)*c*d*x + (a*b - b^2)*c^2)*log(d*x + c + 1) - 8*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*log(d*x + c) + 4*((a*b + b^2)*d^2*x^2 + 2*(a*b + b^2)*c
*d*x + (a*b + b^2)*c^2)*log(d*x + c - 1) + 4*(b^2*d*x + b^2*c + a*b)*log(-
(d*x + c + 1)/(d*x + c - 1)))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(100) = 200.

Time = 1.05 (sec) , antiderivative size = 1102, normalized size of antiderivative = 9.26

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx$$
$$= \begin{cases} -\frac{a^2}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abc^2\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{4abcdx\operatorname{atanh}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} + \frac{2abd^2x^2\operatorname{at}}{2c^2de^3+4cd^2} \\ \frac{x(a+b\operatorname{atanh}(c))^2}{c^3e^3} \end{cases}$$

input
```
integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

output
```
Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*
a*b*c**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**
2) + 4*a*b*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*
e**3*x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
+ 2*a*b*d**2*x**2*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3
*e**3*x**2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**
2) - 2*a*b*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x
**2) + 2*b**2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*
e**3*x**2) - 2*b**2*c**2*log(c/d + x + 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3
*x + 2*d**3*e**3*x**2) + b**2*c**2*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*
d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c**2*atanh(c + d*x)/(2*c**2*d*e**
3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**
2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x
+ 1/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*c*d*x
*atanh(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) +
4*b**2*c*d*x*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3
*x**2) - 2*b**2*c*atanh(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3
*e**3*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3
*x + 2*d**3*e**3*x**2) - 2*b**2*d**2*x**2*log(c/d + x + 1/d)/(2*c**2*d*e**
3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + b**2*d**2*x**2*atanh(c + d*x)...
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(113) = 226$.

Time = 0.21 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.76

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(ce + dex)^3}\,dx =$$
$$-\frac{1}{2}\left(d\left(\frac{2}{d^3e^3x + cd^2e^3} - \frac{\log(dx + c + 1)}{d^2e^3} + \frac{\log(dx + c - 1)}{d^2e^3}\right) + \frac{2\,\text{artanh}(dx + c)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right)ab$$
$$-\frac{1}{8}\left(d^2\left(\frac{\log(dx + c + 1)^2 - 2\,\log(dx + c + 1)\log(dx + c - 1) + \log(dx + c - 1)^2 + 4\,\log(dx + c - 1)}{d^3e^3}\right.\right.$$
$$-\frac{b^2\,\text{artanh}(dx + c)^2}{2\,(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3)} - \frac{a^2}{2\,(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3)}$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/2*(d*(2/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c + 1)/(d^2*e^3) + log(d*x
+ c - 1)/(d^2*e^3)) + 2*arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^
2*d*e^3))*a*b - 1/8*(d^2*((log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d*x
 + c - 1) + log(d*x + c - 1)^2 + 4*log(d*x + c - 1))/(d^3*e^3) + 4*log(d*x
 + c + 1)/(d^3*e^3) - 8*log(d*x + c)/(d^3*e^3)) + 4*d*(2/(d^3*e^3*x + c*d^
2*e^3) - log(d*x + c + 1)/(d^2*e^3) + log(d*x + c - 1)/(d^2*e^3))*arctanh(
d*x + c))*b^2 - 1/2*b^2*arctanh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x +
c^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(113) = 226$.

Time = 0.30 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.15

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(ce + dex)^3}\, dx = \frac{1}{4}\left(\frac{(dx + c + 1)b^2 \log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\left(\frac{(dx+c+1)^2 d^2 e^3}{(dx+c-1)^2} + \frac{2\,(dx+c+1)d^2e^3}{dx+c-1} + d^2e^3\right)(dx + c - 1)} + \frac{2\left(\frac{2\,(dx+c+1)ab}{dx+c-1} + \frac{(dx+c+1)b^2}{dx+c-1} + b^2\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2 d^2e^3}{(dx+c-1)^2} + \frac{2\,(dx+c+1)d^2e^3}{dx+c-1} + d^2e^3}\right.$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
1/4*((d*x + c + 1)*b^2*log(-(d*x + c + 1)/(d*x + c - 1))^2/(((d*x + c + 1)
^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e
^3)*(d*x + c - 1)) + 2*(2*(d*x + c + 1)*a*b/(d*x + c - 1) + (d*x + c + 1)*
b^2/(d*x + c - 1) + b^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^
2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/(d*x + c - 1) + d^2*e^
3) + 4*((d*x + c + 1)*a^2/(d*x + c - 1) + (d*x + c + 1)*a*b/(d*x + c - 1)
+ a*b)/((d*x + c + 1)^2*d^2*e^3/(d*x + c - 1)^2 + 2*(d*x + c + 1)*d^2*e^3/
(d*x + c - 1) + d^2*e^3) + 2*b^2*log(-(d*x + c + 1)/(d*x + c - 1) - 1)/(d^
2*e^3) - 2*b^2*log(-(d*x + c + 1)/(d*x + c - 1))/(d^2*e^3))*((c + 1)*d - (
c - 1)*d)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 776, normalized size of antiderivative = 6.52

$$
\begin{aligned}
&\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx \\
&= \ln(1-d\,x-c)^2\left(\frac{b^2}{8\,d\,e^3}-\frac{b^2}{2\,d\,\left(4\,c^2\,e^3+8\,c\,d\,e^3\,x+4\,d^2\,e^3\,x^2\right)}\right) \\
&\quad+\ln(c+d\,x+1)^2\left(\frac{b^2}{8\,d\,e^3}-\frac{b^2}{8\,d^2\,e^3\,\left(2\,c\,x+d\,x^2+\frac{c^2}{d}\right)}\right)+\ln(1-d\,x-c)\,\Bigg(\ln(c+d\,x \\
&\qquad+1)\left(\frac{b^2}{2\,d\,\left(2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2\right)}-\frac{b^2\,\left(c^2+2\,c\,d\,x+d^2\,x^2\right)}{2\,d\,\left(2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2\right)}\right) \\
&\qquad+\frac{b^2}{2\,d\,\left(4\,c^2\,e^3+8\,c\,d\,e^3\,x+4\,d^2\,e^3\,x^2\right)}+\frac{b\,(4\,a-b)}{2\,d\,\left(4\,c^2\,e^3+8\,c\,d\,e^3\,x+4\,d^2\,e^3\,x^2\right)} \\
&\qquad-\frac{b^2\,\left(x\,\left(4\,c\,d-d+d\,(2\,c-1)\right)-c+c^2+c\,(2\,c-1)+3\,d^2\,x^2+1\right)}{2\,d\,\left(4\,c^2\,e^3+8\,c\,d\,e^3\,x+4\,d^2\,e^3\,x^2\right)} \\
&\quad+\frac{b^2\,\left(x\,\left(2\,d\,e^3+d\,\left(4\,c\,e^3+2\,e^3\right)+8\,c\,d\,e^3\right)+2\,c\,e^3+2\,e^3+c\,\left(4\,c\,e^3+2\,e^3\right)+2\,c^2\,e^3+6\,d^2\,e^3\,x^2\right)}{4\,d\,e^3\,\left(4\,c^2\,e^3+8\,c\,d\,e^3\,x+4\,d^2\,e^3\,x^2\right)}\Bigg) \\
&\quad-\frac{\frac{a^2+2\,b\,c\,a}{2\,d}+a\,b\,x}{c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2} \\
&\quad-\frac{\ln(c+d\,x+1)\,\left(x\,\left(\frac{2\,b^2\,c+b^2}{4\,d\,e^3}+\frac{b^2\,c}{4\,d\,e^3}-\frac{b^2\,(3\,c-1)}{4\,d\,e^3}\right)+\frac{b^2\,c^2+b^2\,c+b^2+4\,a\,b}{8\,d^2\,e^3}-\frac{b^2\,\left(\frac{c^2-c+1}{2\,d}+\frac{c\,(2\,c-1)}{2\,d}\right)}{4\,d\,e^3}+\frac{c\,\left(2\,b^2\,c+b^2\right)}{8\,d^2\,e^3}\right)}{2\,c\,x+d\,x^2+\frac{c^2}{d}} \\
&\quad+\frac{b^2\,\ln(c+d\,x)}{d\,e^3}-\frac{\ln(c+d\,x-1)\,\left(b^2+a\,b\right)}{2\,d\,e^3}+\frac{\ln(c+d\,x+1)\,\left(a\,b-b^2\right)}{2\,d\,e^3}
\end{aligned}
$$

input
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

output
```
log(1 - d*x - c)^2*(b^2/(8*d*e^3) - b^2/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 +
8*c*d*e^3*x))) + log(c + d*x + 1)^2*(b^2/(8*d*e^3) - b^2/(8*d^2*e^3*(2*c*x
 + d*x^2 + c^2/d))) + log(1 - d*x - c)*(log(c + d*x + 1)*(b^2/(2*d*(2*c^2*
e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x)) - (b^2*(c^2 + d^2*x^2 + 2*c*d*x))/(2*d
*(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x))) + b^2/(2*d*(4*c^2*e^3 + 4*d^2
*e^3*x^2 + 8*c*d*e^3*x)) + (b*(4*a - b))/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 +
 8*c*d*e^3*x)) - (b^2*(x*(4*c*d - d + d*(2*c - 1)) - c + c^2 + c*(2*c - 1)
 + 3*d^2*x^2 + 1))/(2*d*(4*c^2*e^3 + 4*d^2*e^3*x^2 + 8*c*d*e^3*x)) + (b^2*
(x*(2*d*e^3 + d*(4*c*e^3 + 2*e^3) + 8*c*d*e^3) + 2*c*e^3 + 2*e^3 + c*(4*c*
e^3 + 2*e^3) + 2*c^2*e^3 + 6*d^2*e^3*x^2))/(4*d*e^3*(4*c^2*e^3 + 4*d^2*e^3
*x^2 + 8*c*d*e^3*x))) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^2*e^3
*x^2 + 2*c*d*e^3*x) - (log(c + d*x + 1)*(x*((2*b^2*c + b^2)/(4*d*e^3) + (b
^2*c)/(4*d*e^3) - (b^2*(3*c - 1))/(4*d*e^3)) + (4*a*b + b^2*c + b^2 + b^2*
c^2)/(8*d^2*e^3) - (b^2*((c^2 - c + 1)/(2*d) + (c*(2*c - 1))/(2*d)))/(4*d*
e^3) + (c*(2*b^2*c + b^2))/(8*d^2*e^3)))/(2*c*x + d*x^2 + c^2/d) + (b^2*lo
g(c + d*x))/(d*e^3) - (log(c + d*x - 1)*(a*b + b^2))/(2*d*e^3) + (log(c +
d*x + 1)*(a*b - b^2))/(2*d*e^3)
```

3.20. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^3}\,dx$

3.21 $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx$

3.21.1 Optimal result

Integrand size = 23, antiderivative size = 180

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx =& -\frac{b^2}{3de^4(c+dx)}+\frac{b^2\text{arctanh}(c+dx)}{3de^4} \\ &-\frac{b(a+b\text{arctanh}(c+dx))}{3de^4(c+dx)^2} \\ &+\frac{(a+b\text{arctanh}(c+dx))^2}{3de^4}-\frac{(a+b\text{arctanh}(c+dx))^2}{3de^4(c+dx)^3} \\ &+\frac{2b(a+b\text{arctanh}(c+dx))\log\left(2-\frac{2}{1+c+dx}\right)}{3de^4} \\ &-\frac{b^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+c+dx}\right)}{3de^4}\end{aligned}$$

output
```
-1/3*b^2/d/e^4/(d*x+c)+1/3*b^2*arctanh(d*x+c)/d/e^4-1/3*b*(a+b*arctanh(d*x
+c))/d/e^4/(d*x+c)^2+1/3*(a+b*arctanh(d*x+c))^2/d/e^4-1/3*(a+b*arctanh(d*x
+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+b*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^4-
1/3*b^2*polylog(2,-1+2/(d*x+c+1))/d/e^4
```

3.21.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx =$$
$$-\frac{a^2-ab\left(-2\text{arctanh}(c+dx)+(c+dx)\left(-1+c^2+2cdx+d^2x^2+2(c+dx)^2\log\left(\frac{c+dx}{\sqrt{1-(c+dx)^2}}\right)\right)\right)+b^2}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
-1/3*(a^2 - a*b*(-2*ArcTanh[c + d*x] + (c + d*x)*(-1 + c^2 + 2*c*d*x + d^2
*x^2 + 2*(c + d*x)^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]])) + b^2*((c + d*
x)^2 + (c + d*x)^2*ArcTanh[c + d*x]^2 + (1 - (c + d*x)^2)*ArcTanh[c + d*x]
^2 + (c + d*x)*ArcTanh[c + d*x]*(1 - (c + d*x)^2 - (c + d*x)^2*ArcTanh[c +
 d*x] - 2*(c + d*x)^2*Log[1 - E^(-2*ArcTanh[c + d*x])]) + (c + d*x)^3*Poly
Log[2, E^(-2*ArcTanh[c + d*x])]))/(d*e^4*(c + d*x)^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6657, 27, 6452, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx$$
$$\Big\downarrow \text{6657}$$
$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{e^4(c+dx)^4}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^4}d(c+dx)}{de^4}$$
$$\Big\downarrow \text{6452}$$

$$\frac{\frac{2}{3}b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^3(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\big\downarrow$ 6544

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^3}d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\big\downarrow$ 6452

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)+\frac{1}{2}b\int \frac{1}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\big\downarrow$ 264

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{1-(c+dx)^2}d(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\big\downarrow$ 219

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}+\frac{1}{2}b\left(\text{arctanh}(c+dx)-\frac{1}{c+dx}\right)\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$\big\downarrow$ 6550

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(c+dx+1)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}+\frac{1}{2}b\left(\text{arctanh}(c+dx)-\frac{1}{c+dx}\right)\right)-\frac{(a-}{}}{de^4}$$

$\big\downarrow$ 6494

$$\frac{\frac{2}{3}b\left(-b\int \frac{\log\left(2-\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+\right.}{de^4}$$

$\big\downarrow$ 2897

$$\frac{\frac{2}{3}b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{2(c+dx)^2}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))+\frac{1}{2}b\left(\text{arctanh}(c+dx)-\right.\right.}{de^4}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

3.21. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx$

output
```
(-1/3*(a + b*ArcTanh[c + d*x])^2/(c + d*x)^3 + (2*b*((b*(-(c + d*x)^(-1) +
 ArcTanh[c + d*x]))/2 - (a + b*ArcTanh[c + d*x])/(2*(c + d*x)^2) + (a + b*
ArcTanh[c + d*x])^2/(2*b) + (a + b*ArcTanh[c + d*x])*Log[2 - 2/(1 + c + d*
x)] - (b*PolyLog[2, -1 + 2/(1 + c + d*x)])/2))/3)/(d*e^4)
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.71

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{3(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}-\frac{\operatorname{arctanh}(dx+c)}{3(dx+c)^2}+\frac{2\ln(dx+c)\operatorname{arctanh}(dx+c)}{3}-\frac{\operatorname{arctanh}(dx+c)\ln}{3}\right.}{}$
default	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{3(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}-\frac{\operatorname{arctanh}(dx+c)}{3(dx+c)^2}+\frac{2\ln(dx+c)\operatorname{arctanh}(dx+c)}{3}-\frac{\operatorname{arctanh}(dx+c)\ln}{3}\right.}{}$
parts	$-\frac{a^2}{3e^4(dx+c)^3d}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{3(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{3}-\frac{\operatorname{arctanh}(dx+c)}{3(dx+c)^2}+\frac{2\ln(dx+c)\operatorname{arctanh}(dx+c)}{3}-\operatorname{arctanh}(\right.}{}$

input
```
int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3/(d*x+c)^3*arctanh(d*x+c)^2-1/3*a
rctanh(d*x+c)*ln(d*x+c-1)-1/3/(d*x+c)^2*arctanh(d*x+c)+2/3*ln(d*x+c)*arcta
nh(d*x+c)-1/3*arctanh(d*x+c)*ln(d*x+c+1)-1/6*ln(d*x+c-1)+1/6*ln(d*x+c+1)-1
/3/(d*x+c)+1/3*dilog(1/2*d*x+1/2*c+1/2)+1/6*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1
/2)-1/12*ln(d*x+c-1)^2+1/12*ln(d*x+c+1)^2-1/6*(ln(d*x+c+1)-ln(1/2*d*x+1/2*
c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/3*dilog(d*x+c)-1/3*dilog(d*x+c+1)-1/3*ln(
d*x+c)*ln(d*x+c+1))+2*a*b/e^4*(-1/3/(d*x+c)^3*arctanh(d*x+c)-1/6*ln(d*x+c-
1)-1/6/(d*x+c)^2+1/3*ln(d*x+c)-1/6*ln(d*x+c+1)))
```

3.21.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(d^4*e^4*
x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.21.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx = \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^2\,\text{atanh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2ab\,\text{atanh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

input
```
integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**2*atanh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atanh(c + d*x)
/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e
**4
```

3.21.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) + log(d*x + c + 1)/
(d^2*e^4) - 2*log(d*x + c)/(d^2*e^4) + log(d*x + c - 1)/(d^2*e^4)) + 2*arc
tanh(d*x + c)/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4
))*a*b - 1/12*b^2*(log(-d*x - c + 1)^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*
c^2*d^2*e^4*x + c^3*d*e^4) + 3*integrate(-1/3*(3*(d*x + c - 1)*log(d*x + c
 + 1)^2 + 2*(d*x - 3*(d*x + c - 1)*log(d*x + c + 1) + c)*log(-d*x - c + 1)
)/(d^5*e^4*x^5 + c^5*e^4 - c^4*e^4 + (5*c*d^4*e^4 - d^4*e^4)*x^4 + 2*(5*c^
2*d^3*e^4 - 2*c*d^3*e^4)*x^3 + 2*(5*c^3*d^2*e^4 - 3*c^2*d^2*e^4)*x^2 + (5*
c^4*d*e^4 - 4*c^3*d*e^4)*x), x)) - 1/3*a^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2
+ 3*c^2*d^2*e^4*x + c^3*d*e^4)
```

3.21.8 Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(a+b\operatorname{atanh}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.22 $\int \frac{(a+b\textbf{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx$

3.22.1 Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx =& -\frac{b^2}{12de^5(c+dx)^2}-\frac{b(a+b\text{arctanh}(c+dx))}{6de^5(c+dx)^3}\\ &-\frac{b(a+b\text{arctanh}(c+dx))}{2de^5(c+dx)}\\ &+\frac{(a+b\text{arctanh}(c+dx))^2}{4de^5}-\frac{(a+b\text{arctanh}(c+dx))^2}{4de^5(c+dx)^4}\\ &+\frac{2b^2\log(c+dx)}{3de^5}-\frac{b^2\log\left(1-(c+dx)^2\right)}{3de^5}\end{aligned}$$

output
```
-1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctanh(d*x+c))/d/e^5/(d*x+c)^3-1/2*b
*(a+b*arctanh(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctanh(d*x+c))^2/d/e^5-1/4*(
a+b*arctanh(d*x+c))^2/d/e^5/(d*x+c)^4+2/3*b^2*ln(d*x+c)/d/e^5-1/3*b^2*ln(1
-(d*x+c)^2)/d/e^5
```

3.22.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx =$$

$$-\frac{\frac{3a^2}{(c+dx)^4}+\frac{2ab}{(c+dx)^3}+\frac{b^2}{(c+dx)^2}+\frac{6ab}{c+dx}+\frac{2b(3a+b(c+3c^3+dx+9c^2dx+9cd^2x^2+3d^3x^3))\text{arctanh}(c+dx)}{(c+dx)^4}-\frac{3b^2(-1+c^4+4c^3dx+6}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^5,x]
```

output
```
-1/12*((3*a^2)/(c + d*x)^4 + (2*a*b)/(c + d*x)^3 + b^2/(c + d*x)^2 + (6*a*
b)/(c + d*x) + (2*b*(3*a + b*(c + 3*c^3 + d*x + 9*c^2*d*x + 9*c*d^2*x^2 +
3*d^3*x^3))*ArcTanh[c + d*x])/(c + d*x)^4 - (3*b^2*(-1 + c^4 + 4*c^3*d*x +
 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTanh[c + d*x]^2)/(c + d*x)^4 +
b*(3*a + 4*b)*Log[1 - c - d*x] - 8*b^2*Log[c + d*x] - (3*a - 4*b)*b*Log[1
+ c + d*x])/(d*e^5)
```

3.22.3 Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6657, 27, 6452, 6544, 6452, 243, 54, 2009, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{e^5(c+dx)^5}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^5}d(c+dx)}{de^5}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{2}b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^4(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{6544}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^4}d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)+\frac{1}{3}b\int \frac{1}{(c+dx)^3(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)+\frac{1}{6}b\int \frac{1}{(-c-dx+1)(c+dx)^4}d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b\text{arctanh}(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{54}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)+\frac{1}{6}b\int \left(\frac{1}{(c+dx)^2}+\frac{1}{(c+dx)^4}+\frac{1}{-c-dx+1}\right)d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b\text{arc}}{4}}{de^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+dx)^2}-\log(-c-dx+1)+\log\left((c+dx)^2\right)\right)\right)-\frac{(a}{}}{de^5}$$

$$\Big\downarrow \text{6544}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2}d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+dx)^2}-\log(-c-d\right.\right.}{de^5}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+b\int \frac{1}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\right.\right.}{de^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\int \frac{1}{(-c-dx+1)(c+dx)^2}d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(\right.\right.}{de^5}$$

3.22. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx$

$\Big\downarrow$ 47

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\int \frac{1}{(c+dx)^2}d(c+dx)^2\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arc}}{3(}\right.}{de^5}$$

$\Big\downarrow$ 14

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\log\left((c+dx)^2\right)\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arctanh}}{3(c+dx)}\right.}{de^5}$$

$\Big\downarrow$ 16

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log(-c-dx+1)\right.\right.}{de^5}$$

$\Big\downarrow$ 6510

$$\frac{\frac{1}{2}b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{c+dx}-\frac{a+b\text{arctanh}(c+dx)}{3(c+dx)^3}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log(-c-dx+1)\right)+\frac{1}{6}b\left(-\right.\right.}{de^5}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(c*e + d*e*x)^5,x]
```

output
```
(-1/4*(a + b*ArcTanh[c + d*x])^2/(c + d*x)^4 + (b*(-1/3*(a + b*ArcTanh[c +
 d*x])/(c + d*x)^3 - (a + b*ArcTanh[c + d*x])/(c + d*x) + (a + b*ArcTanh[c
 + d*x])^2/(2*b) + (b*(-Log[1 - c - d*x] + Log[(c + d*x)^2]))/2 + (b*(-(c
+ d*x)^(-2) - Log[1 - c - d*x] + Log[(c + d*x)^2]))/6))/2)/(d*e^5)
```

3.22.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.22. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5} dx$

3.22.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{4(dx+c)^4}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}-\frac{\operatorname{arctanh}(dx+c)}{6(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{4}\right.}{}$
default	$-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{4(dx+c)^4}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}-\frac{\operatorname{arctanh}(dx+c)}{6(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{4}\right.}{}$
parts	$-\frac{a^2}{4e^5(dx+c)^4d}+\frac{b^2\left(-\frac{\operatorname{arctanh}(dx+c)^2}{4(dx+c)^4}-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{4}-\frac{\operatorname{arctanh}(dx+c)}{6(dx+c)^3}-\frac{\operatorname{arctanh}(dx+c)}{2(dx+c)}+\frac{\operatorname{arctanh}(dx+c)\ln(dx}{4}\right.}{}$
parallelrisch	$\frac{-3b^2\operatorname{arctanh}(dx+c)^2d^5+8\ln(dx+c)x^4b^2d^9-8\ln(dx+c-1)x^4b^2d^9+8\ln(dx+c)b^2c^4d^5-8\ln(dx+c-1)b^2c^4d^5+3b^2d^9\operatorname{arc}}{}$
risch	Expression too large to display

input
```
int((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a^2/e^5/(d*x+c)^4+b^2/e^5*(-1/4/(d*x+c)^4*arctanh(d*x+c)^2-1/4*a
rctanh(d*x+c)*ln(d*x+c-1)-1/6/(d*x+c)^3*arctanh(d*x+c)-1/2/(d*x+c)*arctanh
(d*x+c)+1/4*arctanh(d*x+c)*ln(d*x+c+1)+1/8*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/
2)-1/16*ln(d*x+c-1)^2-1/16*ln(d*x+c+1)^2+1/8*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c
+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/3*ln(d*x+c-1)-1/12/(d*x+c)^2+2/3*ln(d*x+c)
-1/3*ln(d*x+c+1))+2*a*b/e^5*(-1/4/(d*x+c)^4*arctanh(d*x+c)-1/8*ln(d*x+c-1)
-1/12/(d*x+c)^3-1/4/(d*x+c)+1/8*ln(d*x+c+1)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(158) = 316.

Time = 0.30 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.18

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx =$$
$$-\frac{24\,abd^3x^3+24\,abc^3+4\,(18\,abc+b^2)d^2x^2+4\,b^2c^2+8\,abc+8\,(9\,abc^2+b^2c+ab)dx-3\,(b^2d^4x^4+4\,b^2c}{}$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

output
```
-1/48*(24*a*b*d^3*x^3 + 24*a*b*c^3 + 4*(18*a*b*c + b^2)*d^2*x^2 + 4*b^2*c^
2 + 8*a*b*c + 8*(9*a*b*c^2 + b^2*c + a*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*log(-(d*x + c
+ 1)/(d*x + c - 1))^2 + 12*a^2 - 4*((3*a*b - 4*b^2)*d^4*x^4 + 4*(3*a*b - 4
*b^2)*c*d^3*x^3 + 6*(3*a*b - 4*b^2)*c^2*d^2*x^2 + 4*(3*a*b - 4*b^2)*c^3*d*
x + (3*a*b - 4*b^2)*c^4)*log(d*x + c + 1) - 32*(b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d*x + c) + 4*((3*a*
b + 4*b^2)*d^4*x^4 + 4*(3*a*b + 4*b^2)*c*d^3*x^3 + 6*(3*a*b + 4*b^2)*c^2*d
^2*x^2 + 4*(3*a*b + 4*b^2)*c^3*d*x + (3*a*b + 4*b^2)*c^4)*log(d*x + c - 1)
 + 4*(3*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 + 3*b^2*c^3 + b^2*c + (9*b^2*c^2 + b
^2)*d*x + 3*a*b)*log(-(d*x + c + 1)/(d*x + c - 1)))/(d^5*e^5*x^4 + 4*c*d^4
*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5)
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3516 vs. $2(148) = 296$.

Time = 2.44 (sec) , antiderivative size = 3516, normalized size of antiderivative = 20.44

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(ce + dex)^5}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(d*x+c))**2/(d*e*x+c*e)**5,x)
```

output
```
Piecewise((-3*a**2/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e*
*5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 6*a*b*c**4*atanh(c +
d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c
*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 24*a*b*c**3*d*x*atanh(c + d*x)/(12*
c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**
5*x**3 + 12*d**5*e**5*x**4) - 6*a*b*c**3/(12*c**4*d*e**5 + 48*c**3*d**2*e*
*5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) +
 36*a*b*c**2*d**2*x**2*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*
x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 18
*a*b*c**2*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x*
*2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 24*a*b*c*d**3*x**3*atanh(c
 + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 4
8*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 18*a*b*c*d**2*x**2/(12*c**4*d*e*
*5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 +
12*d**5*e**5*x**4) - 2*a*b*c/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c*
*2*d**3*e**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) + 6*a*b*d**4*
x**4*atanh(c + d*x)/(12*c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e
**5*x**2 + 48*c*d**4*e**5*x**3 + 12*d**5*e**5*x**4) - 6*a*b*d**3*x**3/(12*
c**4*d*e**5 + 48*c**3*d**2*e**5*x + 72*c**2*d**3*e**5*x**2 + 48*c*d**4*e**
5*x**3 + 12*d**5*e**5*x**4) - 2*a*b*d*x/(12*c**4*d*e**5 + 48*c**3*d**2*...
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(158) = 316.

Time = 0.21 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.56

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx =$$

$$-\frac{1}{12}\left(d\left(\frac{2\,(3\,d^2x^2+6\,cdx+3\,c^2+1)}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}-\frac{3\,\log\left(dx+c+1\right)}{d^2e^5}+\frac{3\,\log\left(dx+c-1\right)}{d^2e^5}\right)+\frac{}{d^5e^5x^4}\right.$$

$$-\frac{1}{48}\left(d^2\left(\frac{3\,(d^2x^2+2\,cdx+c^2)\log\left(dx+c+1\right)^2+3\,(d^2x^2+2\,cdx+c^2)\log\left(dx+c-1\right)^2+2\,(8\,d^2x^2+}{d^5}\right.\right.$$

$$-\frac{b^2\,\text{artanh}\left(dx+c\right)^2}{4\,(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5)}$$

$$-\frac{a^2}{4\,(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5)}$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

output
```
-1/12*(d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x
^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) - 3*log(d*x + c + 1)/(d^2*e^5) + 3*log
(d*x + c - 1)/(d^2*e^5)) + 6*arctanh(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x
^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5))*a*b - 1/48*(d^2*((3
*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c + 1)^2 + 3*(d^2*x^2 + 2*c*d*x + c^2
)*log(d*x + c - 1)^2 + 2*(8*d^2*x^2 + 16*c*d*x + 8*c^2 - 3*(d^2*x^2 + 2*c*
d*x + c^2)*log(d*x + c - 1))*log(d*x + c + 1) + 16*(d^2*x^2 + 2*c*d*x + c^
2)*log(d*x + c - 1) + 4)/(d^5*e^5*x^2 + 2*c*d^4*e^5*x + c^2*d^3*e^5) - 32*
log(d*x + c)/(d^3*e^5)) + 4*d*(2*(3*d^2*x^2 + 6*c*d*x + 3*c^2 + 1)/(d^5*e^
5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) - 3*log(d*x + c +
 1)/(d^2*e^5) + 3*log(d*x + c - 1)/(d^2*e^5))*arctanh(d*x + c))*b^2 - 1/4*
b^2*arctanh(d*x + c)^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2
+ 4*c^3*d^2*e^5*x + c^4*d*e^5) - 1/4*a^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 +
6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(158) = 316$.

Time = 0.30 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.24

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(ce + dex)^5}\,dx$$
$$= \frac{1}{12}\left((c+1)d - (c-1)d\right)\left(\frac{3\left(\frac{(dx+c+1)^3b^2}{(dx+c-1)^3} + \frac{(dx+c+1)b^2}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)^2}{\frac{(dx+c+1)^4d^2e^5}{(dx+c-1)^4} + \frac{4(dx+c+1)^3d^2e^5}{(dx+c-1)^3} + \frac{6(dx+c+1)^2d^2e^5}{(dx+c-1)^2} + \frac{4(dx+c+1)d^2e^5}{dx+c-1} + d^2e^5} + \frac{2\left(\frac{6(dx+}{(dx+}\right.}{}\right.$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

output
```
1/12*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b^2/(d*x + c - 1)^3 + (d*
x + c + 1)*b^2/(d*x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))^2/((d*x +
c + 1)^4*d^2*e^5/(d*x + c - 1)^4 + 4*(d*x + c + 1)^3*d^2*e^5/(d*x + c - 1)
^3 + 6*(d*x + c + 1)^2*d^2*e^5/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^5/(
d*x + c - 1) + d^2*e^5) + 2*(6*(d*x + c + 1)^3*a*b/(d*x + c - 1)^3 + 6*(d*
x + c + 1)*a*b/(d*x + c - 1) + 3*(d*x + c + 1)^3*b^2/(d*x + c - 1)^3 + 6*(
d*x + c + 1)^2*b^2/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b^2/(d*x + c - 1) + 2
*b^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^2*e^5/(d*x + c
- 1)^4 + 4*(d*x + c + 1)^3*d^2*e^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2
*e^5/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^5/(d*x + c - 1) + d^2*e^5) +
2*(6*(d*x + c + 1)^3*a^2/(d*x + c - 1)^3 + 6*(d*x + c + 1)*a^2/(d*x + c -
1) + 6*(d*x + c + 1)^3*a*b/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*a*b/(d*x +
 c - 1)^2 + 10*(d*x + c + 1)*a*b/(d*x + c - 1) + 4*a*b + (d*x + c + 1)^3*b
^2/(d*x + c - 1)^3 + 2*(d*x + c + 1)^2*b^2/(d*x + c - 1)^2 + (d*x + c + 1)
*b^2/(d*x + c - 1))/((d*x + c + 1)^4*d^2*e^5/(d*x + c - 1)^4 + 4*(d*x + c
+ 1)^3*d^2*e^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^2*e^5/(d*x + c - 1)^2
 + 4*(d*x + c + 1)*d^2*e^5/(d*x + c - 1) + d^2*e^5) + 4*b^2*log(-(d*x + c
+ 1)/(d*x + c - 1) - 1)/(d^2*e^5) - 4*b^2*log(-(d*x + c + 1)/(d*x + c - 1)
)/(d^2*e^5))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 2746, normalized size of antiderivative = 15.97

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx = \text{Too large to display}$$

input
```
int((a + b*atanh(c + d*x))^2/(c*e + d*e*x)^5,x)
```

output
```
log(1 - d*x - c)^2*(b^2/(16*d*e^5) - b^2/(4*d*(4*c^4*e^5 + 4*d^4*e^5*x^4 +
 16*c*d^3*e^5*x^3 + 24*c^2*d^2*e^5*x^2 + 16*c^3*d*e^5*x))) + log(c + d*x +
 1)^2*(b^2/(16*d*e^5) - b^2/(16*d^2*e^5*(4*c^3*x + c^4/d + d^3*x^4 + 6*c^2
*d*x^2 + 4*c*d^2*x^3))) + log(1 - d*x - c)*(log(c + d*x + 1)*(b^2/(4*d*(2*
c^4*e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*c^3*d*e
^5*x)) - (b^2*(c^4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x))/(
4*d*(2*c^4*e^5 + 2*d^4*e^5*x^4 + 8*c*d^3*e^5*x^3 + 12*c^2*d^2*e^5*x^2 + 8*
c^3*d*e^5*x))) + (3*b^2)/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*
x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) + (3*b*(8*a - b))/(4*d*(24*c^
4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3 + 144*c^2*d^2*e^5*x^2 + 96*c^3*d
*e^5*x)) - (b^2*(c*(2*c - 3*c^2 + 4*c^3 + c*(6*c^2 - 3*c + c*(12*c - 3) +
1) - 1) - 3*c + x^2*(d*(2*d - 6*c*d + 12*c^2*d + d*(6*c^2 - 3*c + c*(12*c
- 3) + 1) + c*(24*c*d - 3*d + d*(12*c - 3))) - 9*c*d^2 + c*(30*c*d^2 - 3*d
^2 + d*(24*c*d - 3*d + d*(12*c - 3))) + 3*d^2 + 18*c^2*d^2) + x*(d*(2*c -
3*c^2 + 4*c^3 + c*(6*c^2 - 3*c + c*(12*c - 3) + 1) - 1) - 3*d + 6*c*d + c*
(2*d - 6*c*d + 12*c^2*d + d*(6*c^2 - 3*c + c*(12*c - 3) + 1) + c*(24*c*d -
 3*d + d*(12*c - 3))) - 9*c^2*d + 12*c^3*d) + 3*c^2 - 3*c^3 + 3*c^4 + 25*d
^4*x^4 + x^3*(34*c*d^3 + d*(30*c*d^2 - 3*d^2 + d*(24*c*d - 3*d + d*(12*c -
 3))) - 3*d^3) + 3))/(4*d*(24*c^4*e^5 + 24*d^4*e^5*x^4 + 96*c*d^3*e^5*x^3
+ 144*c^2*d^2*e^5*x^2 + 96*c^3*d*e^5*x)) + (b^2*(c*(c*(6*c*e^5 + 2*e^5 ...
```

3.22. $\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{(ce+dex)^5}\,dx$

3.23 $\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$

3.23.1 Optimal result

Integrand size = 23, antiderivative size = 263

$$\begin{aligned}&\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx\\&=ab^2e^2x+\frac{b^3e^2(c+dx)\text{arctanh}(c+dx)}{d}-\frac{be^2(a+b\text{arctanh}(c+dx))^2}{2d}\\&\quad+\frac{be^2(c+dx)^2(a+b\text{arctanh}(c+dx))^2}{2d}\\&\quad+\frac{e^2(a+b\text{arctanh}(c+dx))^3}{3d}+\frac{e^2(c+dx)^3(a+b\text{arctanh}(c+dx))^3}{3d}\\&\quad-\frac{be^2(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2}{1-c-dx}\right)}{d}+\frac{b^3e^2\log\left(1-(c+dx)^2\right)}{2d}\\&\quad-\frac{b^2e^2(a+b\text{arctanh}(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d}+\frac{b^3e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d}\end{aligned}$$

output
```
a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctanh(d*x+c)/d-1/2*b*e^2*(a+b*arctanh(d*x+c)
)^2/d+1/2*b*e^2*(d*x+c)^2*(a+b*arctanh(d*x+c))^2/d+1/3*e^2*(a+b*arctanh(d*
x+c))^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctanh(d*x+c))^3/d-b*e^2*(a+b*arctanh(d*
x+c))^2*ln(2/(-d*x-c+1))/d+1/2*b^3*e^2*ln(1-(d*x+c)^2)/d-b^2*e^2*(a+b*arct
anh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d+1/2*b^3*e^2*polylog(3,1-2/(-d*x-c+
1))/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.28

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$$
$$=\frac{e^2\Big(3a^2b(c+dx)^2+2a^3(c+dx)^3+6a^2b(c+dx)^3\text{arctanh}(c+dx)+3a^2b\log\left(1-(c+dx)^2\right)+6ab^2(c+d$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(e^2*(3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTan
h[c + d*x] + 3*a^2*b*Log[1 - (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTanh[c +
 d*x] + (c + d*x)^2*ArcTanh[c + d*x] - ArcTanh[c + d*x]^2 + (c + d*x)^3*Ar
cTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*Log[1 + E^(-2*ArcTanh[c + d*x])] + P
olyLog[2, -E^(-2*ArcTanh[c + d*x])]) + b^3*(6*(c + d*x)*ArcTanh[c + d*x] -
 3*(1 - (c + d*x)^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]^3 + 2*(c + d*
x)*ArcTanh[c + d*x]^3 - 2*(c + d*x)*(1 - (c + d*x)^2)*ArcTanh[c + d*x]^3 -
 6*ArcTanh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*Log[1/Sqrt[1 -
(c + d*x)^2]] + 6*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] +
3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])))/(6*d)
```

3.23.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6657, 27, 6452, 6542, 6452, 6542, 2009, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{ 6657}$$
$$\frac{\int e^2(c+dx)^2(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e^2\int (c+dx)^2(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d}$$

$$\downarrow 6452$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\int \frac{(c+dx)^3(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$$\downarrow 6542$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)-\int (c+dx)(a+b\text{arctanh}(c+dx\right.\right.}{d}$$

$$\downarrow 6452$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(b\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(\right.\right.}{d}$$

$$\downarrow 6542$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\int (a+b\text{arctanh}(c+dx))d(c+dx)\right)\right.\right.}{d}$$

$$\downarrow 2009$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)+b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx\right.\right.\right.}{d}$$

$$\downarrow 6510$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+\right.\right.}{d}$$

$$\downarrow 6546$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{-c-dx+1}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-\frac{1}{2}(c+dx)^2(a\right.\right.}{}$$

$$\downarrow 6470$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{arctanh}(c+dx))^3-b\left(-2b\int \frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{3b}\right.\right.}{}$$

$$\downarrow 6620$$

3.23. $\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\operatorname{arctanh}(c+dx))^3-b\left(-2b\left(\frac{1}{2}b\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}\operatorname{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right.\right.\right.\right.$$

$\downarrow$ 7164

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\operatorname{arctanh}(c+dx))^3-b\left(-2b\left(\frac{1}{4}b\operatorname{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTanh[c + d*x])^3)/3 - b*(-1/2*((c + d*x)^2*(a
 + b*ArcTanh[c + d*x])^2) - (a + b*ArcTanh[c + d*x])^3/(3*b) + (a + b*ArcT
anh[c + d*x])^2*Log[2/(1 - c - d*x)] + b*(-(a*(c + d*x)) - b*(c + d*x)*Arc
Tanh[c + d*x] + (a + b*ArcTanh[c + d*x])^2/(2*b) - (b*Log[1 - (c + d*x)^2]
)/2) - 2*b*(-1/2*((a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)]
) + (b*PolyLog[3, 1 - 2/(1 - c - d*x)])/4))))/d
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.23. $\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$

3.23.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.06 (sec) , antiderivative size = 1079, normalized size of antiderivative = 4.10

method	result	size
derivativedivides	Expression too large to display	1079
default	Expression too large to display	1079
parts	Expression too large to display	1087

input
```
int((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arctanh(d*x+c)^3+1/2*(d*
x+c)^2*arctanh(d*x+c)^2+1/2*arctanh(d*x+c)^2*ln(d*x+c-1)+1/2*arctanh(d*x+c
)^2*ln(d*x+c+1)-arctanh(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-arctanh
(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/2*polylog(3,-(d*x+c+1)^2/(
1-(d*x+c)^2))-1/12*arctanh(d*x+c)*(6*I*arctanh(d*x+c)*Pi*csgn(I*(d*x+c+1)/
(1-(d*x+c)^2)^(1/2))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2+6*I*arctanh(d*x+c
)*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^3+3*I*csgn(I*(d*x+c+1)^2/((d*x+
c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2*arctanh(d*x+c)*Pi*csgn(I/(1-(d*x+
c+1)^2/((d*x+c)^2-1)))+6*I*arctanh(d*x+c)*Pi-3*I*csgn(I*(d*x+c+1)^2/((d*x+
c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2*arctanh(d*x+c)*Pi*csgn(I*(d*x+c+1
)^2/((d*x+c)^2-1))+3*I*arctanh(d*x+c)*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))
^3+3*I*arctanh(d*x+c)*Pi*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2*csgn(I*(d
*x+c+1)^2/((d*x+c)^2-1))-6*I*arctanh(d*x+c)*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x
+c)^2-1)))^2-3*I*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^
2-1)))*arctanh(d*x+c)*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(d*x
+c+1)^2/((d*x+c)^2-1))+3*I*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+1)^2
/((d*x+c)^2-1)))^3*arctanh(d*x+c)*Pi-4*arctanh(d*x+c)^2+12*arctanh(d*x+c)*
ln(2)+6*arctanh(d*x+c)-12*d*x-12*c-12)-ln(1+(d*x+c+1)^2/(1-(d*x+c)^2)))+3*
e^2*a*b^2*(1/3*(d*x+c)^3*arctanh(d*x+c)^2+1/3*(d*x+c)^2*arctanh(d*x+c)+1/3
*arctanh(d*x+c)*ln(d*x+c-1)+1/3*arctanh(d*x+c)*ln(d*x+c+1)+1/3*d*x+1/3*...
```

3.23.5 Fricas [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^
2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctanh(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x
^2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctanh(d*x + c)^2 + 3*(a^2*b*d^2*
e^2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctanh(d*x + c), x)
```

3.23.6 Sympy [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx$$
$$=e^2\left(\int a^3c^2\,dx+\int a^3d^2x^2\,dx+\int b^3c^2\,\text{atanh}^3\,(c+dx)\,dx+\int 3ab^2c^2\,\text{atanh}^2\,(c+dx)\,dx\right.$$
$$+\int 3a^2bc^2\,\text{atanh}\,(c+dx)\,dx+\int 2a^3cdx\,dx+\int b^3d^2x^2\,\text{atanh}^3\,(c+dx)\,dx$$
$$+\int 3ab^2d^2x^2\,\text{atanh}^2\,(c+dx)\,dx+\int 3a^2bd^2x^2\,\text{atanh}\,(c+dx)\,dx$$
$$+\int 2b^3cdx\,\text{atanh}^3\,(c+dx)\,dx+\int 6ab^2cdx\,\text{atanh}^2\,(c+dx)\,dx$$
$$\left.+\int 6a^2bcdx\,\text{atanh}\,(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atanh(d*x+c))**3,x)
```

output
```
e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3
*c**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atanh(c + d*x)**2, x)
 + Integral(3*a**2*b*c**2*atanh(c + d*x), x) + Integral(2*a**3*c*d*x, x) +
 Integral(b**3*d**2*x**2*atanh(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**
*2*atanh(c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atanh(c + d*x), x)
+ Integral(2*b**3*c*d*x*atanh(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*at
anh(c + d*x)**2, x) + Integral(6*a**2*b*c*d*x*atanh(c + d*x), x))
```

3.23.7 Maxima [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*(2
*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x +
c - 1)/d^3))*a^2*b*c*d*e^2 + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c
*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*
c - 1)*log(d*x + c - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3/2*(2*(d*x
+ c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*c^2*e^2/d - 1/24*((b^
3*d^3*e^2*x^3 + 3*b^3*c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + (c^3*e^2 - e^2)*
b^3)*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d^3*e^2*x^3 + (6*a*b^2*c*d^2*e^2 + b
^3*d^2*e^2)*x^2 + 2*(3*a*b^2*c^2*d*e^2 + b^3*c*d*e^2)*x + (b^3*d^3*e^2*x^3
 + 3*b^3*c*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + (c^3*e^2 + e^2)*b^3)*log(d*x
+ c + 1))*log(-d*x - c + 1)^2)/d - integrate(-1/8*((b^3*d^3*e^2*x^3 + (3*c
*d^2*e^2 - d^2*e^2)*b^3*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*b^3*x + (c^3*e^2 -
 c^2*e^2)*b^3)*log(d*x + c + 1)^3 + 6*(a*b^2*d^3*e^2*x^3 + (3*c*d^2*e^2 -
d^2*e^2)*a*b^2*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*a*b^2*x + (c^3*e^2 - c^2*e^
2)*a*b^2)*log(d*x + c + 1)^2 - (4*a*b^2*d^3*e^2*x^3 + 2*(6*a*b^2*c*d^2*e^2
 + b^3*d^2*e^2)*x^2 + 3*(b^3*d^3*e^2*x^3 + (3*c*d^2*e^2 - d^2*e^2)*b^3*x^2
 + (3*c^2*d*e^2 - 2*c*d*e^2)*b^3*x + (c^3*e^2 - c^2*e^2)*b^3)*log(d*x + c
+ 1)^2 + 4*(3*a*b^2*c^2*d*e^2 + b^3*c*d*e^2)*x + 2*(6*(c^3*e^2 - c^2*e^2)*
a*b^2 + (c^3*e^2 + e^2)*b^3 + (6*a*b^2*d^3*e^2 + b^3*d^3*e^2)*x^3 + 3*(b^3
*c*d^2*e^2 + 2*(3*c*d^2*e^2 - d^2*e^2)*a*b^2)*x^2 + 3*(b^3*c^2*d*e^2 + ...
```

3.23.8 Giac [F]

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx=\int (dex+ce)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)^2*(b*arctanh(d*x + c) + a)^3, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\text{arctanh}(c+dx))^3\,dx=\int (c\,e+d\,e\,x)^2\,(a+b\,\text{atanh}(c+d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*atanh(c + d*x))^3, x)
```

3.24 $\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx$

3.24.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx = \frac{3be(a+b\text{arctanh}(c+dx))^2}{2d} + \frac{3be(c+dx)(a+b\text{arctanh}(c+dx))^2}{2d} - \frac{e(a+b\text{arctanh}(c+dx))^3}{2d} + \frac{e(c+dx)^2(a+b\text{arctanh}(c+dx))^3}{2d} - \frac{3b^2e(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d} - \frac{3b^3e\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2d}$$

output
```
3/2*b*e*(a+b*arctanh(d*x+c))^2/d+3/2*b*e*(d*x+c)*(a+b*arctanh(d*x+c))^2/d-
1/2*e*(a+b*arctanh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctanh(d*x+c))^3/d-3*
b^2*e*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d-3/2*b^3*e*polylog(2,(-d*x-c-
1)/(-d*x-c+1))/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.38

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx$$
$$=\frac{e\Big(6b^2(-1+c+dx)(b+a(1+c+dx))\text{arctanh}(c+dx)^2+2b^3(-1+c^2+2cdx+d^2x^2)\,\text{arctanh}(c+dx)^3}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(e*(6*b^2*(-1 + c + d*x)*(b + a*(1 + c + d*x))*ArcTanh[c + d*x]^2 + 2*b^3*
(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^3 + 6*b*ArcTanh[c + d*x]*(
a*(c + d*x)*(2*b + a*c + a*d*x) - 2*b^2*Log[1 + E^(-2*ArcTanh[c + d*x])])
+ a*(6*a*b*c + 2*a^2*c^2 + 6*a*b*d*x + 4*a^2*c*d*x + 2*a^2*d^2*x^2 + 3*a*b
*Log[1 - c - d*x] - 3*a*b*Log[1 + c + d*x] - 12*b^2*Log[1/Sqrt[1 - (c + d*
x)^2]]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/(4*d)
```

3.24.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6657, 27, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx$$
$$\Big\downarrow \text{6657}$$
$$\frac{\int e(c+dx)(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{e\int (c+dx)(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \text{6452}$$
$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\int \frac{(c+dx)^2(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)\right)}{d}$$

$\downarrow$ 6542

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(\int\frac{(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)-\int(a+b\text{arctanh}(c+dx))^2d(c+dx)\right.}{d}$$

$\downarrow$ 6436

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\int\frac{(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+\right.}{d}$$

$\downarrow$ 6510

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\int\frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-(c+d\right.}{d}$$

$\downarrow$ 6546

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\left(\int\frac{a+b\text{arctanh}(c+dx)}{-c-dx+1}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}\right)+\frac{(a+b\text{arctan}}{3b}\right.}{d}$$

$\downarrow$ 6470

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\left(-b\int\frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+}\right.\right.\right.}{d}$$

$\downarrow$ 2849

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\left(b\int\frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-\frac{2}{-c-dx+1}}d\frac{1}{-c-dx+1}-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)\right.\right.\right.}{d}$$

$\downarrow$ 2752

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\text{arctanh}(c+dx))^3-\frac{3}{2}b\left(2b\left(-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))+\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

3.24. $\int(ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx$

output
```
(e*(((c + d*x)^2*(a + b*ArcTanh[c + d*x])^3)/2 - (3*b*(-((c + d*x)*(a + b*
ArcTanh[c + d*x])^2) + (a + b*ArcTanh[c + d*x])^3/(3*b) + 2*b*(-1/2*(a + b
*ArcTanh[c + d*x])^2/b + (a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)] + (
b*PolyLog[2, 1 - 2/(1 - c - d*x)])/2)))/2))/d
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(152) = 304$.

Time = 0.65 (sec) , antiderivative size = 932, normalized size of antiderivative = 5.82

method	result
risch	$\frac{3b^3e\,\mathrm{dilog}\left(-\frac{dx}{2}-\frac{c}{2}+\frac{1}{2}\right)}{2d}-\frac{3ed\ln(-dx-c+1)a^2b\,x^2}{4}+\frac{3e\ln(-dx-c+1)^2a\,b^2cx}{4}-\frac{3e\ln(-dx-c+1)a^2bcx}{2}-\frac{3ea\,b}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/2*b^3*e/d*dilog(-1/2*d*x-1/2*c+1/2)-3/4*e*d*ln(-d*x-c+1)*a^2*b*x^2+3/4*e
*ln(-d*x-c+1)^2*a*b^2*c*x-3/2*e*ln(-d*x-c+1)*a^2*b*c*x-3/2*e/d*a*b^2*ln(-d
*x-c+1)*c+3/8*e/d*ln(-d*x-c+1)^2*a*b^2*c^2-3/4*e/d*ln(-d*x-c+1)*a^2*b*c^2+
3/8*e*d*ln(-d*x-c+1)^2*a*b^2*x^2+3/4*e*a^2*b/d*ln(-d*x-c-1)*c^2+3/2*e*a*b^
2/d*ln(-d*x-c-1)*c+e*a^3*c*x-1/16*e*d*ln(-d*x-c+1)^3*b^3*x^2-1/16*e/d*ln(-
d*x-c+1)^3*b^3*c^2+3/2*e/d*ln(-d*x-c+1)*a*b^2-3/8*e/d*a*b^2*ln(-d*x-c+1)^2
+3/4*e/d*a^2*b*ln(-d*x-c+1)+3/8*e/d*b^3*ln(-d*x-c+1)^2*c-3/2*e*a*b^2*ln(-d
*x-c+1)*x-1/8*e*ln(-d*x-c+1)^3*b^3*c*x+1/2*e*d*a^3*x^2+3/8*e*b^3*ln(-d*x-c
+1)^2*x-3/8*e/d*ln(-d*x-c+1)^2*b^3+1/16*e/d*b^3*ln(-d*x-c+1)^3+3/2*e*a^2*b
*x+1/16*e*b^3*(d^2*x^2+2*c*d*x+c^2-1)/d*ln(d*x+c+1)^3+3/16*e*b^2*(-b*d^2*x
^2*ln(-d*x-c+1)+2*a*d^2*x^2-2*b*c*d*x*ln(-d*x-c+1)+4*a*c*d*x-ln(-d*x-c+1)*
b*c^2+2*a*c^2+2*b*d*x+2*b*c+b*ln(-d*x-c+1)-2*a+2*b)/d*ln(d*x+c+1)^2-1/2*e/
d*a^3-3/4*e*a^2*b/d*ln(-d*x-c-1)+3/2*e*a*b^2/d*ln(-d*x-c-1)+3/2*b^3*e/d*ln
(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)-3/2*b^3*e/d*ln(1/2*d*x+1/2*c+1/
2)*ln(-d*x-c+1)+(3/16*e*b^3*(d^2*x^2+2*c*d*x+c^2-1)/d*ln(-d*x-c+1)^2-3/4*e
*b^2*x*(a*d*x+2*a*c+b)*ln(-d*x-c+1)-3/4*b*e*(-a^2*d^2*x^2-2*a^2*c*d*x+ln(-
d*x-c+1)*a*b*c^2-2*a*b*d*x+b^2*c*ln(-d*x-c+1)-ln(-d*x-c+1)*a*b-ln(-d*x-c+1
)*b^2)/d)*ln(d*x+c+1)+1/2*e/d*a^3*c^2-3/2*e/d*a^2*b+3/2*e/d*a^2*b*c
```

3.24.5 Fricas [F]

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx=\int (dex+ce)(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctanh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arctanh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*
e)*arctanh(d*x + c), x)
```

3.24.6 Sympy [F]

$$\int (ce + dex)(a + b\operatorname{arctanh}(c + dx))^3\,dx = e\left(\int a^3c\,dx + \int a^3dx\,dx + \int b^3c\operatorname{atanh}^3(c + dx)\,dx + \int 3ab^2c\operatorname{atanh}^2(c + dx)\,dx + \int 3a^2bc\operatorname{atanh}(c + dx)\,dx + \int b^3dx\operatorname{atanh}^3(c + dx)\,dx + \int 3ab^2dx\operatorname{atanh}^2(c + dx)\,dx + \int 3a^2bdx\operatorname{atanh}(c + dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))**3,x)
```

output
```
e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atanh(c +
 d*x)**3, x) + Integral(3*a*b**2*c*atanh(c + d*x)**2, x) + Integral(3*a**2
*b*c*atanh(c + d*x), x) + Integral(b**3*d*x*atanh(c + d*x)**3, x) + Integr
al(3*a*b**2*d*x*atanh(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atanh(c + d*
x), x))
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(142) = 284$.

Time = 0.40 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.93

$$\int (ce + dex)(a + b\text{arctanh}(c + dx))^3\,dx = \frac{1}{2}a^3dex^2 + \frac{3}{4}\left(2x^2\,\text{artanh}(dx + c) + d\left(\frac{2x}{d^2} - \frac{(c^2 + 2c + 1)\log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1)\log(dx + c - 1)}{d^3}\right)\right) + a^3cex + \frac{3\left(2(dx + c)\,\text{artanh}(dx + c) + \log\left(-(dx + c)^2 + 1\right)\right)a^2bce}{2d} + \frac{3\left(\log(dx + c + 1)\log\left(-\frac{1}{2}dx - \frac{1}{2}c + \frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}dx + \frac{1}{2}c + \frac{1}{2}\right)\right)b^3e}{2d} + \frac{3(ce + e)ab^2\log(dx + c + 1)}{2d} - \frac{3(ce - e)ab^2\log(dx + c - 1)}{2d} + \frac{24\,ab^2dex\log(dx + c + 1) + (b^3d^2ex^2 + 2b^3cdex + (c^2e - e)b^3)\log(dx + c + 1)^3 - (b^3d^2ex^2 + 2b^3cdex}{}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c +
1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*d*e
 + a^3*c*e*x + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*
a^2*b*c*e/d + 3/2*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/
2*d*x + 1/2*c + 1/2))*b^3*e/d + 3/2*(c*e + e)*a*b^2*log(d*x + c + 1)/d - 3
/2*(c*e - e)*a*b^2*log(d*x + c - 1)/d + 1/16*(24*a*b^2*d*e*x*log(d*x + c +
 1) + (b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (c^2*e - e)*b^3)*log(d*x + c + 1)^3
 - (b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (c^2*e - e)*b^3)*log(-d*x - c + 1)^3 +
 6*(a*b^2*d^2*e*x^2 + (c^2*e - e)*a*b^2 + (c*e + e)*b^3 + (2*a*b^2*c*d*e +
 b^3*d*e)*x)*log(d*x + c + 1)^2 + 3*(2*a*b^2*d^2*e*x^2 + 2*(c^2*e - e)*a*b
^2 + 2*(c*e - e)*b^3 + 2*(2*a*b^2*c*d*e + b^3*d*e)*x + (b^3*d^2*e*x^2 + 2*
b^3*c*d*e*x + (c^2*e - e)*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)^2 - 3*(
8*a*b^2*d*e*x + (b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (c^2*e - e)*b^3)*log(d*x
+ c + 1)^2 + 4*(a*b^2*d^2*e*x^2 + (c^2*e - e)*a*b^2 + (c*e + e)*b^3 + (2*a
*b^2*c*d*e + b^3*d*e)*x)*log(d*x + c + 1))*log(-d*x - c + 1))/d
```

3.24.8 Giac [F]

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx = \int (dex+ce)(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((d*e*x + c*e)*(b*arctanh(d*x + c) + a)^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)(a+b\text{arctanh}(c+dx))^3\,dx = \int (c\,e+d\,e\,x)\,\left(a+b\,\text{atanh}(c+d\,x)\right)^3\,dx$$

input
```
int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)*(a + b*atanh(c + d*x))^3, x)
```

3.25 $\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}\,dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 257

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}\,dx = {} & \frac{2(a+b\text{arctanh}(c+dx))^3\text{arctanh}\left(1-\frac{2}{1-c-dx}\right)}{de} \\ & - \frac{3b(a+b\text{arctanh}(c+dx))^2\,\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{2de} \\ & + \frac{3b(a+b\text{arctanh}(c+dx))^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-c-dx}\right)}{2de} \\ & + \frac{3b^2(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2de} \\ & - \frac{3b^2(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(3,-1+\frac{2}{1-c-dx}\right)}{2de} \\ & - \frac{3b^3\,\text{PolyLog}\left(4,1-\frac{2}{1-c-dx}\right)}{4de} \\ & + \frac{3b^3\,\text{PolyLog}\left(4,-1+\frac{2}{1-c-dx}\right)}{4de}\end{aligned}$$

output
```
-2*(a+b*arctanh(d*x+c))^3*arctanh(-1+2/(-d*x-c+1))/d/e-3/2*b*(a+b*arctanh(
d*x+c))^2*polylog(2,1-2/(-d*x-c+1))/d/e+3/2*b*(a+b*arctanh(d*x+c))^2*polyl
og(2,-1+2/(-d*x-c+1))/d/e+3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2/(-d*x
-c+1))/d/e-3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,-1+2/(-d*x-c+1))/d/e-3/4
*b^3*polylog(4,1-2/(-d*x-c+1))/d/e+3/4*b^3*polylog(4,-1+2/(-d*x-c+1))/d/e
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.26

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}\,dx$$
$$= \frac{4a^3\log(c+dx)+12a^2b\text{arctanh}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+\log\left(\frac{i(c+dx)}{\sqrt{1-(c+dx)^2}}\right)\right)-\frac{3}{2}a^2b\Big(\pi^2-4i\pi\text{arctan}}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(4*a^3*Log[c + d*x] + 12*a^2*b*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)
^2]] + Log[(I*(c + d*x))/Sqrt[1 - (c + d*x)^2]]) - (3*a^2*b*(Pi^2 - (4*I)*
Pi*ArcTanh[c + d*x] - 8*ArcTanh[c + d*x]^2 - 8*ArcTanh[c + d*x]*Log[1 - E^
(-2*ArcTanh[c + d*x])] + (4*I)*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] + 8*ArcT
anh[c + d*x]*Log[1 + E^(2*ArcTanh[c + d*x])] - (4*I)*Pi*Log[2/Sqrt[1 - (c
+ d*x)^2]] - 8*ArcTanh[c + d*x]*Log[2/Sqrt[1 - (c + d*x)^2]] + 8*ArcTanh[c
 + d*x]*Log[((2*I)*(c + d*x))/Sqrt[1 - (c + d*x)^2]] + 4*PolyLog[2, E^(-2*
ArcTanh[c + d*x])] + 4*PolyLog[2, -E^(2*ArcTanh[c + d*x])]))/2 + 6*a*b^2*(
2*ArcTanh[c + d*x]^2*Log[1 - E^(-2*ArcTanh[c + d*x])] - 2*ArcTanh[c + d*x]
^2*Log[1 + E^(-2*ArcTanh[c + d*x])] + 2*ArcTanh[c + d*x]*PolyLog[2, -E^(-2
*ArcTanh[c + d*x])] - 2*ArcTanh[c + d*x]*PolyLog[2, E^(-2*ArcTanh[c + d*x]
)] + PolyLog[3, -E^(-2*ArcTanh[c + d*x])] - PolyLog[3, E^(-2*ArcTanh[c + d
*x])]) + b^3*(4*ArcTanh[c + d*x]^3*Log[1 - E^(-2*ArcTanh[c + d*x])] - 4*Ar
cTanh[c + d*x]^3*Log[1 + E^(-2*ArcTanh[c + d*x])] + 6*ArcTanh[c + d*x]^2*P
olyLog[2, -E^(-2*ArcTanh[c + d*x])] - 6*ArcTanh[c + d*x]^2*PolyLog[2, E^(-
2*ArcTanh[c + d*x])] + 6*ArcTanh[c + d*x]*PolyLog[3, -E^(-2*ArcTanh[c + d*
x])] - 6*ArcTanh[c + d*x]*PolyLog[3, E^(-2*ArcTanh[c + d*x])] + 3*PolyLog[
4, -E^(-2*ArcTanh[c + d*x])] - 3*PolyLog[4, E^(-2*ArcTanh[c + d*x])]))/(4*
d*e)
```

3.25.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6657, 27, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{6448}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^3-6b\int \frac{(a+b\text{arctanh}(c+dx))^2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)}{de}$$

$$\Big\downarrow \text{6614}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^3-6b\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(c+dx))^2\log\left(2-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}\int \frac{(a+\cdots}{\cdots}\right.}{de}$$

$$\Big\downarrow \text{6620}$$

$$2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-\cdots\right.\right.$$

$$\Big\downarrow \text{6624}$$

$$2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-\cdots\right.\right.$$

$\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))^2-\right.\right.$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(2*(a + b*ArcTanh[c + d*x])^3*ArcTanh[1 - 2/(1 - c - d*x)] - 6*b*((((a + b
*ArcTanh[c + d*x])^2*PolyLog[2, 1 - 2/(1 - c - d*x)])/2 - b*(((a + b*ArcTa
nh[c + d*x])*PolyLog[3, 1 - 2/(1 - c - d*x)])/2 - (b*PolyLog[4, 1 - 2/(1 -
 c - d*x)])/4))/2 + (-1/2*((a + b*ArcTanh[c + d*x])^2*PolyLog[2, -1 + 2/(1
 - c - d*x)]) + b*(((a + b*ArcTanh[c + d*x])*PolyLog[3, -1 + 2/(1 - c - d*
x)])/2 - (b*PolyLog[4, -1 + 2/(1 - c - d*x)])/4))/2))/(d*e)
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.25.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 1442, normalized size of antiderivative = 5.61

method	result	size
derivativedivides	Expression too large to display	1442
default	Expression too large to display	1442
parts	Expression too large to display	1450

input
```
int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3/e*ln(d*x+c)+b^3/e*(ln(d*x+c)*arctanh(d*x+c)^3-arctanh(d*x+c)^3*ln
((d*x+c+1)^2/(1-(d*x+c)^2)-1)+arctanh(d*x+c)^3*ln(1-(d*x+c+1)/(1-(d*x+c)^2
)^(1/2))+3*arctanh(d*x+c)^2*polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*arc
tanh(d*x+c)*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+6*polylog(4,(d*x+c+1)
/(1-(d*x+c)^2)^(1/2))+arctanh(d*x+c)^3*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))
+3*arctanh(d*x+c)^2*polylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-6*arctanh(d*
x+c)*polylog(3,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+6*polylog(4,-(d*x+c+1)/(1-(
d*x+c)^2)^(1/2))+1/2*I*Pi*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+
1)^2/((d*x+c)^2-1)))*(csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csgn(I/(1-(d*
x+c+1)^2/((d*x+c)^2-1)))-csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csgn(I*(-(
d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))-csgn(I*(-(d*x+c
+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I/(1-(d*x+c+1)^
2/((d*x+c)^2-1)))+csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d
*x+c)^2-1)))^2)*arctanh(d*x+c)^3-3/2*arctanh(d*x+c)^2*polylog(2,-(d*x+c+1)
^2/(1-(d*x+c)^2))+3/2*arctanh(d*x+c)*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2))
-3/4*polylog(4,-(d*x+c+1)^2/(1-(d*x+c)^2)))+3*a*b^2/e*(ln(d*x+c)*arctanh(d
*x+c)^2-arctanh(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/2*polylog(3
,-(d*x+c+1)^2/(1-(d*x+c)^2))-arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)
-1)+arctanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*arctanh(d*x+c)*
polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*polylog(3,(d*x+c+1)/(1-(d*x+...
```

3.25.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

3.25.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{ce + dex}\, dx = \frac{\int \frac{a^3}{c+dx}\, dx + \int \frac{b^3 \operatorname{atanh}^3(c+dx)}{c+dx}\, dx + \int \frac{3ab^2 \operatorname{atanh}^2(c+dx)}{c+dx}\, dx + \int \frac{3a^2 b \operatorname{atanh}(c+dx)}{c+dx}\, dx}{e}$$

input
```
integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e),x)
```

output
```
(Integral(a**3/(c + d*x), x) + Integral(b**3*atanh(c + d*x)**3/(c + d*x),
x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b
*atanh(c + d*x)/(c + d*x), x))/e
```

3.25.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{ce + dex}\, dx = \int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{dex + ce}\, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^3*log(d*e*x + c*e)/(d*e) + integrate(1/8*b^3*(log(d*x + c + 1) - log(-d*
x - c + 1))^3/(d*e*x + c*e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c +
 1))^2/(d*e*x + c*e) + 3/2*a^2*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(d
*e*x + c*e), x)
```

3.25.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{ce + dex}\, dx = \int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{dex + ce}\, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{ce+dex}\,dx = \int \frac{(a+b\,\text{atanh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x),x)
```

output
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x), x)
```

3.26 $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx$

3.26.1 Optimal result

Integrand size = 23, antiderivative size = 143

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx &= \frac{(a+b\text{arctanh}(c+dx))^3}{de^2}-\frac{(a+b\text{arctanh}(c+dx))^3}{de^2(c+dx)}\\&\quad+\frac{3b(a+b\text{arctanh}(c+dx))^2\log\left(2-\frac{2}{1+c+dx}\right)}{de^2}\\&\quad-\frac{3b^2(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+c+dx}\right)}{de^2}\\&\quad-\frac{3b^3\,\text{PolyLog}\left(3,-1+\frac{2}{1+c+dx}\right)}{2de^2}\end{aligned}$$

output
```
(a+b*arctanh(d*x+c))^3/d/e^2-(a+b*arctanh(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b
*arctanh(d*x+c))^2*ln(2-2/(d*x+c+1))/d/e^2-3*b^2*(a+b*arctanh(d*x+c))*poly
log(2,-1+2/(d*x+c+1))/d/e^2-3/2*b^3*polylog(3,-1+2/(d*x+c+1))/d/e^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.62

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx$$
$$= \frac{-\frac{2a^3}{c+dx}-\frac{6a^2b\text{arctanh}(c+dx)}{c+dx}+6a^2b\log(c+dx)-3a^2b\log\left(1-c^2-2cdx-d^2x^2\right)+6ab^2\left(\text{arctanh}(c+dx)\,(\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTanh[c + d*x])/(c + d*x) + 6*a^2*b*Log[c
 + d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*x^2] + 6*a*b^2*(ArcTanh[c +
d*x]*((1 - (c + d*x)^(-1))*ArcTanh[c + d*x] + 2*Log[1 - E^(-2*ArcTanh[c +
d*x])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x])]) + 2*b^3*(ArcTanh[c + d*x]^2
*((1 - (c + d*x)^(-1))*ArcTanh[c + d*x] + 3*Log[1 - E^(-2*ArcTanh[c + d*x]
)]) - 3*ArcTanh[c + d*x]*PolyLog[2, E^(-2*ArcTanh[c + d*x])] - (3*PolyLog[
3, E^(-2*ArcTanh[c + d*x])])/2))/(2*d*e^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6657, 27, 6452, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2} \, dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{3b\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6550}$$

$$\frac{3b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(c+dx+1)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}\right)-\frac{(a+b\text{arctanh}(c+dx))^3}{c+dx}}{de^2}$$

$$\Big\downarrow \text{6494}$$

$$\frac{3b\left(-2b\int \frac{(a+b\text{arctanh}(c+dx))\log\left(2-\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+\right.}{de^2}$$

$$\Big\downarrow \; 6618$$

$$\frac{3b\left(-2b\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)(a+b\text{arctanh}(c+dx))-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)}{1-(c+dx)^2}d(c+dx)\right)+\frac{(a+b\text{arctanh}}{3b}\right.}{de^2}$$

$$\Big\downarrow \; 7164$$

$$\frac{3b\left(-2b\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)(a+b\text{arctanh}(c+dx))+\frac{1}{4}b\,\text{PolyLog}\left(3,\frac{2}{c+dx+1}-1\right)\right)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}\right.}{de^2}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTanh[c + d*x])^3/(c + d*x)) + 3*b*((a + b*ArcTanh[c + d*x])^3
/(3*b) + (a + b*ArcTanh[c + d*x])^2*Log[2 - 2/(1 + c + d*x)] - 2*b*(((a +
b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d*x)])/2 + (b*PolyLog[3, -1
 + 2/(1 + c + d*x)])/4)))/(d*e^2)
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.26.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 1466, normalized size of antiderivative = 10.25

method	result	size
derivativedivides	Expression too large to display	1466
default	Expression too large to display	1466
parts	Expression too large to display	1474

input
```
int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/e^2/(d*x+c)+b^3/e^2*(-1/(d*x+c)*arctanh(d*x+c)^3-3/2*arctanh(d*x
+c)^2*ln(d*x+c-1)+3*ln(d*x+c)*arctanh(d*x+c)^2-3/2*arctanh(d*x+c)^2*ln(d*x
+c+1)+3*arctanh(d*x+c)^2*ln((d*x+c+1)/(1-(d*x+c)^2)^(1/2))-arctanh(d*x+c)^
3+3/4*(-2*I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(-(d*x+c+1)^2/
((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2-I*Pi*csgn(I/(1-(d*x+c+1)
^2/((d*x+c)^2-1)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((
d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))+I*Pi*csgn(I*(d*x+c+1)^2/((d*x+c
)^2-1))^3+2*I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^3-I*Pi*csgn(I*(d*x+
c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*
x+c)^2-1)))^2+I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(d*x+c+1)^
2/((d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2+2*I*Pi*csgn(I*(-(d*x+c+1)
^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^3-2*I*Pi*csgn(I/(1-(d*x
+c+1)^2/((d*x+c)^2-1)))^2+I*Pi*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+
1)^2/((d*x+c)^2-1)))^3+I*Pi*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2*csgn(I
*(d*x+c+1)^2/((d*x+c)^2-1))+2*I*Pi*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))*c
sgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2+2*I*Pi-2*I*Pi*csgn(I*(-(d*x+c+1)^2/((d*
x+c)^2-1)-1))*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*x+c
)^2-1)))^2+2*I*Pi*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csgn(I/(1-(d*x+c+
1)^2/((d*x+c)^2-1)))*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/
((d*x+c)^2-1)))+4*ln(2))*arctanh(d*x+c)^2-3*arctanh(d*x+c)^2*ln((d*x+c+...
```

3.26.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.26.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\,\text{atanh}^3\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\,\text{atanh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\,\text{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atanh(c +
d*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atanh(c + d*x
)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atanh(c + d*x)/(
c**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.26.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-3/2*(d*(log(d*x + c + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2) + log(d*x +
 c - 1)/(d^2*e^2)) + 2*arctanh(d*x + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3
/(d^2*e^2*x + c*d*e^2) - 1/8*((b^3*d*x + b^3*(c - 1))*log(-d*x - c + 1)^3
+ 3*(2*a*b^2 + (b^3*d*x + b^3*(c + 1))*log(d*x + c + 1))*log(-d*x - c + 1)
^2)/(d^2*e^2*x + c*d*e^2) - integrate(-1/8*((b^3*d*x + b^3*(c - 1))*log(d*
x + c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c - 1))*log(d*x + c + 1)^2 + 3*(4*a*b
^2*d*x + 4*a*b^2*c - (b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^2 + 2*(b^3*d
^2*x^2 + (c^2 + c)*b^3 - 2*a*b^2*(c - 1) + ((2*c*d + d)*b^3 - 2*a*b^2*d)*x
)*log(d*x + c + 1))*log(-d*x - c + 1))/(d^3*e^2*x^3 + c^3*e^2 - c^2*e^2 +
(3*c*d^2*e^2 - d^2*e^2)*x^2 + (3*c^2*d*e^2 - 2*c*d*e^2)*x), x)
```

3.26.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{(a+b\,\text{atanh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.27 $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx$

3.27.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx &= \frac{3b(a+b\text{arctanh}(c+dx))^2}{2de^3}-\frac{3b(a+b\text{arctanh}(c+dx))^2}{2de^3(c+dx)}\\&+\frac{(a+b\text{arctanh}(c+dx))^3}{2de^3}-\frac{(a+b\text{arctanh}(c+dx))^3}{2de^3(c+dx)^2}\\&+\frac{3b^2(a+b\text{arctanh}(c+dx))\log\left(2-\frac{2}{1+c+dx}\right)}{de^3}\\&-\frac{3b^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+c+dx}\right)}{2de^3}\end{aligned}$$

output
```
3/2*b*(a+b*arctanh(d*x+c))^2/d/e^3-3/2*b*(a+b*arctanh(d*x+c))^2/d/e^3/(d*x
+c)+1/2*(a+b*arctanh(d*x+c))^3/d/e^3-1/2*(a+b*arctanh(d*x+c))^3/d/e^3/(d*x
+c)^2+3*b^2*(a+b*arctanh(d*x+c))*ln(2-2/(d*x+c+1))/d/e^3-3/2*b^3*polylog(2
,-1+2/(d*x+c+1))/d/e^3
```

3.27.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.02

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$= \frac{-4a^3-12a^2bc+ib^3c^3\pi^3-12a^2bdx+2ib^3c^2d\pi^3x+ib^3cd^2\pi^3x^2+12b^2(-1+c+dx)(b(c+dx)+a(1+c}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
(-4*a^3 - 12*a^2*b*c + I*b^3*c^3*Pi^3 - 12*a^2*b*d*x + (2*I)*b^3*c^2*d*Pi^
3*x + I*b^3*c*d^2*Pi^3*x^2 + 12*b^2*(-1 + c + d*x)*(b*(c + d*x) + a*(1 + c
 + d*x))*ArcTanh[c + d*x]^2 + 4*b^3*(-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh
[c + d*x]^3 + 12*b*ArcTanh[c + d*x]*(a*(-2*b*(c + d*x) + a*(-1 + c^2 + 2*c
*d*x + d^2*x^2)) + 2*b^2*(c + d*x)^2*Log[1 - E^(-2*ArcTanh[c + d*x])]) + 2
4*a*b^2*c^2*Log[(c + d*x)/Sqrt[1 - (c + d*x)^2]] + 48*a*b^2*c*d*x*Log[(c +
 d*x)/Sqrt[1 - (c + d*x)^2]] + 24*a*b^2*d^2*x^2*Log[(c + d*x)/Sqrt[1 - (c
+ d*x)^2]] - 12*b^3*(c + d*x)^2*PolyLog[2, E^(-2*ArcTanh[c + d*x])])/(8*d*
e^3*(c + d*x)^2)
```

3.27.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6657, 27, 6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$\Big\downarrow \text{6657}$$
$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{e^3(c+dx)^3}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{3}{2}b\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6544}$$

$$\frac{\frac{3}{2}b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^2}d(c+dx)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)\right)-\frac{(a+b\text{arctanh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6452}$$

$$\frac{\frac{3}{2}b\left(2b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}\right)-\frac{(a+b\text{arctanh}(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6510}$$

$$\frac{\frac{3}{2}b\left(2b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(1-(c+dx)^2)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-\frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}\right)-\frac{(a+b\text{arctanh}(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{6550}$$

$$\frac{\frac{3}{2}b\left(2b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)(c+dx+1)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}\right)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-\frac{(a+b\text{arctanh}(c+dx))^2}{c+dx}\right)-\frac{(a+b}{}}{de^3}$$

$$\Big\downarrow \text{6494}$$

$$\frac{\frac{3}{2}b\left(2b\left(-b\int \frac{\log\left(2-\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))\right)+\frac{(a+ba}{}}{de^3}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\frac{3}{2}b\left(2b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(2-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)\right)+\frac{(a+barct}{}}{de^3}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

3.27. $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx$

output
```
(-1/2*(a + b*ArcTanh[c + d*x])^3/(c + d*x)^2 + (3*b*(-((a + b*ArcTanh[c +
d*x])^2/(c + d*x)) + (a + b*ArcTanh[c + d*x])^3/(3*b) + 2*b*((a + b*ArcTan
h[c + d*x])^2/(2*b) + (a + b*ArcTanh[c + d*x])*Log[2 - 2/(1 + c + d*x)] -
(b*PolyLog[2, -1 + 2/(1 + c + d*x)])/2)))/2)/(d*e^3)
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 4949, normalized size of antiderivative = 29.81

method	result	size
derivativedivides	Expression too large to display	4949
default	Expression too large to display	4949
parts	Expression too large to display	4957

input
```
int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(3/4*I*Pi*arctanh(d*x+c)^2-3/8*I*Pi*cs
gn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c
+1)^2/((d*x+c)^2-1)))^2*arctanh(d*x+c)*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))
-3/2/(d*x+c)*arctanh(d*x+c)^2-3/8*I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)
))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(
d*x+c+1)^2/((d*x+c)^2-1)))*arctanh(d*x+c)*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/
2))-3/8*I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(d*x+c+1)^2/((d*
x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))
*polylog(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-3/8*I*Pi*csgn(I/(1-(d*x+c+1)^2/(
(d*x+c)^2-1)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+
c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*polylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^
(1/2))-3/8*I*Pi*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(d*x+c+1)^2/(
(d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1
)))*dilog((d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3/8*I*Pi*csgn(I/(1-(d*x+c+1)^2/((
d*x+c)^2-1)))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))*csgn(I*(d*x+c+1)^2/((d*x+c
)^2-1)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*dilog(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2
))+3/8*I*Pi*csgn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))^2*csgn(I*(d*x+c+1)^2/((d
*x+c)^2-1))*arctanh(d*x+c)*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3/4*I*Pi*cs
gn(I*(d*x+c+1)/(1-(d*x+c)^2)^(1/2))*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1))^2*ar
ctanh(d*x+c)*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+3/8*I*Pi*csgn(I/(1-(d*...
```

3.27.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3
*e^3), x)
```

3.27.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx$$
$$= \frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{atanh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{atanh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{atanh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**3*atanh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(3*a*b**2*atanh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3), x) + Integral(3*a**2*b*atanh(c + d*x)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.27.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\operatorname{artanh}(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-3/4*(d*(2/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c + 1)/(d^2*e^3) + log(d*x
 + c - 1)/(d^2*e^3)) + 2*arctanh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^
2*d*e^3))*a^2*b - 3/8*(d^2*((log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*log(d
*x + c - 1) + log(d*x + c - 1)^2 + 4*log(d*x + c - 1))/(d^3*e^3) + 4*log(d
*x + c + 1)/(d^3*e^3) - 8*log(d*x + c)/(d^3*e^3)) + 4*d*(2/(d^3*e^3*x + c*
d^2*e^3) - log(d*x + c + 1)/(d^2*e^3) + log(d*x + c - 1)/(d^2*e^3))*arctan
h(d*x + c))*a*b^2 - 1/16*b^3*(((d^2*x^2 + 2*c*d*x + c^2 - 1)*log(-d*x - c
 + 1)^3 + 3*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + 2*c)*
log(-d*x - c + 1)^2)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) + 2*integra
te(-((d*x + c - 1)*log(d*x + c + 1)^3 + 3*(2*d^2*x^2 + 4*c*d*x - (d*x + c
 - 1)*log(d*x + c + 1)^2 + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d
 - d)*x - c)*log(d*x + c + 1))*log(-d*x - c + 1))/(d^4*e^3*x^4 + c^4*e^3 -
 c^3*e^3 + (4*c*d^3*e^3 - d^3*e^3)*x^3 + 3*(2*c^2*d^2*e^3 - c*d^2*e^3)*x^2
 + (4*c^3*d*e^3 - 3*c^2*d*e^3)*x), x)) - 3/2*a*b^2*arctanh(d*x + c)^2/(d^3*
e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*
x + c^2*d*e^3)
```

3.27.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\text{atanh}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.28 $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.28.1 Optimal result

Integrand size = 23, antiderivative size = 269

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx = &-\frac{b^2(a+b\text{arctanh}(c+dx))}{de^4(c+dx)}+\frac{b(a+b\text{arctanh}(c+dx))^2}{2de^4}\\&-\frac{b(a+b\text{arctanh}(c+dx))^2}{2de^4(c+dx)^2}\\&+\frac{(a+b\text{arctanh}(c+dx))^3}{3de^4}-\frac{(a+b\text{arctanh}(c+dx))^3}{3de^4(c+dx)^3}\\&+\frac{b^3\log(c+dx)}{de^4}-\frac{b^3\log\left(1-(c+dx)^2\right)}{2de^4}\\&+\frac{b(a+b\text{arctanh}(c+dx))^2\log\left(2-\frac{2}{1+c+dx}\right)}{de^4}\\&-\frac{b^2(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+c+dx}\right)}{de^4}\\&-\frac{b^3\,\text{PolyLog}\left(3,-1+\frac{2}{1+c+dx}\right)}{2de^4}\end{aligned}$$

output
```
-b^2*(a+b*arctanh(d*x+c))/d/e^4/(d*x+c)+1/2*b*(a+b*arctanh(d*x+c))^2/d/e^4
-1/2*b*(a+b*arctanh(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*(a+b*arctanh(d*x+c))^3/d
/e^4-1/3*(a+b*arctanh(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^
3*ln(1-(d*x+c)^2)/d/e^4+b*(a+b*arctanh(d*x+c))^2*ln(2-2/(d*x+c+1))/d/e^4-b
^2*(a+b*arctanh(d*x+c))*polylog(2,-1+2/(d*x+c+1))/d/e^4-1/2*b^3*polylog(3,
-1+2/(d*x+c+1))/d/e^4
```

3.28.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$$

$$= \frac{-\frac{2a^3}{(c+dx)^3}-\frac{3a^2b}{(c+dx)^2}-\frac{6a^2b\text{arctanh}(c+dx)}{(c+dx)^3}+6a^2b\log(c+dx)-3a^2b\log\left(1-c^2-2cdx-d^2x^2\right)+6ab^2\Big(-\frac{(c+}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
((-2*a^3)/(c + d*x)^3 - (3*a^2*b)/(c + d*x)^2 - (6*a^2*b*ArcTanh[c + d*x])
/(c + d*x)^3 + 6*a^2*b*Log[c + d*x] - 3*a^2*b*Log[1 - c^2 - 2*c*d*x - d^2*
x^2] + 6*a*b^2*(-(((c + d*x)^2 + ArcTanh[c + d*x]^2)/(c + d*x)^3) + ArcTan
h[c + d*x]*(-((1 - (c + d*x)^2)/(c + d*x)^2) + ArcTanh[c + d*x] + 2*Log[1
- E^(-2*ArcTanh[c + d*x])]) - PolyLog[2, E^(-2*ArcTanh[c + d*x])]) + 6*b^3
*((I/24)*Pi^3 - ArcTanh[c + d*x]/(c + d*x) - ((1 + c^3 + 3*c^2*d*x + 3*c*d
^2*x^2 + d^3*x^3)*ArcTanh[c + d*x]^3)/(3*(c + d*x)^3) + (ArcTanh[c + d*x]^
2*(-1 + c^2 + 2*c*d*x + d^2*x^2 + 2*(c + d*x)^2*Log[1 - E^(2*ArcTanh[c + d
*x])]))/(2*(c + d*x)^2) + Log[c + d*x] + Log[1/Sqrt[1 - (c + d*x)^2]] + Ar
cTanh[c + d*x]*PolyLog[2, E^(2*ArcTanh[c + d*x])] - PolyLog[3, E^(2*ArcTan
h[c + d*x])]/2))/(6*d*e^4)
```

3.28.3 Rubi [A] (warning: unable to verify)

Time = 1.84 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {6657, 27, 6452, 6544, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$$

$$\Big\downarrow \text{6657}$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{e^4(c+dx)^4}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\text{arctanh}(c+dx))^3}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \; 6452$$

$$\frac{b\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^3(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 6544$$

$$\frac{b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)^3}d(c+dx)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)\right)-\frac{(a+b\text{arctanh}(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 6452$$

$$\frac{b\left(b\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2(1-(c+dx)^2)}d(c+dx)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}\right)-\frac{(a+b\text{arctanh}(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 6544$$

$$\frac{b\left(b\left(\int \frac{a+b\text{arctanh}(c+dx)}{(c+dx)^2}d(c+dx)+\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)\right)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctan}}{2(c+d}}{de^4}$$

$$\Big\downarrow \; 6452$$

$$\frac{b\left(b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+b\int \frac{1}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(}{de^4}$$

$$\Big\downarrow \; 243$$

$$\frac{b\left(b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\int \frac{1}{(-c-dx+1)(c+dx)^2}d(c+dx)^2-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)+\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}}{de^4}$$

$$\Big\downarrow \; 47$$

$$\frac{b\left(b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\int \frac{1}{(c+dx)^2}d(c+dx)^2\right)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}\right)+\int \frac{(a}{(}}{de^4}$$

$$\Big\downarrow \; 14$$

3.28. $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$

$$\frac{b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)+b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{-c-dx+1}d(c+dx)^2+\log\left((c+dx)^2\right)\right.\right.\right.}{de^4}$$

$\downarrow$ 16

$$\frac{b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)+b\left(\int \frac{a+b\text{arctanh}(c+dx)}{1-(c+dx)^2}d(c+dx)-\frac{a+b\text{arctanh}(c+dx)}{c+dx}+\frac{1}{2}b(\log\left((c+dx)^2\right)-\text{lo}\right.\right.}{de^4}$$

$\downarrow$ 6510

$$\frac{b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(1-(c+dx)^2)}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}+b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arctanh}(c+dx)}{c+dx}+\frac{1}{2}b(\log\left((c\right.\right.\right.}{de^4}$$

$\downarrow$ 6550

$$\frac{b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{(c+dx)(c+dx+1)}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}+b\left(\frac{(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{a+b\text{arcta}}{c-}\right.\right.}{de^4}$$

$\downarrow$ 6494

$$b\left(-2b\int \frac{(a+b\text{arctanh}(c+dx))\log\left(2-\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}-\frac{(a+b\text{arctanh}(c+dx))^2}{2(c+dx)^2}+\log\left(2-\frac{2}{c+dx+1}\right.\right.$$

$\downarrow$ 6618

$$b\left(-2b\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)(a+b\text{arctanh}(c+dx))-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)}{1-(c+dx)^2}d(c+dx)\right)+\frac{(a+b\text{arctanh}(c}{3b}\right.$$

$\downarrow$ 7164

$$b\left(-2b\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{c+dx+1}-1\right)(a+b\text{arctanh}(c+dx))+\frac{1}{4}b\,\text{PolyLog}\left(3,\frac{2}{c+dx+1}-1\right)\right)+\frac{(a+b\text{arctanh}(c+dx))^3}{3b}\right.$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

3.28. $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$

output
```
(-1/3*(a + b*ArcTanh[c + d*x])^3/(c + d*x)^3 + b*(-1/2*(a + b*ArcTanh[c +
d*x])^2/(c + d*x)^2 + (a + b*ArcTanh[c + d*x])^3/(3*b) + b*(-((a + b*ArcTa
nh[c + d*x])/(c + d*x)) + (a + b*ArcTanh[c + d*x])^2/(2*b) + (b*(-Log[1 -
c - d*x] + Log[(c + d*x)^2]))/2) + (a + b*ArcTanh[c + d*x])^2*Log[2 - 2/(1
 + c + d*x)] - 2*b*(((a + b*ArcTanh[c + d*x])*PolyLog[2, -1 + 2/(1 + c + d
*x)])/2 + (b*PolyLog[3, -1 + 2/(1 + c + d*x)])/4)))/(d*e^4)
```

3.28.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.28. $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.28.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.94 (sec) , antiderivative size = 1776, normalized size of antiderivative = 6.60

method	result	size
derivativedivides	Expression too large to display	1776
default	Expression too large to display	1776
parts	Expression too large to display	1784

input
```
int((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3/e^4/(d*x+c)^3+b^3/e^4*(-1/3/(d*x+c)^3*arctanh(d*x+c)^3-1/2*a
rctanh(d*x+c)^2*ln(d*x+c-1)-1/2/(d*x+c)^2*arctanh(d*x+c)^2+ln(d*x+c)*arcta
nh(d*x+c)^2-1/2*arctanh(d*x+c)^2*ln(d*x+c+1)+arctanh(d*x+c)^2*ln((d*x+c+1)
/(1-(d*x+c)^2)^(1/2))-arctanh(d*x+c)^2*ln((d*x+c+1)^2/(1-(d*x+c)^2)-1)+arc
tanh(d*x+c)^2*ln(1-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*arctanh(d*x+c)*polylog
(2,(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*polylog(3,(d*x+c+1)/(1-(d*x+c)^2)^(1/2
))+arctanh(d*x+c)^2*ln(1+(d*x+c+1)/(1-(d*x+c)^2)^(1/2))+2*arctanh(d*x+c)*p
olylog(2,-(d*x+c+1)/(1-(d*x+c)^2)^(1/2))-2*polylog(3,-(d*x+c+1)/(1-(d*x+c)
^2)^(1/2))+1/12*arctanh(d*x+c)*(6*I*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^
3*Pi*arctanh(d*x+c)*(d*x+c)+6*I*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1))*csg
n(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(
1-(d*x+c+1)^2/((d*x+c)^2-1)))*Pi*arctanh(d*x+c)*(d*x+c)-6*I*csgn(I/(1-(d*x
+c+1)^2/((d*x+c)^2-1)))^2*Pi*arctanh(d*x+c)*(d*x+c)-6*I*csgn(I/(1-(d*x+c+1
)^2/((d*x+c)^2-1)))*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/(
(d*x+c)^2-1)))^2*Pi*arctanh(d*x+c)*(d*x+c)+3*I*Pi*arctanh(d*x+c)*csgn(I*(d
*x+c+1)^2/((d*x+c)^2-1))^3*(d*x+c)+3*I*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1)/(1
-(d*x+c+1)^2/((d*x+c)^2-1)))^3*Pi*arctanh(d*x+c)*(d*x+c)+6*I*csgn(I*(d*x+c
+1)/(1-(d*x+c)^2)^(1/2))*Pi*arctanh(d*x+c)*csgn(I*(d*x+c+1)^2/((d*x+c)^2-1
))^2*(d*x+c)+6*I*csgn(I*(-(d*x+c+1)^2/((d*x+c)^2-1)-1)/(1-(d*x+c+1)^2/((d*
x+c)^2-1)))^3*Pi*arctanh(d*x+c)*(d*x+c)+6*I*Pi*arctanh(d*x+c)*(d*x+c)-3...
```

3.28.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 +
 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.28.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$$
$$=\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^3\,\text{atanh}^3\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3ab^2\,\text{atanh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{}{c}}{e^4}$$

input
```
integrate((a+b*atanh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**3*atanh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atanh(c + d
*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),
 x) + Integral(3*a**2*b*atanh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x*
*2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.28.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/2*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) + log(d*x + c + 1)/
(d^2*e^4) - 2*log(d*x + c)/(d^2*e^4) + log(d*x + c - 1)/(d^2*e^4)) + 2*arc
tanh(d*x + c)/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4
))*a^2*b - 1/3*a^3/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*
d*e^4) - 1/24*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 - 1)*
b^3)*log(-d*x - c + 1)^3 + 3*(b^3*d*x + b^3*c + 2*a*b^2 + (b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + (c^3 + 1)*b^3)*log(d*x + c + 1))*log(-d*x
 - c + 1)^2)/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
 - integrate(-1/8*((b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^3 + 6*(a*b^2*d
*x + a*b^2*(c - 1))*log(d*x + c + 1)^2 + (2*b^3*d^2*x^2 + 2*b^3*c^2 + 4*a*
b^2*c - 3*(b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^2 + 4*(b^3*c*d + a*b^2*
d)*x + 2*(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + (c^4 + c)*b^
3 - 6*a*b^2*(c - 1) + ((4*c^3*d + d)*b^3 - 6*a*b^2*d)*x)*log(d*x + c + 1))
*log(-d*x - c + 1))/(d^5*e^4*x^5 + c^5*e^4 - c^4*e^4 + (5*c*d^4*e^4 - d^4*
e^4)*x^4 + 2*(5*c^2*d^3*e^4 - 2*c*d^3*e^4)*x^3 + 2*(5*c^3*d^2*e^4 - 3*c^2*
d^2*e^4)*x^2 + (5*c^4*d*e^4 - 4*c^3*d*e^4)*x), x)
```

3.28.8 Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx=\int \frac{(a+b\operatorname{atanh}(c+dx))^3}{(ce+dex)^4}\,dx$$

input
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*atanh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.28. $\int \frac{(a+b\,\text{arctanh}(c+dx))^3}{(ce+dex)^4}\,dx$

3.29 $\int \frac{\operatorname{arctanh}(1+x)}{2+2x} \, dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x} \, dx = -\frac{1}{4} \operatorname{PolyLog}(2,-1-x) + \frac{\operatorname{PolyLog}(2,1+x)}{4}$$

output
```
-1/4*polylog(2,-1-x)+1/4*polylog(2,1+x)
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x} \, dx = -\frac{1}{4} \operatorname{PolyLog}\left(2, \frac{1}{2}(-2-2x)\right) + \frac{1}{4} \operatorname{PolyLog}\left(2, \frac{1}{2}(2+2x)\right)$$

input
```
Integrate[ArcTanh[1 + x]/(2 + 2*x),x]
```

output
```
-1/4*PolyLog[2, (-2 - 2*x)/2] + PolyLog[2, (2 + 2*x)/2]/4
```

3.29.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6657, 27, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(x+1)}{2x+2}\,dx$$

$$\Big\downarrow \text{ 6657}$$

$$\int \frac{\operatorname{arctanh}(x+1)}{2(x+1)}d(x+1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\operatorname{arctanh}(x+1)}{x+1}d(x+1)$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{1}{2}\left(\frac{\operatorname{PolyLog}(2,x+1)}{2}-\frac{\operatorname{PolyLog}(2,-x-1)}{2}\right)$$

input
```
Int[ArcTanh[1 + x]/(2 + 2*x),x]
```

output
```
(-1/2*PolyLog[2, -1 - x] + PolyLog[2, 1 + x]/2)/2
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\operatorname{dilog}(-x)}{4}-\frac{\operatorname{dilog}(x+2)}{4}$	14
derivativedivides	$\frac{\ln(1+x)\operatorname{arctanh}(1+x)}{2}-\frac{\operatorname{dilog}(1+x)}{4}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}$	34
default	$\frac{\ln(1+x)\operatorname{arctanh}(1+x)}{2}-\frac{\operatorname{dilog}(1+x)}{4}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}$	34
parts	$\frac{\ln(1+x)\operatorname{arctanh}(1+x)}{2}-\frac{\operatorname{dilog}(1+x)}{4}-\frac{\operatorname{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}$	34

input
```
int(arctanh(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*dilog(-x)-1/4*dilog(x+2)
```

3.29.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x}\,dx=\int \frac{\operatorname{artanh}\left(x+1\right)}{2\left(x+1\right)}\,dx$$

input
```
integrate(arctanh(1+x)/(2+2*x),x, algorithm="fricas")
```

output
```
integral(1/2*arctanh(x + 1)/(x + 1), x)
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x}\,dx = \frac{\int \frac{\operatorname{atanh}(x+1)}{x+1}\,dx}{2}$$

input
```
integrate(atanh(1+x)/(2+2*x),x)
```

output
```
Integral(atanh(x + 1)/(x + 1), x)/2
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\begin{aligned}\int \frac{\operatorname{arctanh}(1+x)}{2+2x}\,dx = &-\frac{1}{4}(\log(x+2)-\log(x))\log(x+1) \\ &+\frac{1}{2}\operatorname{artanh}(x+1)\log(x+1)-\frac{1}{4}\log(x+1)\log(x) \\ &+\frac{1}{4}\log(x+2)\log(-x-1)-\frac{1}{4}\operatorname{Li}_2(-x)+\frac{1}{4}\operatorname{Li}_2(x+2)\end{aligned}$$

input
```
integrate(arctanh(1+x)/(2+2*x),x, algorithm="maxima")
```

output
```
-1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arctanh(x + 1)*log(x + 1) - 1/
4*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dil
og(x + 2)
```

3.29.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{artanh}(x+1)}{2(x+1)}\,dx$$

input
```
integrate(arctanh(1+x)/(2+2*x),x, algorithm="giac")
```

output
```
integrate(1/2*arctanh(x + 1)/(x + 1), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(1+x)}{2+2x}\,dx = \int \frac{\operatorname{atanh}(x+1)}{2\,x+2}\,dx$$

input
```
int(atanh(x + 1)/(2*x + 2),x)
```

output
```
int(atanh(x + 1)/(2*x + 2), x)
```

3.30 $\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.30.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx=-\frac{\operatorname{PolyLog}(2,-a-bx)}{2d}+\frac{\operatorname{PolyLog}(2,a+bx)}{2d}$$

output
```
-1/2*polylog(2,-b*x-a)/d+1/2*polylog(2,b*x+a)/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx=b\left(-\frac{\operatorname{PolyLog}\left(2,-\frac{ad+bdx}{d}\right)}{2bd}+\frac{\operatorname{PolyLog}\left(2,\frac{ad+bdx}{d}\right)}{2bd}\right)$$

input
```
Integrate[ArcTanh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
b*(-1/2*PolyLog[2, -((a*d + b*d*x)/d)]/(b*d) + PolyLog[2, (a*d + b*d*x)/d]
/(2*b*d))
```

3.30.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6657, 27, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \ 6657$$

$$\frac{\int \frac{b\operatorname{arctanh}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\operatorname{arctanh}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \ 6446$$

$$\frac{\frac{1}{2}\operatorname{PolyLog}(2,a+bx)-\frac{1}{2}\operatorname{PolyLog}(2,-a-bx)}{d}$$

input
```
Int[ArcTanh[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(-1/2*PolyLog[2, -a - b*x] + PolyLog[2, a + b*x]/2)/d
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6657
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTanh[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\operatorname{dilog}(bx+a+1)}{2d}+\frac{\operatorname{dilog}(-bx-a+1)}{2d}$	29
parts	$\frac{\ln(bx+a)\operatorname{arctanh}(bx+a)}{d}-\frac{\frac{\operatorname{dilog}(bx+a+1)}{2}+\frac{\ln(bx+a)\ln(bx+a+1)}{2}+\frac{\operatorname{dilog}(bx+a)}{2}}{d}$	56
derivativedivides	$\frac{\frac{b\ln(bx+a)\operatorname{arctanh}(bx+a)}{d}-\frac{b\left(\frac{\operatorname{dilog}(bx+a+1)}{2}+\frac{\ln(bx+a)\ln(bx+a+1)}{2}+\frac{\operatorname{dilog}(bx+a)}{2}\right)}{d}}{b}$	62
default	$\frac{\frac{b\ln(bx+a)\operatorname{arctanh}(bx+a)}{d}-\frac{b\left(\frac{\operatorname{dilog}(bx+a+1)}{2}+\frac{\ln(bx+a)\ln(bx+a+1)}{2}+\frac{\operatorname{dilog}(bx+a)}{2}\right)}{d}}{b}$	62

input
```
int(arctanh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*dilog(b*x+a+1)+1/2/d*dilog(-b*x-a+1)
```

3.30.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arctanh(b*x + a)/(b*d*x + a*d), x)
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{atanh}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(atanh(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(atanh(a + b*x)/(a + b*x), x)/d
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\operatorname{Li}_2(-bx-a+1)}{bd} - \frac{\log(bx+a+1)\log(-bx-a)+\operatorname{Li}_2(bx+a+\right.$$
$$-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{2\,d} + \frac{\operatorname{artanh}(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d}$$

input
```
integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
-1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log
(b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x
+ a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arctanh(b*x + a)*log
(d*x + a*d/b)/d
```

3.30.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(d*x + a*d/b), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(atanh(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(atanh(a + b*x)/(d*x + (a*d)/b), x)
```

3.31 $\int (e+fx)^3(a+b\,\mathbf{arctanh}(c+dx))\,dx$

3.31.1 Optimal result

Integrand size = 18, antiderivative size = 168

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx = \frac{bf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{bf^3(c+dx)^3}{12d^4} + \frac{(e+fx)^4(a+b\text{arctanh}(c+dx))}{4f} + \frac{b(de+f-cf)^4\log(1-c-dx)}{8d^4f} - \frac{b(de-f-cf)^4\log(1+c+dx)}{8d^4f}$$

output
```
1/4*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d
*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arctanh(d*x+c))/f+
1/8*b*(-c*f+d*e+f)^4*ln(-d*x-c+1)/d^4/f-1/8*b*(-c*f+d*e-f)^4*ln(d*x+c+1)/d
^4/f
```

3.31.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.61

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx$$
$$= \frac{6d(4ad^3e^3+bf(6d^2e^2-8cdef+(1+3c^2)f^2))\,x+6d^2f(6ad^2e^2+bf(2de-cf))\,x^2+2d^3f^2(12ade+bf)}{}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d
^2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^
3 + 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3
)*ArcTanh[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-
1 + c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*
e^3 + 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c
 + d*x])/(24*d^4)
```

3.31.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6661, 27, 6478, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3(a+b\text{arctanh}(c+dx))}{d^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^3(a+b\text{arctanh}(c+dx))d(c+dx)}{d^4}$$

$$\Big\downarrow \text{6478}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\text{arctanh}(c+dx))}{4f}-\frac{b\int \frac{(de-cf+f(c+dx))^4}{1-(c+dx)^2}d(c+dx)}{4f}}{d^4}$$

$\downarrow$ 477

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\text{arctanh}(c+dx))}{4f}-\frac{b\int \left(-(c+dx)^2f^4-4(de-cf)(c+dx)f^3-(6d^2e^2-12cdfe+(6c^2+1)f^2)f^2+\frac{(de-cf+f)^4}{2(-c-dx+1)}+\frac{(de-cf-f)}{2(c+dx+1}\right.}{4f}}{d^4}$$

$\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\text{arctanh}(c+dx))}{4f}-\frac{b(-f^2(c+dx)((6c^2+1)f^2-12cdef+6d^2e^2)-2f^3(c+dx)^2(de-cf)-\frac{1}{2}(-cf+de+f)^4\log(-c-dx+1}{4f}}{d^4}$$

input
```
Int[(e + f*x)^3*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcTanh[c + d*x]))/(4*f) - (b*(-(f^2*
(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*(c + d*x)) - 2*f^3*(d*e - c*f)*
(c + d*x)^2 - (f^4*(c + d*x)^3)/3 - ((d*e + f - c*f)^4*Log[1 - c - d*x])/2
 + ((d*e - f - c*f)^4*Log[1 + c + d*x])/2))/(4*f))/d^4
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(156) = 312$.

Time = 0.14 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.57

method	result
parallelrisch	$-\frac{-3x^4 a d^4 f^3 - 12xa d^4 e^3 - 3xbd f^3 - x^3 b d^3 f^3 + 3 \operatorname{arctanh}(dx+c) b c^4 f^3 - 12 \operatorname{arctanh}(dx+c) b d^3 e^3 + 18 \operatorname{arctanh}(dx+c) b}{}$
derivativedivides	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3 f} - \frac{b\left(-\frac{f^3 \operatorname{arctanh}(dx+c) c^4}{4} + f^2 \operatorname{arctanh}(dx+c) c^3 de + f^3 \operatorname{arctanh}(dx+c) c^3 (dx+c) - \frac{3f \operatorname{arctanh}(dx+c) c^2 d^2 e^2}{2} - 3\right.}{}}{}$
default	$\frac{\frac{a(cf-de-f(dx+c))^4}{4d^3 f} - \frac{b\left(-\frac{f^3 \operatorname{arctanh}(dx+c) c^4}{4} + f^2 \operatorname{arctanh}(dx+c) c^3 de + f^3 \operatorname{arctanh}(dx+c) c^3 (dx+c) - \frac{3f \operatorname{arctanh}(dx+c) c^2 d^2 e^2}{2} - 3\right.}{}}{}$
parts	$\frac{a(fx+e)^4}{4f} + \frac{b\left(\frac{f^3 \operatorname{arctanh}(dx+c)(dx+c)^4}{4d^3} - \frac{f^3 \operatorname{arctanh}(dx+c)(dx+c)^3 c}{d^3} + \frac{f^2 \operatorname{arctanh}(dx+c)(dx+c)^3 e}{d^2} + \frac{3f^3 \operatorname{arctanh}(dx+c)(dx+c}{2d^3}\right.}{}$
risch	$\frac{e^3 b \ln(-dx-c+1)}{2d} + \frac{e^3 b \ln(dx+c+1)}{2d} + \frac{f^3 a x^4}{4} + \frac{f^3 b x^3}{12d} + \frac{f^3 bx}{4d^3} + \frac{f^3 \ln(-dx-c+1) b}{8d^4} - \frac{f^3 \ln(dx+c+1) b}{8d^4} -$

input
```
int((f*x+e)^3*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*x^4*a*d^4*f^3-12*x*a*d^4*e^3-3*x*b*d*f^3-x^3*b*d^3*f^3+3*arctanh
(d*x+c)*b*c^4*f^3-12*arctanh(d*x+c)*b*d^3*e^3+18*arctanh(d*x+c)*b*c^2*f^3+
12*arctanh(d*x+c)*b*c*f^3+12*arctanh(d*x+c)*b*c^3*f^3+12*ln(d*x+c-1)*b*c^3
*f^3-12*ln(d*x+c-1)*b*d^3*e^3+12*ln(d*x+c-1)*b*c*f^3-42*b*c^2*d*e*f^2+36*b
*c*d^2*e^2*f+18*a*c^2*d^2*e^2*f+24*a*c*d^3*e^3+15*b*c^3*f^3-18*f*e^2*a*d^2
-6*b*e*f^2*d+9*b*c*f^3+3*arctanh(d*x+c)*b*f^3-12*x^3*a*d^4*e*f^2-18*x^2*a*
d^4*e^2*f-12*ln(d*x+c-1)*b*d*e*f^2-12*arctanh(d*x+c)*b*c*d^3*e^3+18*arctan
h(d*x+c)*b*d^2*e^2*f-12*arctanh(d*x+c)*b*d*e*f^2-3*x^4*arctanh(d*x+c)*b*d^
4*f^3-12*x*arctanh(d*x+c)*b*d^4*e^3+3*x^2*b*c*d^2*f^3-6*x^2*b*d^3*e*f^2-9*
x*b*c^2*d*f^3-18*x*b*d^3*e^2*f-36*ln(d*x+c-1)*b*c^2*d*e*f^2+36*ln(d*x+c-1)
*b*c*d^2*e^2*f+18*arctanh(d*x+c)*b*c^2*d^2*e^2*f-12*x^3*arctanh(d*x+c)*b*d
^4*e*f^2-18*x^2*arctanh(d*x+c)*b*d^4*e^2*f+24*x*b*c*d^2*e*f^2-12*arctanh(d
*x+c)*b*c^3*d*e*f^2-36*arctanh(d*x+c)*b*c^2*d*e*f^2+36*arctanh(d*x+c)*b*c*
d^2*e^2*f-36*arctanh(d*x+c)*b*c*d*e*f^2)/d^4
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(156) = 312$.

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.30

$$\int (e+fx)^3(a+b\operatorname{arctanh}(c+dx))\,dx$$
$$= \frac{6\,ad^4f^3x^4 + 2\,(12\,ad^4ef^2 + bd^3f^3)x^3 + 6\,(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3)x^2 + 6\,(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcd}{}$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(6*a*d^4*f^3*x^4 + 2*(12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 6*(6*a*d^4*e^
2*f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 6*(4*a*d^4*e^3 + 6*b*d^3*e^2*f -
8*b*c*d^2*e*f^2 + (3*b*c^2 + b)*d*f^3)*x + 3*(4*(b*c + b)*d^3*e^3 - 6*(b*c
^2 + 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*e*f^2 - (b*c
^4 + 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3)*log(d*x + c + 1) - 3*(4*(b*c - b)
*d^3*e^3 - 6*(b*c^2 - 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c^2 + 3*b*c -
b)*d*e*f^2 - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3)*log(d*x + c - 1)
 + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*
x)*log(-(d*x + c + 1)/(d*x + c - 1)))/d^4
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(151) = 302$.

Time = 0.72 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.83

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx$$
$$=\begin{cases} ae^3x+\frac{3ae^2fx^2}{2}+aef^2x^3+\frac{af^3x^4}{4}-\frac{bc^4f^3\,\text{atanh}\,(c+dx)}{4d^4}+\frac{bc^3ef^2\,\text{atanh}\,(c+dx)}{d^3}-\frac{bc^3f^3\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^4}+\frac{bc^3f^3\,\text{atanh}\,(c+dx)}{d^4} \\ (a+b\,\text{atanh}\,(c))\left(e^3x+\frac{3e^2fx^2}{2}+ef^2x^3+\frac{f^3x^4}{4}\right) \end{cases}$$

input
```
integrate((f*x+e)**3*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 -
b*c**4*f**3*atanh(c + d*x)/(4*d**4) + b*c**3*e*f**2*atanh(c + d*x)/d**3 -
b*c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*atanh(c + d*x)/d**4 - 3*
b*c**2*e**2*f*atanh(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)
/d**3 - 3*b*c**2*e*f**2*atanh(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3
*b*c**2*f**3*atanh(c + d*x)/(2*d**4) + b*c*e**3*atanh(c + d*x)/d - 3*b*c*e
**2*f*log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*atanh(c + d*x)/d**2 - 2*b*c*e
*f**2*x/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*atanh(c + d*x)/d**3 -
 b*c*f**3*log(c/d + x + 1/d)/d**4 + b*c*f**3*atanh(c + d*x)/d**4 + b*e**3*
x*atanh(c + d*x) + 3*b*e**2*f*x**2*atanh(c + d*x)/2 + b*e*f**2*x**3*atanh(
c + d*x) + b*f**3*x**4*atanh(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*
e**3*atanh(c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*
x**3/(12*d) - 3*b*e**2*f*atanh(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x +
1/d)/d**3 - b*e*f**2*atanh(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*atan
h(c + d*x)/(4*d**4), Ne(d, 0)), ((a + b*atanh(c))*(e**3*x + 3*e**2*f*x**2/
2 + e*f**2*x**3 + f**3*x**4/4), True))
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(156) = 312$.

Time = 0.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.98

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx = \frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2$$
$$+\frac{3}{4}\left(2\,x^2\,\text{artanh}\,(dx+c)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3}+\frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right)$$
$$+\frac{1}{2}\left(2\,x^3\,\text{artanh}\,(dx+c)+d\left(\frac{dx^2-4\,cx}{d^3}+\frac{(c^3+3\,c^2+3\,c+1)\log{(dx+c+1)}}{d^4}-\frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+\frac{1}{24}\left(6\,x^4\,\text{artanh}\,(dx+c)+d\left(\frac{2\,(d^2x^3-3\,cdx^2+3\,(3\,c^2+1)x)}{d^4}-\frac{3\,(c^4+4\,c^3+6\,c^2+4\,c+1)\log{(dx}}{d^5}\right.\right.$$
$$+ae^3x+\frac{\left(2\,(dx+c)\,\text{artanh}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)be^3}{2\,d}$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/4*(2*x^2*arctanh(d*x + c
) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*lo
g(d*x + c - 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 +
3*c - 1)*log(d*x + c - 1)/d^4))*b*e*f^2 + 1/24*(6*x^4*arctanh(d*x + c) + d
*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 +
 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x
 + c - 1)/d^5))*b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(
-(d*x + c)^2 + 1))*b*e^3/d
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2336 vs. $2(156) = 312$.

Time = 0.34 (sec) , antiderivative size = 2336, normalized size of antiderivative = 13.90

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*d^3*e^3/(d*x + c - 1)^3
- 3*(d*x + c + 1)^2*b*d^3*e^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*d^3*e^3/
(d*x + c - 1) - b*d^3*e^3 - 3*(d*x + c + 1)^3*b*c*d^2*e^2*f/(d*x + c - 1)^
3 + 9*(d*x + c + 1)^2*b*c*d^2*e^2*f/(d*x + c - 1)^2 - 9*(d*x + c + 1)*b*c*
d^2*e^2*f/(d*x + c - 1) + 3*b*c*d^2*e^2*f + 3*(d*x + c + 1)^3*b*c^2*d*e*f^
2/(d*x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*d*e*f^2/(d*x + c - 1)^2 + 9*(d
*x + c + 1)*b*c^2*d*e*f^2/(d*x + c - 1) - 3*b*c^2*d*e*f^2 - (d*x + c + 1)^
3*b*c^3*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c^3*f^3/(d*x + c - 1)^2
- 3*(d*x + c + 1)*b*c^3*f^3/(d*x + c - 1) + b*c^3*f^3 + 3*(d*x + c + 1)^3*
b*d^2*e^2*f/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*d^2*e^2*f/(d*x + c - 1)^
2 + 3*(d*x + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 6*(d*x + c + 1)^3*b*c*d*e*
f^2/(d*x + c - 1)^3 + 12*(d*x + c + 1)^2*b*c*d*e*f^2/(d*x + c - 1)^2 - 6*(
d*x + c + 1)*b*c*d*e*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*c^2*f^3/(d*x
+ c - 1)^3 - 6*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)
*b*c^2*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 3
*(d*x + c + 1)^2*b*d*e*f^2/(d*x + c - 1)^2 + (d*x + c + 1)*b*d*e*f^2/(d*x
+ c - 1) - b*d*e*f^2 - 3*(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 3*(d*x
+ c + 1)^2*b*c*f^3/(d*x + c - 1)^2 - (d*x + c + 1)*b*c*f^3/(d*x + c - 1) +
 b*c*f^3 + (d*x + c + 1)^3*b*f^3/(d*x + c - 1)^3 + (d*x + c + 1)*b*f^3/(d*
x + c - 1))*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^5/(d*x...
```

3.31. $\int (e+fx)^3(a+b\text{arctanh}(c+dx))\,dx$

3.31.9 Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 737, normalized size of antiderivative = 4.39

$$\int (e+fx)^3(a+b\operatorname{arctanh}(c+dx))\,dx$$

$$= \ln(c+d\,x+1)\left(\frac{b\,e^3\,x}{2}+\frac{3\,b\,e^2\,f\,x^2}{4}+\frac{b\,e\,f^2\,x^3}{2}+\frac{b\,f^3\,x^4}{8}\right) - \ln(1-d\,x-c)\left(\frac{b\,e^3\,x}{2}+\frac{3\,b\,e^2\,f\,x^2}{4}+\frac{b\,e\,f^2\,x^3}{2}+\frac{b\,f^3\,x^4}{8}\right) + x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2+3\,b\,d\,e\,f-6\,a\,f^2)}{2\,d^2} - \frac{(4\,c^2-4)\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2} + \frac{2\,c\left(\frac{2\,c\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2-4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2-4)}{4\,d^2}\right)}{d}\right) - x^2\left(\frac{c\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d} - \frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f+4\,b\,d\,e\,f^2-4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2-4)}{8\,d^2}\right) + x^3\left(\frac{f^2\,(b\,f+8\,a\,c\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4} + \frac{\ln(c+d\,x-1)\,(b\,c^4\,f^3-4\,b\,c^3\,d\,e\,f^2-4\,b\,c^3\,f^3+6\,b\,c^2\,d^2\,e^2\,f+12\,b\,c^2\,d\,e\,f^2+6\,b\,c^2\,f^3-4\,b\,c\,d^3\,e^3-}{8\,d^4} - \frac{\ln(c+d\,x+1)\,(b\,c^4\,f^3-4\,b\,c^3\,d\,e\,f^2+4\,b\,c^3\,f^3+6\,b\,c^2\,d^2\,e^2\,f-12\,b\,c^2\,d\,e\,f^2+6\,b\,c^2\,f^3-4\,b\,c\,d^3\,e^3+}{8\,d^4}$$

input
```
int((e + f*x)^3*(a + b*atanh(c + d*x)),x)
```

output

```
log(c + d*x + 1)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f
^2*x^3)/2) - log(1 - d*x - c)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^
2)/4 + (b*e*f^2*x^3)/2) + x*((e*(6*a*c^2*f^2 - 6*a*f^2 + 2*a*d^2*e^2 + 3*b
*d*e*f + 12*a*c*d*e*f))/(2*d^2) - ((4*c^2 - 4)*((f^2*(b*f + 8*a*c*f + 12*a
*d*e))/(4*d) - (2*a*c*f^3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f +
 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f
^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 - 4))/(4*d^2
)))/d) - x^2*((c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))
/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^
2)/(8*d^2) + (a*f^3*(4*c^2 - 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*
a*d*e))/(12*d) - (2*a*c*f^3)/(3*d)) + (a*f^3*x^4)/4 + (log(c + d*x - 1)*(b
*f^3 + 6*b*c^2*f^3 - 4*b*c^3*f^3 + 4*b*d^3*e^3 + b*c^4*f^3 - 4*b*c*f^3 + 4
*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f - 12*b*c*d^2*e^2*f + 12*b*c^2*d
*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4) -
(log(c + d*x + 1)*(b*f^3 + 6*b*c^2*f^3 + 4*b*c^3*f^3 - 4*b*d^3*e^3 + b*c^4
*f^3 + 4*b*c*f^3 - 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*c*d^
2*e^2*f - 12*b*c^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*
d*e*f^2))/(8*d^4)
```

3.31. $\int (e+fx)^3(a+b\operatorname{arctanh}(c+dx))\,dx$

3.32 $\int (e+fx)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx$

3.32.1 Optimal result

Integrand size = 18, antiderivative size = 120

$$\int (e+fx)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx = \frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3(a+b\,\mathrm{arctanh}(c+dx))}{3f} + \frac{b(de+f-cf)^3\log(1-c-dx)}{6d^3f} - \frac{b(de-(1+c)f)^3\log(1+c+dx)}{6d^3f}$$

output
```
b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*
x+c))/f+1/6*b*(-c*f+d*e+f)^3*ln(-d*x-c+1)/d^3/f-1/6*b*(d*e-(1+c)*f)^3*ln(d
*x+c+1)/d^3/f
```

3.32.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.45

$$\int (e+fx)^2(a+b\,\mathrm{arctanh}(c+dx))\,dx$$

$$= \frac{2d(3ad^2e^2+bf(3de-2cf))\,x+d^2f(6ade+bf)x^2+2ad^3f^2x^3+2bd^3x(3e^2+3efx+f^2x^2)\,\mathrm{arctanh}(c+}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2
*a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c + d*x] -
b*(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x
] + b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*
x])/(6*d^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6661, 27, 6478, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\text{arctanh}(c+dx))}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\text{arctanh}(c+dx))d(c+dx)}{d^3}$$

$$\Big\downarrow \; 6478$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))}{3f}-\frac{b\int \frac{(de-cf+f(c+dx))^3}{1-(c+dx)^2}d(c+dx)}{3f}}{d^3}$$

$$\Big\downarrow \; 477$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))}{3f}-\frac{b\int \left(-\left((c+dx)f^3\right)-3(de-cf)f^2+\frac{(de-cf+f)^3}{2(-c-dx+1)}+\frac{(de-(c+1)f)^3}{2(c+dx+1)}\right)d(c+dx)}{3f}}{d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))}{3f}-\frac{b\left(-3f^2(c+dx)(de-cf)-\frac{1}{2}(-cf+de+f)^3\log(-c-dx+1)+\frac{1}{2}(de-(c+1)f)^3\log(c+dx+1)-\frac{1}{2}f^3(c+\right.}{3f}}{d^3}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTanh[c + d*x]))/(3*f) - (b*(-3*f^2
*(d*e - c*f)*(c + d*x) - (f^3*(c + d*x)^2)/2 - ((d*e + f - c*f)^3*Log[1 -
c - d*x])/2 + ((d*e - (1 + c)*f)^3*Log[1 + c + d*x])/2))/(3*f))/d^3
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(112) = 224$.

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.92

method	result
parallelrisch	$\frac{-12ac\,e^2d^2+6x^2a\,d^3ef+6x\,\operatorname{arctanh}(dx+c)b\,d^3e^2+2x^3\operatorname{arctanh}(dx+c)b\,d^3f^2+6\operatorname{arctanh}(dx+c)bc\,d^2e^2-6\operatorname{arctanh}(dx+\cdots}{}$
parts	$\frac{a(fx+e)^3}{3f}+\frac{b\left(\frac{f^2\operatorname{arctanh}(dx+c)(dx+c)^3}{3d^2}-\frac{f^2\operatorname{arctanh}(dx+c)(dx+c)^2c}{d^2}+\frac{f\operatorname{arctanh}(dx+c)(dx+c)^2e}{d}+\frac{f^2\operatorname{arctanh}(dx+c)(dx+c)\cdots}{d^2}\right.}{}$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arctanh}(dx+c)c^3}{3}+f\operatorname{arctanh}(dx+c)c^2de+f^2\operatorname{arctanh}(dx+c)c^2(dx+c)-\operatorname{arctanh}(dx+c)c\,d^2e^2-2f\ \operatorname{ar}\cdots\right.}{}}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f}+\frac{b\left(-\frac{f^2\operatorname{arctanh}(dx+c)c^3}{3}+f\operatorname{arctanh}(dx+c)c^2de+f^2\operatorname{arctanh}(dx+c)c^2(dx+c)-\operatorname{arctanh}(dx+c)c\,d^2e^2-2f\ \operatorname{ar}\cdots\right.}{}}{}$
risch	$\frac{f^2a\,x^3}{3}+\frac{f^2b\,x^2}{6d}+\frac{f^2\ln(dx+c-1)b}{6d^3}+\frac{f^2\ln(-dx-c-1)b}{6d^3}-\frac{b\,e^2x\ln(-dx-c+1)}{2}+\frac{\ln(dx+c-1)b\,e^3}{6f}-\frac{\ln(-dx-\cdots}{6\cdots}$

input

```
int((f*x+e)^2*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/6*(-12*a*c*e^2*d^2+6*x^2*a*d^3*e*f+6*x*arctanh(d*x+c)*b*d^3*e^2+2*x^3*ar
ctanh(d*x+c)*b*d^3*f^2+6*arctanh(d*x+c)*b*c*d^2*e^2-6*arctanh(d*x+c)*b*d*e
*f-4*x*b*c*d*f^2+6*x*b*d^2*e*f-6*a*c^2*e*f*d+b*f^2+7*b*c^2*f^2+6*f*e*a*d-1
2*b*c*d*e*f+6*x^2*arctanh(d*x+c)*b*d^3*e*f-12*arctanh(d*x+c)*b*c*d*e*f-6*a
rctanh(d*x+c)*b*c^2*d*e*f-12*ln(d*x+c-1)*b*c*d*e*f+2*arctanh(d*x+c)*b*f^2+
2*ln(d*x+c-1)*b*f^2+x^2*b*d^2*f^2+2*arctanh(d*x+c)*b*c^3*f^2+6*arctanh(d*x
+c)*b*c^2*f^2+6*arctanh(d*x+c)*b*d^2*e^2+6*arctanh(d*x+c)*b*c*f^2+6*x*a*d^
3*e^2+2*x^3*a*d^3*f^2+6*ln(d*x+c-1)*b*c^2*f^2+6*ln(d*x+c-1)*b*d^2*e^2)/d^3
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(112) = 224$.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.02

$$\int (e+fx)^2(a+b\operatorname{arctanh}(c+dx))\,dx$$
$$=\frac{2\,ad^3f^2x^3+(6\,ad^3ef+bd^2f^2)x^2+2\,(3\,ad^3e^2+3\,bd^2ef-2\,bcdf^2)x+(3\,(bc+b)d^2e^2-3\,(bc^2+2\,bc+b)\cdots}{}$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*
b*d^2*e*f - 2*b*c*d*f^2)*x + (3*(b*c + b)*d^2*e^2 - 3*(b*c^2 + 2*b*c + b)*
d*e*f + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2)*log(d*x + c + 1) - (3*(b*c - b)
*d^2*e^2 - 3*(b*c^2 - 2*b*c + b)*d*e*f + (b*c^3 - 3*b*c^2 + 3*b*c - b)*f^2
)*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^2*x)*log
(-(d*x + c + 1)/(d*x + c - 1)))/d^3
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(105) = 210$.

Time = 0.53 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.08

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))\,dx$$
$$= \begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\,\text{atanh}\,(c+dx)}{3d^3} - \frac{bc^2ef\,\text{atanh}\,(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bc^2f^2\,\text{atanh}\,(c+dx)}{d^3} + \frac{bce^2\,\text{atanh}}{d} \\ (a+b\,\text{atanh}\,(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) \end{cases}$$

input
```
integrate((f*x+e)**2*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atanh(c + d
*x)/(3*d**3) - b*c**2*e*f*atanh(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x +
1/d)/d**3 - b*c**2*f**2*atanh(c + d*x)/d**3 + b*c*e**2*atanh(c + d*x)/d -
2*b*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*atanh(c + d*x)/d**2 - 2*b*c*
f**2*x/(3*d**2) + b*c*f**2*atanh(c + d*x)/d**3 + b*e**2*x*atanh(c + d*x) +
 b*e*f*x**2*atanh(c + d*x) + b*f**2*x**3*atanh(c + d*x)/3 + b*e**2*log(c/d
 + x + 1/d)/d - b*e**2*atanh(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) -
b*e*f*atanh(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*at
anh(c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*atanh(c))*(e**2*x + e*f*x**2 + f
**2*x**3/3), True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))\,dx = \frac{1}{3}\,af^2x^3 + aefx^2 + \frac{1}{2}\left(2\,x^2\,\text{artanh}\,(dx+c) + d\left(\frac{2\,x}{d^2} - \frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3} + \frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right) + \frac{1}{6}\left(2\,x^3\,\text{artanh}\,(dx+c) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{(c^3+3\,c^2+3\,c+1)\log{(dx+c+1)}}{d^4} - \frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right. + ae^2x + \frac{(2\,(dx+c)\,\text{artanh}\,(dx+c) + \log{(-(dx+c)^2+1)})be^2}{2\,d}$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^
2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))
*b*e*f + 1/6*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c
^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c -
 1)/d^4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x
+ c)^2 + 1))*b*e^2/d
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(112) = 224.

Time = 0.31 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.13

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
1/6*((c + 1)*d - (c - 1)*d)*((3*(d*x + c + 1)^2*b*d^2*e^2/(d*x + c - 1)^2
- 6*(d*x + c + 1)*b*d^2*e^2/(d*x + c - 1) + 3*b*d^2*e^2 - 6*(d*x + c + 1)^
2*b*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*c*d*e*f/(d*x + c - 1) - 6
*b*c*d*e*f + 3*(d*x + c + 1)^2*b*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)
*b*c^2*f^2/(d*x + c - 1) + 3*b*c^2*f^2 + 6*(d*x + c + 1)^2*b*d*e*f/(d*x +
c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*c*f
^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^2/(d*x + c - 1) + 3*(d*x + c +
1)^2*b*f^2/(d*x + c - 1)^2 + b*f^2)*log(-(d*x + c + 1)/(d*x + c - 1))/((d*
x + c + 1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c - 1)^2 +
 3*(d*x + c + 1)*d^4/(d*x + c - 1) - d^4) + 2*(3*(d*x + c + 1)^2*a*d^2*e^2
/(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*d^2*e^2/(d*x + c - 1) + 3*a*d^2*e^2 -
 6*(d*x + c + 1)^2*a*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*a*c*d*e*f/
(d*x + c - 1) - 6*a*c*d*e*f + 3*(d*x + c + 1)^2*a*c^2*f^2/(d*x + c - 1)^2
- 6*(d*x + c + 1)*a*c^2*f^2/(d*x + c - 1) + 3*a*c^2*f^2 + 6*(d*x + c + 1)^
2*a*d*e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*a*d*e*f/(d*x + c - 1) + 3*(d*x
 + c + 1)^2*b*d*e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1
) + 3*b*d*e*f - 6*(d*x + c + 1)^2*a*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1
)*a*c*f^2/(d*x + c - 1) - 3*(d*x + c + 1)^2*b*c*f^2/(d*x + c - 1)^2 + 6*(d
*x + c + 1)*b*c*f^2/(d*x + c - 1) - 3*b*c*f^2 + 3*(d*x + c + 1)^2*a*f^2/(d
*x + c - 1)^2 + a*f^2 + (d*x + c + 1)^2*b*f^2/(d*x + c - 1)^2 - (d*x + ...
```

3.32.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.18

$$\int (e+fx)^2(a+b\operatorname{arctanh}(c+dx))\,dx = x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right) - \ln(1-d\,x-c)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right) - x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d} - \frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f-3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2-3)}{3\,d^2}\right) + \ln(c+d\,x+1)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)+\frac{a\,f^2\,x^3}{3} + \frac{\ln(c+d\,x-1)\left(\frac{b\,f^2}{6}+d\left(\frac{b\,e\,f\,c^2}{2}-b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}-\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}-\frac{b\,c^3\,f^2}{6}-\frac{b\,c\,f^2}{2}\right)}{d^3} + \frac{\ln(c+d\,x+1)\left(\frac{b\,f^2}{6}-d\left(\frac{b\,e\,f\,c^2}{2}+b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}+\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}+\frac{b\,c^3\,f^2}{6}+\frac{b\,c\,f^2}{2}\right)}{d^3}$$

input
```
int((e + f*x)^2*(a + b*atanh(c + d*x)),x)
```

output
```
x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - log(1 - d*x - c)
*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) - x*((2*c*((f*(b*f + 6*a*c*
f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*c^2*f^2 - 3*a*f^2 + 3*a*d^2
*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 - 3))/(3*d^2)) +
log(c + d*x + 1)*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) + (a*f^2*x^
3)/3 + (log(c + d*x - 1)*((b*f^2)/6 + d*((b*e*f)/2 + (b*c^2*e*f)/2 - b*c*e
*f) + d^2*((b*e^2)/2 - (b*c*e^2)/2) + (b*c^2*f^2)/2 - (b*c^3*f^2)/6 - (b*c
*f^2)/2))/d^3 + (log(c + d*x + 1)*((b*f^2)/6 - d*((b*e*f)/2 + (b*c^2*e*f)/
2 + b*c*e*f) + d^2*((b*e^2)/2 + (b*c*e^2)/2) + (b*c^2*f^2)/2 + (b*c^3*f^2)
/6 + (b*c*f^2)/2))/d^3
```

3.32. $\int (e+fx)^2(a+b\text{arctanh}(c+dx))\,dx$

3.33 $\int (e+fx)(a+b\,\mathbf{arctanh}(c+dx))\,dx$

3.33.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx)(a+b\operatorname{arctanh}(c+dx))\,dx = \frac{bfx}{2d}+\frac{(e+fx)^2(a+b\operatorname{arctanh}(c+dx))}{2f} + \frac{b(de+f-cf)^2\log(1-c-dx)}{4d^2f} - \frac{b(de-(1+c)f)^2\log(1+c+dx)}{4d^2f}$$

output
```
1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arctanh(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*ln(-
d*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*ln(d*x+c+1)/d^2/f
```

3.33.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

$$\begin{aligned}&\int (e+fx)(a+b\operatorname{arctanh}(c+dx))\,dx\\&= aex+\frac{bfx}{2d}+\frac{1}{2}afx^2+bex\operatorname{arctanh}(c+dx)+\frac{1}{2}bfx^2\operatorname{arctanh}(c+dx)\\&\quad+\frac{b(1-2c+c^2)\,f\log(1-c-dx)}{4d^2}+\frac{b(-1-2c-c^2)\,f\log(1+c+dx)}{4d^2}\\&\quad+\frac{be(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}\end{aligned}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]
```

output
```
a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcTanh[c + d*x] + (b*f*x^2*Ar
cTanh[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-
1 - 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c -
d*x]) + (1 + c)*Log[1 + c + d*x]))/(2*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6661, 27, 6478, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\text{arctanh}(c+dx))}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))(a+b\text{arctanh}(c+dx))d(c+dx)}{d^2}$$

$$\Big\downarrow \text{6478}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))}{2f}-\frac{b\int \frac{(de-cf+f(c+dx))^2}{1-(c+dx)^2}d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{477}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))}{2f}-\frac{b\int \left(-f^2+\frac{(de-cf+f)^2}{2(-c-dx+1)}+\frac{(de-(c+1)f)^2}{2(c+dx+1)}\right)d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))}{2f}-\frac{b\left(-\frac{1}{2}(-cf+de+f)^2\log(-c-dx+1)+\frac{1}{2}(de-(c+1)f)^2\log(c+dx+1)-\left(f^2(c+dx)\right)\right)}{2f}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcTanh[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTanh[c + d*x]))/(2*f) - (b*(-(f^2*
(c + d*x)) - ((d*e + f - c*f)^2*Log[1 - c - d*x])/2 + ((d*e - (1 + c)*f)^2
*Log[1 + c + d*x])/2))/(2*f))/d^2
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

method	result
parts	$a\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b\left(\frac{\operatorname{arctanh}(dx+c)(dx+c)^2 f}{2d}-\frac{\operatorname{arctanh}(dx+c)cf(dx+c)}{d}+\operatorname{arctanh}(dx+c)e(dx+c)-\frac{-f(dx+c)-\frac{(-2cf+2}{}}{}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\operatorname{arctanh}(dx+c)fc(dx+c)-\operatorname{arctanh}(dx+c)ed(dx+c)-\frac{\operatorname{arctanh}(dx+c)f(dx+c)^2}{2}-\frac{f(dx+c)}{2}\right.}{d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\operatorname{arctanh}(dx+c)fc(dx+c)-\operatorname{arctanh}(dx+c)ed(dx+c)-\frac{\operatorname{arctanh}(dx+c)f(dx+c)^2}{2}-\frac{f(dx+c)}{2}\right.}{d}}{d}$
parallelrisch	$-\frac{-\operatorname{arctanh}(dx+c)b\,d^2f\,x^2-a\,d^2f\,x^2-2x\operatorname{arctanh}(dx+c)b\,d^2e-2a\,d^2ex+\operatorname{arctanh}(dx+c)b\,c^2f-2\operatorname{arctanh}(dx+c)bcde+}{}$
risch	$\frac{bx(fx+2e)\ln(dx+c+1)}{4}-\frac{bf\,x^2\ln(-dx-c+1)}{4}-\frac{bex\ln(-dx-c+1)}{2}+\frac{af\,x^2}{2}+\frac{\ln(-dx-c+1)b\,c^2f}{4d^2}-\frac{\ln(-dx-c}{2d}$

input
```
int((f*x+e)*(a+b*arctanh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arctanh(d*x+c)*(d*x+c)^2*f-1/d*arctanh(d*x+c)
*c*f*(d*x+c)+arctanh(d*x+c)*e*(d*x+c)-1/2/d*(-f*(d*x+c)-1/2*(-2*c*f+2*d*e+
f)*ln(d*x+c-1)+1/2*(2*c*f-2*d*e+f)*ln(d*x+c+1)))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int (e+fx)(a+b\operatorname{arctanh}(c+dx))\,dx$$
$$=\frac{2\,ad^2fx^2+2\,(2\,ad^2e+bdf)x+(2\,(bc+b)de-(bc^2+2\,bc+b)f)\log(dx+c+1)-(2\,(bc-b)de-(bc^2-}{4\,d^2}$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d^2*f*x^2 + 2*(2*a*d^2*e + b*d*f)*x + (2*(b*c + b)*d*e - (b*c^2 +
 2*b*c + b)*f)*log(d*x + c + 1) - (2*(b*c - b)*d*e - (b*c^2 - 2*b*c + b)*f
)*log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*e*x)*log(-(d*x + c + 1)/(d*x +
 c - 1)))/d^2
```

3.33.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.

Time = 0.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int (e+fx)(a+b\text{arctanh}(c+dx))\,dx$$
$$= \begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2f\operatorname{atanh}(c+dx)}{2d^2} + \frac{bce\operatorname{atanh}(c+dx)}{d} - \frac{bcf\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{bcf\operatorname{atanh}(c+dx)}{d^2} + bex\operatorname{atanh}(c+dx) + \frac{bfx^2 \cdots}{} \\ (a+b\operatorname{atanh}(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

input
```
integrate((f*x+e)*(a+b*atanh(d*x+c)),x)
```

output
```
Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atanh(c + d*x)/(2*d**2) + b*c*e*a
tanh(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*atanh(c + d*x)/d**
2 + b*e*x*atanh(c + d*x) + b*f*x**2*atanh(c + d*x)/2 + b*e*log(c/d + x + 1
/d)/d - b*e*atanh(c + d*x)/d + b*f*x/(2*d) - b*f*atanh(c + d*x)/(2*d**2),
Ne(d, 0)), ((a + b*atanh(c))*(e*x + f*x**2/2), True))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int (e+fx)(a+b\text{arctanh}(c+dx))\,dx = \frac{1}{2}afx^2$$
$$+\frac{1}{4}\left(2x^2\operatorname{artanh}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)\cdots$$
$$+aex+\frac{\left(2(dx+c)\operatorname{artanh}(dx+c)+\log\left(-(dx+c)^2+1\right)\right)be}{2d}$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 + 1/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*l
og(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f + a*e*x +
 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b*e/d
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.52

$$\int (e+fx)(a+b\text{arctanh}(c+dx))\,dx$$
$$=\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{\left(\frac{(dx+c+1)bde}{dx+c-1}-bde-\frac{(dx+c+1)bcf}{dx+c-1}+bcf+\frac{(dx+c+1)bf}{dx+c-1}\right)\log\left(-\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2d^3}{(dx+c-1)^2}-\frac{2\,(dx+c+1)d^3}{dx+c-1}+d^3}+\frac{2\,(dx+c+1)ac}{dx+c-1}\right.$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*((((d*x + c + 1)*b*d*e/(d*x + c - 1) - b*d*e -
(d*x + c + 1)*b*c*f/(d*x + c - 1) + b*c*f + (d*x + c + 1)*b*f/(d*x + c - 1
))*log(-(d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^2*d^3/(d*x + c - 1)^2
- 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) + (2*(d*x + c + 1)*a*d*e/(d*x +
 c - 1) - 2*a*d*e - 2*(d*x + c + 1)*a*c*f/(d*x + c - 1) + 2*a*c*f + 2*(d*x
 + c + 1)*a*f/(d*x + c - 1) + (d*x + c + 1)*b*f/(d*x + c - 1) - b*f)/((d*x
 + c + 1)^2*d^3/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3)
 - (b*d*e - b*c*f)*log(-(d*x + c + 1)/(d*x + c - 1) + 1)/d^3 + (b*d*e - b*
c*f)*log(-(d*x + c + 1)/(d*x + c - 1))/d^3)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int (e+fx)(a+b\text{arctanh}(c+dx))\,dx = a\,e\,x+\frac{a\,f\,x^2}{2}+\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2\,d}-\frac{b\,f\,\text{atanh}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\text{atanh}(c+d\,x)}{2}+\frac{b\,f\,x}{2\,d}+b\,e\,x\,\text{atanh}(c+d\,x)-\frac{b\,c^2\,f\,\text{atanh}(c+d\,x)}{2\,d^2}-\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2\,d^2}+\frac{b\,c\,e\,\text{atanh}(c+d\,x)}{d}$$

input
```
int((e + f*x)*(a + b*atanh(c + d*x)),x)
```

output
```
a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/(2*d) - (b*f*
atanh(c + d*x))/(2*d^2) + (b*f*x^2*atanh(c + d*x))/2 + (b*f*x)/(2*d) + b*e
*x*atanh(c + d*x) - (b*c^2*f*atanh(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^
2*x^2 + 2*c*d*x - 1))/(2*d^2) + (b*c*e*atanh(c + d*x))/d
```

3.33. $\int (e + fx)(a + b\text{arctanh}(c + dx))\,dx$

3.34 $\int (a + b\text{arctanh}(c + dx))\,dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 40

$$\int (a + b\text{arctanh}(c + dx))\,dx = ax + \frac{b(c + dx)\text{arctanh}(c + dx)}{d} + \frac{b\log\left(1 - (c + dx)^2\right)}{2d}$$

output
```
a*x+b*(d*x+c)*arctanh(d*x+c)/d+1/2*b*ln(1-(d*x+c)^2)/d
```

3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a + b\text{arctanh}(c + dx))\,dx = ax + bx\text{arctanh}(c + dx) + \frac{b(-((-1 + c)\log(1 - c - dx)) + (1 + c)\log(1 + c + dx))}{2d}$$

input
```
Integrate[a + b*ArcTanh[c + d*x],x]
```

output
```
a*x + b*x*ArcTanh[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Lo
g[1 + c + d*x]))/(2*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b(c+dx)\text{arctanh}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}$$

input
```
Int[a + b*ArcTanh[c + d*x],x]
```

output
```
a*x + (b*(c + d*x)*ArcTanh[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result
default	$ax+\frac{b\left((dx+c)\,\text{arctanh}(dx+c)+\frac{\ln\left(1-(dx+c)^2\right)}{2}\right)}{d}$
parts	$ax+\frac{b\left((dx+c)\,\text{arctanh}(dx+c)+\frac{\ln\left(1-(dx+c)^2\right)}{2}\right)}{d}$
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\,\text{arctanh}(dx+c)+\frac{\ln\left(1-(dx+c)^2\right)}{2}\right)}{d}$
parallelrisch	$-\frac{b(-x\,\text{arctanh}(dx+c)d^2-\text{arctanh}(dx+c)cd-\ln(dx+c-1)d-d\,\text{arctanh}(dx+c))}{d^2}+ax$
risch	$ax+\frac{bx\ln(dx+c+1)}{2}-\frac{bx\ln(-dx-c+1)}{2}-\frac{b\ln(dx+c-1)c}{2d}+\frac{b\ln(-dx-c-1)c}{2d}+\frac{b\ln(dx+c-1)}{2d}+\frac{b\ln(-dx-c}{2d}$

input
```
int(a+b*arctanh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*((d*x+c)*arctanh(d*x+c)+1/2*ln(1-(d*x+c)^2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int (a+b\operatorname{arctanh}(c+dx))\,dx = \frac{bdx\log\left(-\frac{dx+c+1}{dx+c-1}\right)+2\,adx+(bc+b)\log(dx+c+1)-(bc-b)\log(dx+c-1)}{2\,d}$$

input
```
integrate(a+b*arctanh(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*d*x*log(-(d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x
 + c + 1) - (b*c - b)*log(d*x + c - 1))/d
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int (a+b\operatorname{arctanh}(c+dx))\,dx = ax+b\left(\begin{cases} \frac{c\operatorname{atanh}(c+dx)}{d}+x\operatorname{atanh}(c+dx)+\frac{\log(c+dx+1)}{d}-\frac{\operatorname{atanh}(c+dx)}{d} & \text{for } d\neq 0 \\ x\operatorname{atanh}(c) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*atanh(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*atanh(c + d*x)/d + x*atanh(c + d*x) + log(c + d*x + 1
)/d - atanh(c + d*x)/d, Ne(d, 0)), (x*atanh(c), True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int (a + b\text{arctanh}(c + dx))\,dx = ax + \frac{\left(2\,(dx + c)\,\text{artanh}\,(dx + c) + \log\left(-(dx + c)^2 + 1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arctanh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*b/d
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.00

$$\int (a + b\text{arctanh}(c + dx))\,dx$$

$$= \frac{1}{2}\left((c+1)d - (c-1)d\right)b\left(\frac{\log\left(\frac{|-dx-c-1|}{|dx+c-1|}\right)}{d^2} - \frac{\log\left(\left|-\frac{dx+c+1}{dx+c-1} + 1\right|\right)}{d^2} + \frac{\log\left(-\frac{c - \frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1} - c - 1\right)d}{\frac{(dx+c+1)d}{dx+c-1} - d} + 1}{c - \frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1} - c - 1\right)d}{\frac{(dx+c+1)d}{dx+c-1} - d} - 1}\right)}{d^2\left(\frac{dx+c+1}{dx+c-1} - 1\right)}\right)$$
$$+ ax$$

input
```
integrate(a+b*arctanh(d*x+c),x, algorithm="giac")
```

output
```
1/2*((c + 1)*d - (c - 1)*d)*b*(log(abs(-d*x - c - 1)/abs(d*x + c - 1))/d^2
 - log(abs(-(d*x + c + 1)/(d*x + c - 1) + 1))/d^2 + log(-(c - ((d*x + c +
1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d) +
1)/(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(
d*x + c - 1) - d) - 1))/(d^2*((d*x + c + 1)/(d*x + c - 1) - 1))) + a*x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a + b\text{arctanh}(c + dx))\,dx = a\,x + \frac{\frac{b\,\ln(c^2+2\,c\,d\,x+d^2\,x^2-1)}{2} + b\,c\,\text{atanh}(c + d\,x)}{d} + b\,x\,\text{atanh}(c + d\,x)$$

input
```
int(a + b*atanh(c + d*x),x)
```

output
```
a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*atanh(c + d*x))/d + b*
x*atanh(c + d*x)
```

3.35 $\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx$

3.35.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx = -\frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f} + \frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f} - \frac{b\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}$$

output
```
-(a+b*arctanh(d*x+c))*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))*ln(2*d*(f*x+e
)/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*polylog(2,1-2/(d*x+c+1))/f-1/2*b*polylog
(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

3.35.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx=\frac{a\log(e+fx)}{f}-\frac{b\log(1-c-dx)\log\left(\frac{d(e+fx)}{de+f-cf}\right)}{2f}+\frac{b\log(1+c+dx)\log\left(\frac{d(e+fx)}{de-(1+c)f}\right)}{2f}-\frac{b\,\text{PolyLog}\left(2,\frac{f(1-c-dx)}{de+f-cf}\right)}{2f}+\frac{b\,\text{PolyLog}\left(2,-\frac{f(1+c+dx)}{de-f-cf}\right)}{2f}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x),x]
```

output
```
(a*Log[e + f*x])/f - (b*Log[1 - c - d*x]*Log[(d*(e + f*x))/(d*e + f - c*f)
])/(2*f) + (b*Log[1 + c + d*x]*Log[(d*(e + f*x))/(d*e - (1 + c)*f)])/(2*f)
 - (b*PolyLog[2, (f*(1 - c - d*x))/(d*e + f - c*f)])/(2*f) + (b*PolyLog[2,
 -((f*(1 + c + d*x))/(d*e - f - c*f))])/(2*f)
```

3.35.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6661, 27, 6472, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}dx$$

$$\Big\downarrow \text{ 6661}$$

$$\frac{\int \frac{d(a+b\text{arctanh}(c+dx))}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{a+b\text{arctanh}(c+dx)}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{ 6472}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{b\int \frac{\log\left(\frac{2}{c+dx+1}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))}{f}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{b\int \frac{\log\left(\frac{2}{c+dx+1}\right)}{1-\frac{2}{c+dx+1}}d\frac{1}{c+dx+1}}{f}+\frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))}{f}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{1-(c+dx)^2}d(c+dx)}{f}+\frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))}{f}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}$$

$$\Big\downarrow \text{2897}$$

$$\frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))}{f}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(e + f*x),x]
```

output
```
-(((a + b*ArcTanh[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c +
 d*x])*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*(d*e
 - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

3.35. $\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6472
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.48

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\ln(f(dx+c)-cf+de)\operatorname{arctanh}(dx+c)}{f}+\frac{b\ln(f(dx+c)-cf+de)\ln\left(\frac{f(dx+c)-f}{cf-de-f}\right)}{2f}+\frac{b\operatorname{dilog}\left(\frac{f(dx+c)-f}{cf-de-f}\right.}{2f}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arctanh}(dx+c)}{f}-\frac{f\left(\operatorname{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)+\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)}{-cf+de+}\right.\right.}{2}\right.}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\operatorname{arctanh}(dx+c)}{f}-\frac{f\left(\operatorname{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)+\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)}{-cf+de+}\right.\right.}{2}\right.}{d}$
risch	$-\frac{b\operatorname{dilog}\left(\frac{(-dx-c+1)f+cf-de-f}{cf-de-f}\right)}{2f}-\frac{b\ln(-dx-c+1)\ln\left(\frac{(-dx-c+1)f+cf-de-f}{cf-de-f}\right)}{2f}+\frac{a\ln((-dx-c+1)f+cf-de-f)}{f}$

input

```
int((a+b*arctanh(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output

```
a*ln(f*x+e)/f+b*ln(f*(d*x+c)-c*f+d*e)/f*arctanh(d*x+c)+1/2*b/f*ln(f*(d*x+c
)-c*f+d*e)*ln((f*(d*x+c)-f)/(c*f-d*e-f))+1/2*b/f*dilog((f*(d*x+c)-f)/(c*f-
d*e-f))-1/2*b/f*ln(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)+f)/(c*f-d*e+f))-1/2*b/
f*dilog((f*(d*x+c)+f)/(c*f-d*e+f))
```

3.35.5 Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{e+fx}\,dx=\int \frac{b\operatorname{artanh}\left(dx+c\right)+a}{fx+e}\,dx$$

input

```
integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output

```
integral((b*arctanh(d*x + c) + a)/(f*x + e), x)
```

3.35.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx=\int \frac{a+b\,\text{atanh}\,(c+dx)}{e+fx}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))/(f*x+e),x)
```

output
```
Integral((a + b*atanh(c + d*x))/(e + f*x), x)
```

3.35.7 Maxima [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx=\int \frac{b\,\text{artanh}\,(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x) + a*l
og(f*x + e)/f
```

3.35.8 Giac [F]

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx=\int \frac{b\,\text{artanh}\,(dx+c)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)/(f*x + e), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(c+dx)}{e+fx}\,dx=\int \frac{a+b\,\mathrm{atanh}(c+d\,x)}{e+f\,x}\,dx$$

input
```
int((a + b*atanh(c + d*x))/(e + f*x),x)
```

output
```
int((a + b*atanh(c + d*x))/(e + f*x), x)
```

3.36 $\int \frac{a+b\operatorname{arctanh}(c+dx)}{(e+fx)^2}\,dx$

3.36.1 Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\operatorname{arctanh}(c+dx)}{f(e+fx)} - \frac{bd\log(1-c-dx)}{2f(de+f-cf)} + \frac{bd\log(1+c+dx)}{2f(de-f-cf)} - \frac{bd\log(e+fx)}{(de+f-cf)(de-(1+c)f)}$$

output
```
(-a-b*arctanh(d*x+c))/f/(f*x+e)-1/2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*
d*ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)
```

3.36.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(e+fx)^2}\,dx = \frac{1}{2}\left(-\frac{2a}{f(e+fx)} - \frac{2b\operatorname{arctanh}(c+dx)}{f(e+fx)} + \frac{bd\log(1-c-dx)}{f(-de+(-1+c)f)} - \frac{bd\log(1+c+dx)}{f(-de+f+cf)} - \frac{2bd\log(e+fx)}{d^2e^2-2cdef+(-1+c^2)f^2}\right)$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^2,x]
```

output
```
((-2*a)/(f*(e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (b*d*Log[1
- c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e)
+ f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))
/2
```

3.36.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6659, 2081, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2}dx$$

$$\Big\downarrow \ 6659$$

$$\frac{bd\int \frac{1}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{a+b\text{arctanh}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 2081$$

$$\frac{bd\int \frac{1}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{a+b\text{arctanh}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 1141$$

$$-\frac{bd^3\int \left(\frac{f^2}{d^2(de-cf+f)(de-(c+1)f)(e+fx)}-\frac{1}{2d(de-cf+f)(-c-dx+1)}-\frac{1}{2d(de-cf-f)(c+dx+1)}\right)dx}{f}-\frac{a+b\text{arctanh}(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a+b\text{arctanh}(c+dx)}{f(e+fx)}-\frac{bd^3\left(\frac{\log(-c-dx+1)}{2d^2(-cf+de+f)}-\frac{\log(c+dx+1)}{2d^2(de-(c+1)f)}+\frac{f\log(e+fx)}{d^2(-cf+de+f)(de-(c+1)f)}\right)}{f}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^2,x]
```

output
```
-((a + b*ArcTanh[c + d*x])/(f*(e + f*x))) - (b*d^3*(Log[1 - c - d*x]/(2*d^
2*(d*e + f - c*f)) - Log[1 + c + d*x]/(2*d^2*(d*e - (1 + c)*f)) + (f*Log[e
 + f*x])/(d^2*(d*e + f - c*f)*(d*e - (1 + c)*f))))/f
```

3.36.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.36.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

method	result
parts	$-\frac{a}{(fx+e)f}-\frac{bd\,\mathrm{arctanh}(dx+c)}{(dfx+de)f}-\frac{bd\ln(f(dx+c)-cf+de)}{(cf-de-f)(cf-de+f)}+\frac{bd\ln(dx+c-1)}{f(2cf-2de-2f)}-\frac{bd\ln(dx+c+1)}{f(2cf-2de+2f)}$
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\mathrm{arctanh}(dx+c)}{(cf-de-f(dx+c))f}-\frac{-\frac{\ln(dx+c-1)}{2cf-2de-2f}+\frac{\ln(dx+c+1)}{2cf-2de+2f}+\frac{f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}}{f}\right)}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+b\,d^2\left(\frac{\mathrm{arctanh}(dx+c)}{(cf-de-f(dx+c))f}-\frac{-\frac{\ln(dx+c-1)}{2cf-2de-2f}+\frac{\ln(dx+c+1)}{2cf-2de+2f}+\frac{f\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}}{f}\right)}{d}$
parallelrisch	$-\frac{-a\,d^2f^2+x\ \mathrm{arctanh}(dx+c)bc\,d^3f^2-x\ \mathrm{arctanh}(dx+c)b\,d^4ef-\mathrm{arctanh}(dx+c)bc\,d^3ef-\mathrm{arctanh}(dx+c)b\,d^2f^2-\ln(dx+}{}$
risch	$-\frac{b\ln(dx+c+1)}{2f(fx+e)}+\frac{\ln(-dx-c+1)bcd\,f^2x-\ln(-dx-c+1)b\,d^2efx-\ln(dx+c+1)bcd\,f^2x+\ln(dx+c+1)b\,d^2efx-\ln(-dx-}{}$

input
```
int((a+b*arctanh(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(f*x+e)/f-b*d/(d*f*x+d*e)/f*arctanh(d*x+c)-b*d/(c*f-d*e-f)/(c*f-d*e+f)*
ln(f*(d*x+c)-c*f+d*e)+b*d/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-b*d/f/(2*c*f-2*d
*e+2*f)*ln(d*x+c+1)
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(112) = 224$.

Time = 0.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.29

$$\int \frac{a+b\mathrm{arctanh}(c+dx)}{(e+fx)^2}\,dx = -\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2-a)f^2-(bd^2e^2-(bc-b)def+(bd^2ef-(bc-b)df^2)x)\log(dx+c+1)+}{2\,(d^2e^3f-2\,c}$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 - a)*f^2 - (b*d^2*e^2 - (b*c -
b)*d*e*f + (b*d^2*e*f - (b*c - b)*d*f^2)*x)*log(d*x + c + 1) + (b*d^2*e^2
- (b*c + b)*d*e*f + (b*d^2*e*f - (b*c + b)*d*f^2)*x)*log(d*x + c - 1) + 2*
(b*d*f^2*x + b*d*e*f)*log(f*x + e) + (b*d^2*e^2 - 2*b*c*d*e*f + (b*c^2 - b
)*f^2)*log(-(d*x + c + 1)/(d*x + c - 1)))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^
2 - 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 - 1)*f^4)*x)
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1605 vs. $2(92) = 184$.

Time = 2.47 (sec) , antiderivative size = 1605, normalized size of antiderivative = 13.96

$$\int \frac{a + b\text{arctanh}(c + dx)}{(e + fx)^2}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(d*x+c))/(f*x+e)**2,x)
```

output
```
Piecewise((-(a + b*atanh(c))/(e*f + f**2*x), Eq(d, 0)), ((a*x + b*c*atanh(
c + d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x + 1/d)/d - b*atanh(c + d*x
)/d)/e**2, Eq(f, 0)), (-2*a*f/(2*e*f**2 + 2*f**3*x) + b*d*e*atanh(d*e/f +
d*x - 1)/(2*e*f**2 + 2*f**3*x) + b*d*f*x*atanh(d*e/f + d*x - 1)/(2*e*f**2
+ 2*f**3*x) - 2*b*f*atanh(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - b*f/(2*
e*f**2 + 2*f**3*x), Eq(c, (d*e - f)/f)), (-2*a*f/(2*e*f**2 + 2*f**3*x) - b
*d*e*atanh(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - b*d*f*x*atanh(d*e/f +
d*x + 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*atanh(d*e/f + d*x + 1)/(2*e*f**2 +
2*f**3*x) + b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e + f)/f)), (-a*c**2*f**2/
(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*
f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*a*c*d*e*f/(c**2*e*f**3 + c**2*
f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x
 - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*
f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x)
+ a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d
**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c**2*f**2*atanh(c + d
*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e
**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d*e*f*atanh(c + d*x)/(c*
*2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f +
 d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*atanh(c + d*x)/(c**...
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2}\,dx$$
$$=\frac{1}{2}\left(d\left(\frac{\log(dx+c+1)}{def-(c+1)f^2}-\frac{\log(dx+c-1)}{def-(c-1)f^2}-\frac{2\log(fx+e)}{d^2e^2-2\,cdef+(c^2-1)f^2}\right)-\frac{2\,\text{artanh}(dx+c)}{f^2x+ef}\right)b$$
$$-\frac{a}{f^2x+ef}$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
 (c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*
arctanh(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(112) = 224.

Time = 0.31 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.12

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2}\,dx =$$
$$-\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{b\log\left(-\frac{(dx+c+1)de}{dx+c-1}+de+\frac{(dx+c+1)cf}{dx+c-1}-cf-\frac{(dx+c+1)f}{dx+c-1}-f\right)}{d^2e^2-2\,cdef+c^2f^2-f^2}-\frac{}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d}\right.$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)*d*e/(d*x + c - 1) + d*e
 + (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*f/(d*x + c - 1) -
 f)/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - b*log(-(d*x + c + 1)/(d*x + c
- 1))/((d*x + c + 1)*d^2*e^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d
*e*f/(d*x + c - 1) + 2*c*d*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2
*f^2 + 2*(d*x + c + 1)*d*e*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x +
c - 1) + (d*x + c + 1)*f^2/(d*x + c - 1) + f^2) - b*log(-(d*x + c + 1)/(d*
x + c - 1))/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - 2*a/((d*x + c + 1)*d^2
*e^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f/(d*x + c - 1) + 2*c
*d*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2 + 2*(d*x + c + 1)*d
*e*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1) + (d*x + c + 1)*f
^2/(d*x + c - 1) + f^2))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.48

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2}\,dx = \ln(e+f\,x)\,\left(\frac{b\,(c-1)}{2\,e\,(d\,e-f\,(c-1))}-\frac{b\,(c+1)}{2\,e\,(d\,e-f\,(c+1))}\right) - \frac{a}{x\,f^2+e\,f}+\frac{b\,\ln(1-d\,x-c)}{f\,(2\,e+2\,f\,x)}-\frac{b\,\ln(c+d\,x+1)}{2\,f\,(e+f\,x)} - \frac{b\,d\,\ln(c+d\,x-1)}{2\,f^2-2\,c\,f^2+2\,d\,e\,f}-\frac{b\,d\,\ln(c+d\,x+1)}{2\,c\,f^2+2\,f^2-2\,d\,e\,f}$$

input
```
int((a + b*atanh(c + d*x))/(e + f*x)^2,x)
```

output
```
log(e + f*x)*((b*(c - 1))/(2*e*(d*e - f*(c - 1))) - (b*(c + 1))/(2*e*(d*e
- f*(c + 1)))) - a/(e*f + f^2*x) + (b*log(1 - d*x - c))/(f*(2*e + 2*f*x))
- (b*log(c + d*x + 1))/(2*f*(e + f*x)) - (b*d*log(c + d*x - 1))/(2*f^2 - 2
*c*f^2 + 2*d*e*f) - (b*d*log(c + d*x + 1))/(2*c*f^2 + 2*f^2 - 2*d*e*f)
```

3.37 $\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^3}\,dx$

3.37.1 Optimal result

Integrand size = 18, antiderivative size = 167

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^3}\,dx = \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{a+b\text{arctanh}(c+dx)}{2f(e+fx)^2} - \frac{bd^2\log(1-c-dx)}{4f(de+f-cf)^2} + \frac{bd^2\log(1+c+dx)}{4f(de-f-cf)^2} - \frac{bd^2(de-cf)\log(e+fx)}{(de+f-cf)^2(de-(1+c)f)^2}$$

output
```
1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*arctanh(d*x+c))/f/(f*x
+e)^2-1/4*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*ln(d*x+c+1)/f/(-c*
f+d*e-f)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

3.37.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^3}\,dx = \frac{1}{4}\left(-\frac{2a}{f(e+fx)^2} + \frac{2bd}{(d^2e^2-2cdef+(-1+c^2)f^2)(e+fx)} - \frac{2b\text{arctanh}(c+dx)}{f(e+fx)^2} - \frac{bd^2\log(1-c-dx)}{f(de+f-cf)^2} + \frac{bd^2\log(1+c+dx)}{f(-de+f+cf)^2} - \frac{4bd^2(de-cf)\log(e+fx)}{(d^2e^2-2cdef+(-1+c^2)f^2)^2}\right)$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])/(e + f*x)^3,x]
```

output
```
((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*
(e + f*x)) - (2*b*ArcTanh[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d
*x])/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f
)^2) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2
)*f^2)^2)/4
```

3.37.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6659, 2081, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^3}dx$$

$$\Big\downarrow \ 6659$$

$$\frac{bd\int \frac{1}{(e+fx)^2(1-(c+dx)^2)}dx}{2f}-\frac{a+b\text{arctanh}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \ 2081$$

$$\frac{bd\int \frac{1}{(e+fx)^2(-c^2-2dxc-d^2x^2+1)}dx}{2f}-\frac{a+b\text{arctanh}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \ 1141$$

$$-\frac{bd^3\int \left(\frac{2(de-cf)f^2}{d(de-cf+f)^2(de-(c+1)f)^2(e+fx)}+\frac{f^2}{d^2(de-cf+f)(de-(c+1)f)(e+fx)^2}-\frac{1}{2(de-cf+f)^2(-c-dx+1)}-\frac{1}{2(de-(c+1)f)^2(c+dx+}}{2f}$$
$$\frac{a+b\text{arctanh}(c+dx)}{2f(e+fx)^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a+b\text{arctanh}(c+dx)}{2f(e+fx)^2}-$$
$$\frac{bd^3\left(-\frac{f}{d^2(e+fx)(-cf+de+f)(de-(c+1)f)}+\frac{2f(de-cf)\log(e+fx)}{d(-cf+de+f)^2(de-(c+1)f)^2}+\frac{\log(-c-dx+1)}{2d(-cf+de+f)^2}-\frac{\log(c+dx+1)}{2d(de-(c+1)f)^2}\right)}{2f}$$

input
```
Int[(a + b*ArcTanh[c + d*x])/(e + f*x)^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c + d*x])/(f*(e + f*x)^2) - (b*d^3*(-(f/(d^2*(d*e + f
- c*f)*(d*e - (1 + c)*f)*(e + f*x))) + Log[1 - c - d*x]/(2*d*(d*e + f - c*
f)^2) - Log[1 + c + d*x]/(2*d*(d*e - (1 + c)*f)^2) + (2*f*(d*e - c*f)*Log[
e + f*x])/(d*(d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)))/(2*f)
```

3.37.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.37.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3 \operatorname{arctanh}(dx+c)}{2(f(dx+c)-cf+de)^2 f}+\frac{d^3\left(\frac{f}{(cf-de-f)(cf-de+f)(f(dx+c)-cf+de)}+\frac{2f(cf-de)\ln(f(dx+c)-cf+de)}{(cf-de-f)^2(cf-de+f)^2}-\right.}{2f}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arctanh}(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{-\frac{\ln(dx+c-1)}{2(cf-de-f)^2}-\frac{f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}+\frac{2f(cf-de)\ln}{(cf-de-}}{2f}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\operatorname{arctanh}(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{-\frac{\ln(dx+c-1)}{2(cf-de-f)^2}-\frac{f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}+\frac{2f(cf-de)\ln}{(cf-de-}}{2f}\right.}{d}$
parallelrisch	$-a\,c^4d^2f^5+2a\,c^2d^2f^5+b\,d^5e^3f^2-b\,d^3e\,f^4-a\,d^6e^4f+x^2\operatorname{arctanh}(dx+c)b\,d^6e^2f^3-2x^2\operatorname{arctanh}(dx+c)bc\,d^4f^5+2x^2\operatorname{arcta}$
risch	Expression too large to display

input
```
int((a+b*arctanh(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arctanh(d*x+c)+1/
2*d^3/f*(f/(c*f-d*e-f)/(c*f-d*e+f)/(f*(d*x+c)-c*f+d*e)+2*f*(c*f-d*e)/(c*f-
d*e-f)^2/(c*f-d*e+f)^2*ln(f*(d*x+c)-c*f+d*e)-1/2/(c*f-d*e-f)^2*ln(d*x+c-1)
+1/2/(c*f-d*e+f)^2*ln(d*x+c+1)))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(160) = 320$.

Time = 0.83 (sec) , antiderivative size = 834, normalized size of antiderivative = 4.99

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(e+fx)^3}\,dx =$$
$$-\frac{2\,ad^4e^4-2\,(4\,ac+b)d^3e^3f+4\,(3\,ac^2+bc-a)d^2e^2f^2-2\,(4\,ac^3+bc^2-4\,ac-b)def^3+2\,(ac^4-2\,ac^2}{}$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d^4*e^4 - 2*(4*a*c + b)*d^3*e^3*f + 4*(3*a*c^2 + b*c - a)*d^2*e^
2*f^2 - 2*(4*a*c^3 + b*c^2 - 4*a*c - b)*d*e*f^3 + 2*(a*c^4 - 2*a*c^2 + a)*
f^4 - 2*(b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 - b)*d*f^4)*x - (b*d^4*e
^4 - 2*(b*c - b)*d^3*e^3*f + (b*c^2 - 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*
f^2 - 2*(b*c - b)*d^3*e*f^3 + (b*c^2 - 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*
e^3*f - 2*(b*c - b)*d^3*e^2*f^2 + (b*c^2 - 2*b*c + b)*d^2*e*f^3)*x)*log(d*
x + c + 1) + (b*d^4*e^4 - 2*(b*c + b)*d^3*e^3*f + (b*c^2 + 2*b*c + b)*d^2*
e^2*f^2 + (b*d^4*e^2*f^2 - 2*(b*c + b)*d^3*e*f^3 + (b*c^2 + 2*b*c + b)*d^2
*f^4)*x^2 + 2*(b*d^4*e^3*f - 2*(b*c + b)*d^3*e^2*f^2 + (b*c^2 + 2*b*c + b)
*d^2*e*f^3)*x)*log(d*x + c - 1) + 4*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^
3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x
+ e) + (b*d^4*e^4 - 4*b*c*d^3*e^3*f + 2*(3*b*c^2 - b)*d^2*e^2*f^2 - 4*(b*c
^3 - b*c)*d*e*f^3 + (b*c^4 - 2*b*c^2 + b)*f^4)*log(-(d*x + c + 1)/(d*x + c
 - 1)))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 - 1)*d^2*e^4*f^3 - 4*(c^3
- c)*d*e^3*f^4 + (c^4 - 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^
4 + 2*(3*c^2 - 1)*d^2*e^2*f^5 - 4*(c^3 - c)*d*e*f^6 + (c^4 - 2*c^2 + 1)*f^
7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 - 1)*d^2*e^3*f^4 - 4*
(c^3 - c)*d*e^2*f^5 + (c^4 - 2*c^2 + 1)*e*f^6)*x)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19859 vs. $2(143) = 286$.

Time = 9.02 (sec) , antiderivative size = 19859, normalized size of antiderivative = 118.92

$$\int \frac{a + b\operatorname{arctanh}(c + dx)}{(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(d*x+c))/(f*x+e)**3,x)
```

output
```
Piecewise((-(a + b*atanh(c))/(2*e**2*f + 4*e*f**2*x + 2*f**3*x**2), Eq(d,
0)), ((a*x + b*c*atanh(c + d*x)/d + b*x*atanh(c + d*x) + b*log(c/d + x + 1
/d)/d - b*atanh(c + d*x)/d)/e**3, Eq(f, 0)), (-4*a*f**2/(8*e**2*f**3 + 16*
e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*atanh(d*e/f + d*x - 1)/(8*e**2*f**3
+ 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*atanh(d*e/f + d*x - 1)/(8*e*
*2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*x**2*atanh(d*e/f + d*x
- 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*e*f/(8*e**2*f**3 + 16
*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*
x**2) - 4*b*f**2*atanh(d*e/f + d*x - 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**
5*x**2) - b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2), Eq(c, (d*e - f
)/f)), (-4*a*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*
atanh(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**
2*e*f*x*atanh(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) +
 b*d**2*f**2*x**2*atanh(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f*
*5*x**2) - b*d*e*f/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/
(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - 4*b*f**2*atanh(d*e/f + d*x + 1
)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*f**2/(8*e**2*f**3 + 16*e*f
**4*x + 8*f**5*x**2), Eq(c, (d*e + f)/f)), (-a*c**4*f**4/(2*c**4*e**2*f**5
 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**
2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2...
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int \frac{a + b\text{arctanh}(c + dx)}{(e + fx)^3}\,dx$$
$$= \frac{1}{4}\left(d\left(\frac{d\log(dx + c + 1)}{d^2e^2f - 2(c+1)def^2 + (c^2 + 2c + 1)f^3} - \frac{d\log(dx + c - 1)}{d^2e^2f - 2(c-1)def^2 + (c^2 - 2c + 1)f^3} - \frac{}{d^4e^4 - 4cd\ldots}\right.\right.$$
$$- \frac{a}{2(f^3x^2 + 2ef^2x + e^2f)}$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1
)*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c +
1)*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c
^2 - 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^
2*e^3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 -
1)*f^3)*x)) - 2*arctanh(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*b - 1/2*a/
(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 2567, normalized size of antiderivative = 15.37

$$\int \frac{a + b\text{arctanh}(c + dx)}{(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctanh(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/2*((c + 1)*d - (c - 1)*d)*((b*d^2*e - b*c*d*f)*log(-(d*x + c + 1)*d*e/(
d*x + c - 1) + d*e + (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)
*f/(d*x + c - 1) - f)/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3
*d*e*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - ((d*
x + c + 1)*b*d^2*e/(d*x + c - 1) - b*d^2*e - (d*x + c + 1)*b*c*d*f/(d*x +
c - 1) + b*c*d*f + (d*x + c + 1)*b*d*f/(d*x + c - 1))*log(-(d*x + c + 1)/(
d*x + c - 1))/((d*x + c + 1)^2*d^4*e^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d
^4*e^4/(d*x + c - 1) + d^4*e^4 - 4*(d*x + c + 1)^2*c*d^3*e^3*f/(d*x + c -
1)^2 + 8*(d*x + c + 1)*c*d^3*e^3*f/(d*x + c - 1) - 4*c*d^3*e^3*f + 6*(d*x
+ c + 1)^2*c^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*
f^2/(d*x + c - 1) + 6*c^2*d^2*e^2*f^2 - 4*(d*x + c + 1)^2*c^3*d*e*f^3/(d*x
 + c - 1)^2 + 8*(d*x + c + 1)*c^3*d*e*f^3/(d*x + c - 1) - 4*c^3*d*e*f^3 +
(d*x + c + 1)^2*c^4*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^4/(d*x + c
 - 1) + c^4*f^4 + 4*(d*x + c + 1)^2*d^3*e^3*f/(d*x + c - 1)^2 - 4*(d*x + c
 + 1)*d^3*e^3*f/(d*x + c - 1) - 12*(d*x + c + 1)^2*c*d^2*e^2*f^2/(d*x + c
- 1)^2 + 12*(d*x + c + 1)*c*d^2*e^2*f^2/(d*x + c - 1) + 12*(d*x + c + 1)^2
*c^2*d*e*f^3/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d*e*f^3/(d*x + c - 1)
- 4*(d*x + c + 1)^2*c^3*f^4/(d*x + c - 1)^2 + 4*(d*x + c + 1)*c^3*f^4/(d*x
 + c - 1) + 6*(d*x + c + 1)^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 2*d^2*e^2*f^2
- 12*(d*x + c + 1)^2*c*d*e*f^3/(d*x + c - 1)^2 + 4*c*d*e*f^3 + 6*(d*x +...
```

3.37.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.50

$$\int \frac{a+b\operatorname{arctanh}(c+dx)}{(e+fx)^3}\,dx = \frac{b\,d^2\,\ln(c+d\,x+1)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2+8\,c\,f^3+4\,d^2\,e^2\,f-8\,d\,e\,f^2+4\,f^3} - \frac{\ln(e+f\,x)\,\left(b\,d^3\,e-b\,c\,d^2\,f\right)}{c^4\,f^4-4\,c^3\,d\,e\,f^3+6\,c^2\,d^2\,e^2\,f^2-2\,c^2\,f^4-4\,c\,d^3\,e^3\,f+4\,c\,d\,e\,f^3+d^4\,e^4-2\,d^2\,e^2\,f^2+f^4} - \frac{b\,\ln(c+d\,x+1)}{4\,f\,\left(e^2+2\,e\,f\,x+f^2\,x^2\right)} - \frac{b\,d^2\,\ln(c+d\,x-1)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2-8\,c\,f^3+4\,d^2\,e^2\,f+8\,d\,e\,f^2+4\,f^3} - \frac{\frac{-a\,c^2\,f^2+2\,a\,c\,d\,e\,f-a\,d^2\,e^2+b\,d\,e\,f+a\,f^2}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}+\frac{b\,d\,f^2\,x}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}}{2\,e^2\,f+4\,e\,f^2\,x+2\,f^3\,x^2} + \frac{b\,\ln(1-d\,x-c)}{2\,f\,\left(2\,e^2+4\,e\,f\,x+2\,f^2\,x^2\right)}$$

input
```
int((a + b*atanh(c + d*x))/(e + f*x)^3,x)
```

output
```
(b*d^2*log(c + d*x + 1))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*
e*f^2 - 8*c*d*e*f^2) - (log(e + f*x)*(b*d^3*e - b*c*d^2*f))/(f^4 - 2*c^2*f
^4 + c^4*f^4 + d^4*e^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 + 6*c^2*d^2*e^2*f^2 -
 4*c*d^3*e^3*f - 4*c^3*d*e*f^3) - (b*log(c + d*x + 1))/(4*f*(e^2 + f^2*x^2
 + 2*e*f*x)) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*f^3 + 4*c^2*f^3 + 4*d
^2*e^2*f + 8*d*e*f^2 - 8*c*d*e*f^2) - ((a*f^2 - a*c^2*f^2 - a*d^2*e^2 + b*
d*e*f + 2*a*c*d*e*f)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(
f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f))/(2*e^2*f + 2*f^3*x^2 + 4*e*f^2*x) +
(b*log(1 - d*x - c))/(2*f*(2*e^2 + 2*f^2*x^2 + 4*e*f*x))
```

3.38 $\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 562

$$\begin{aligned}\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx &= \frac{b^2f^2(de-cf)x}{d^3}\\
&+\frac{abf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{2d^3}+\frac{b^2f^3(c+dx)^2}{12d^4}-\frac{b^2f^2(de-cf)\text{arctanh}(c+dx)}{d^4}\\
&+\frac{b^2f\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)(c+dx)\text{arctanh}(c+dx)}{2d^4}\\
&+\frac{bf^2(de-cf)(c+dx)^2(a+b\text{arctanh}(c+dx))}{d^4}+\frac{bf^3(c+dx)^3(a+b\text{arctanh}(c+dx))}{6d^4}\\
&+\frac{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(a+b\text{arctanh}(c+dx))^2}{d^4}\\
&-\frac{\left(d^4e^4-4cd^3e^3f+6\left(1+c^2\right)d^2e^2f^2-4c\left(3+c^2\right)def^3+\left(1+6c^2+c^4\right)f^4\right)(a+b\text{arctanh}(c+dx))^2}{4d^4f}\\
&+\frac{(e+fx)^4(a+b\text{arctanh}(c+dx))^2}{4f}\\
&-\frac{2b(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d^4}\\
&+\frac{b^2f^3\log\left(1-(c+dx)^2\right)}{12d^4}+\frac{b^2f\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)\log\left(1-(c+dx)^2\right)}{4d^4}\\
&-\frac{b^2(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d^4}\end{aligned}$$

output
```
b^2*f^2*(-c*f+d*e)*x/d^3+1/2*a*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/
d^3+1/12*b^2*f^3*(d*x+c)^2/d^4-b^2*f^2*(-c*f+d*e)*arctanh(d*x+c)/d^4+1/2*b
^2*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*(d*x+c)*arctanh(d*x+c)/d^4+b*f^2
*(-c*f+d*e)*(d*x+c)^2*(a+b*arctanh(d*x+c))/d^4+1/6*b*f^3*(d*x+c)^3*(a+b*ar
ctanh(d*x+c))/d^4+(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(
d*x+c))^2/d^4-1/4*(d^4*e^4-4*c*d^3*e^3*f+6*(c^2+1)*d^2*e^2*f^2-4*c*(c^2+3)
*d*e*f^3+(c^4+6*c^2+1)*f^4)*(a+b*arctanh(d*x+c))^2/d^4/f+1/4*(f*x+e)^4*(a+
b*arctanh(d*x+c))^2/f-2*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*
arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^4+1/12*b^2*f^3*ln(1-(d*x+c)^2)/d^4+1/4*
b^2*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*ln(1-(d*x+c)^2)/d^4-b^2*(-c*f+d
*e)*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^4
```

3.38.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 1082, normalized size of antiderivative = 1.93

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx = \frac{1}{12}\Bigg(12a^2e^3x+18a^2e^2fx^2+12a^2ef^2x^3+3a^2f^3x^4$$

$$+ab\Big(6x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\text{arctanh}(c+dx)$$

$$-\frac{-2dfx(3(1+3c^2)f^2-3cdf(8e+fx)+d^2(18e^2+6efx+f^2x^2))+3(-1+c)\left(4d^3e^3-6(-1+c)d^2e^2\right.}{}$$

$$+\frac{12b^2e^3\left(\text{arctanh}(c+dx)\left((-1+c+dx)\text{arctanh}(c+dx)-2\log\left(1+e^{-2\text{arctanh}(c+dx)}\right)\right)+\text{PolyLog}\left(2,-\right.\right.}{d}$$

$$-\frac{18b^2e^2f\Big((1-2c+c^2-d^2x^2)\text{arctanh}(c+dx)^2-2\text{arctanh}(c+dx)\left(c+dx+2c\log\left(1+e^{-2\text{arctanh}(c+d}\right.\right.}{d^2}$$

$$+\frac{b^2f^3\Big(-1-11c^2-10cdx+d^2x^2-3(1-4c+6c^2-4c^3+c^4-d^4x^4)\text{arctanh}(c+dx)^2+2\text{arctanh}(c+}{}$$

$$-\frac{3b^2ef^2(1-(c+dx)^2)^{3/2}\left(-\frac{c+dx}{\sqrt{1-(c+dx)^2}}+\frac{6c(c+dx)\text{arctanh}(c+dx)}{\sqrt{1-(c+dx)^2}}+\frac{3(c+dx)\text{arctanh}(c+dx)^2}{\sqrt{1-(c+dx)^2}}-\frac{3c^2(c+dx)\text{arcta}}{\sqrt{1-(c+d}}\right.}{}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(12*a^2*e^3*x + 18*a^2*e^2*f*x^2 + 12*a^2*e*f^2*x^3 + 3*a^2*f^3*x^4 + a*b*
(6*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[c + d*x] - (-2*d*
f*x*(3*(1 + 3*c^2)*f^2 - 3*c*d*f*(8*e + f*x) + d^2*(18*e^2 + 6*e*f*x + f^2
*x^2)) + 3*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 + c)^2*d*e*f
^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] + 3*(1 + c)*(-4*d^3*e^3 + 6*(1 + c)*
d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x])/d^4) +
(12*b^2*e^3*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 +
 E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d - (1
8*b^2*e^2*f*((1 - 2*c + c^2 - d^2*x^2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c +
d*x]*(c + d*x + 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 2*Log[1/Sqrt[1 - (
c + d*x)^2]] + 2*c*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^2 + (b^2*f^3*(
-1 - 11*c^2 - 10*c*d*x + d^2*x^2 - 3*(1 - 4*c + 6*c^2 - 4*c^3 + c^4 - d^4*
x^4)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(9*c + 13*c^3 + 3*d*x + 9*c^2
*d*x - 3*c*d^2*x^2 + d^3*x^3 + 12*(c + c^3)*Log[1 + E^(-2*ArcTanh[c + d*x]
)]) - 8*Log[1/Sqrt[1 - (c + d*x)^2]] - 36*c^2*Log[1/Sqrt[1 - (c + d*x)^2]]
 - 12*(c + c^3)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^4 - (3*b^2*e*f^2*
(1 - (c + d*x)^2)^(3/2)*(-((c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*c*(c + d*
x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]
^2)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 -
(c + d*x)^2] + ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcT...
```

3.38.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6661}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3(a+b\text{arctanh}(c+dx))^2}{d^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (de - cf + f(c+dx))^3 (a + b\text{arctanh}(c+dx))^2 d(c+dx)}{d^4}$$

$$\Big\downarrow \text{6480}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\text{arctanh}(c+dx))^2}{4f} - \frac{b\int\Big(-(c+dx)^2(a+b\text{arctanh}(c+dx))f^4-4(de-cf)(c+dx)(a+b\text{arctanh}(c+dx))f^3-(6d^2e^2-1}{}}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\text{arctanh}(c+dx))^2}{4f} - \frac{b\Big(-\frac{2f(de-cf)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)(a+b\text{arctanh}(c+dx))^2}{b}+4f(de-cf)\left(\left(c^2+1\right)f^2-2cdef+d^2\right.}{}}{}$$

input
```
Int[(e + f*x)^3*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcTanh[c + d*x])^2)/(4*f) - (b*(-2*b
*f^3*(d*e - c*f)*(c + d*x) - a*f^2*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f
^2)*(c + d*x) - (b*f^4*(c + d*x)^2)/6 + 2*b*f^3*(d*e - c*f)*ArcTanh[c + d*
x] - b*f^2*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*(c + d*x)*ArcTanh[c
+ d*x] - 2*f^3*(d*e - c*f)*(c + d*x)^2*(a + b*ArcTanh[c + d*x]) - (f^4*(c
+ d*x)^3*(a + b*ArcTanh[c + d*x]))/3 - (2*f*(d*e - c*f)*(d^2*e^2 - 2*c*d*e
*f + (1 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/b + ((d^4*e^4 - 4*c*d^3*e^
3*f + 6*(1 + c^2)*d^2*e^2*f^2 - 4*c*(3 + c^2)*d*e*f^3 + (1 + 6*c^2 + c^4)*
f^4)*(a + b*ArcTanh[c + d*x])^2)/(2*b) + 4*f*(d*e - c*f)*(d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)] - (b*f^
4*Log[1 - (c + d*x)^2])/6 - (b*f^2*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f
^2)*Log[1 - (c + d*x)^2])/2 + 2*b*f*(d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))]))/(2*f))/d^4
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2317 vs. $2(548) = 1096$.

Time = 0.28 (sec) , antiderivative size = 2318, normalized size of antiderivative = 4.12

method	result	size
derivativedivides	Expression too large to display	2318
default	Expression too large to display	2318
parts	Expression too large to display	2339
risch	Expression too large to display	3412

input
```
int((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*a^2/d^3*(c*f-d*e-f*(d*x+c))^4/f-b^2/d^3*(-1/4*f^3*arctanh(d*x+c)^
2*c^4+f^2*arctanh(d*x+c)^2*c^3*d*e+f^3*arctanh(d*x+c)^2*c^3*(d*x+c)-3/2*f*
arctanh(d*x+c)^2*c^2*d^2*e^2-3*f^2*arctanh(d*x+c)^2*c^2*d*e*(d*x+c)-3/2*f^
3*arctanh(d*x+c)^2*c^2*(d*x+c)^2+arctanh(d*x+c)^2*c*d^3*e^3+3*f*arctanh(d*
x+c)^2*c*d^2*e^2*(d*x+c)+3*f^2*arctanh(d*x+c)^2*c*d*e*(d*x+c)^2+f^3*arctan
h(d*x+c)^2*c*(d*x+c)^3-1/4/f*arctanh(d*x+c)^2*d^4*e^4-arctanh(d*x+c)^2*d^3
*e^3*(d*x+c)-3/2*f*arctanh(d*x+c)^2*d^2*e^2*(d*x+c)^2-f^2*arctanh(d*x+c)^2
*d*e*(d*x+c)^3-1/4*f^3*arctanh(d*x+c)^2*(d*x+c)^4+1/2/f*(-6*arctanh(d*x+c)
*ln(d*x+c+1)*c^2*d*e*f^3+6*arctanh(d*x+c)*ln(d*x+c+1)*c*d^2*e^2*f^2-6*arct
anh(d*x+c)*ln(d*x+c+1)*c*d*e*f^3+6*arctanh(d*x+c)*ln(d*x+c-1)*c*d^2*e^2*f^
2+6*arctanh(d*x+c)*ln(d*x+c-1)*c*d*e*f^3-2*arctanh(d*x+c)*ln(d*x+c+1)*c^3*
d*e*f^3+3*arctanh(d*x+c)*ln(d*x+c+1)*c^2*d^2*e^2*f^2-2*arctanh(d*x+c)*ln(d
*x+c+1)*c*d^3*e^3*f+12*arctanh(d*x+c)*c*d*e*f^3*(d*x+c)-6*arctanh(d*x+c)*l
n(d*x+c-1)*c^2*d*e*f^3+2*arctanh(d*x+c)*ln(d*x+c-1)*c^3*d*e*f^3-3*arctanh(
d*x+c)*ln(d*x+c-1)*c^2*d^2*e^2*f^2+2*arctanh(d*x+c)*ln(d*x+c-1)*c*d^3*e^3*
f-arctanh(d*x+c)*f^4*(d*x+c)-1/2*arctanh(d*x+c)*ln(d*x+c-1)*f^4+1/2*arctan
h(d*x+c)*ln(d*x+c+1)*f^4-1/3*arctanh(d*x+c)*f^4*(d*x+c)^3-1/3*f^2*(-6*c*f^
2*(d*x+c)+6*d*e*f*(d*x+c)+1/2*f^2*(d*x+c)^2+1/2*(18*c^2*f^2-36*c*d*e*f+18*
d^2*e^2-6*c*f^2+6*d*e*f+4*f^2)*ln(d*x+c-1)-1/2*(-18*c^2*f^2+36*c*d*e*f-18*
d^2*e^2-6*c*f^2+6*d*e*f-4*f^2)*ln(d*x+c+1))-1/6*(-3*c^4*f^4+12*c^3*d*e*...
```

3.38.5 Fricas [F]

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)^3(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^3*x^3 + 3*a^2*e*f^2*x^2 + 3*a^2*e^2*f*x + a^2*e^3 + (b^2*f^
3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x + b^2*e^3)*arctanh(d*x + c)^2 + 2*
(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^2*f*x + a*b*e^3)*arctanh(d*x + c)
, x)
```

3.38.6 Sympy [F]

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx=\int\left(a+b\,\text{atanh}\,(c+dx)\right)^2(e+fx)^3\,dx$$

input
```
integrate((f*x+e)**3*(a+b*atanh(d*x+c))**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2*(e + f*x)**3, x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(538) = 1076$.

Time = 0.40 (sec) , antiderivative size = 1363, normalized size of antiderivative = 2.43

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*f^3*x^4 + a^2*e*f^2*x^3 + 3/2*a^2*e^2*f*x^2 + 3/2*(2*x^2*arctanh(d
*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c +
 1)*log(d*x + c - 1)/d^3))*a*b*e^2*f + (2*x^3*arctanh(d*x + c) + d*((d*x^2
 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^
2 + 3*c - 1)*log(d*x + c - 1)/d^4))*a*b*e*f^2 + 1/12*(6*x^4*arctanh(d*x +
c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6
*c^2 + 4*c + 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*l
og(d*x + c - 1)/d^5))*a*b*f^3 + a^2*e^3*x + (2*(d*x + c)*arctanh(d*x + c)
+ log(-(d*x + c)^2 + 1))*a*b*e^3/d + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 +
d*e*f^2 - (3*d^2*e^2*f + f^3)*c)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c +
1/2) + dilog(1/2*d*x + 1/2*c + 1/2))*b^2/d^4 + 1/12*(13*c^3*f^3 + 18*d^2*e
^2*f - 6*d*e*f^2 - 6*(5*d*e*f^2 - 3*f^3)*c^2 + 4*f^3 + 9*(2*d^2*e^2*f - 4*
d*e*f^2 + f^3)*c)*b^2*log(d*x + c + 1)/d^4 - 1/12*(13*c^3*f^3 - 18*d^2*e^2
*f - 6*d*e*f^2 - 6*(5*d*e*f^2 + 3*f^3)*c^2 - 4*f^3 + 9*(2*d^2*e^2*f + 4*d*
e*f^2 + f^3)*c)*b^2*log(d*x + c - 1)/d^4 + 1/48*(4*b^2*d^2*f^3*x^2 + 8*(6*
d^2*e*f^2 - 5*c*d*f^3)*b^2*x + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*e*f^2*x^3 +
6*b^2*d^4*e^2*f*x^2 + 4*b^2*d^4*e^3*x - (c^4*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f
 - 4*(d*e*f^2 - f^3)*c^3 - 4*d*e*f^2 + 6*(d^2*e^2*f - 2*d*e*f^2 + f^3)*c^2
 + f^3 - 4*(d^3*e^3 - 3*d^2*e^2*f + 3*d*e*f^2 - f^3)*c)*b^2)*log(d*x + c +
 1)^2 + 3*(b^2*d^4*f^3*x^4 + 4*b^2*d^4*e*f^2*x^3 + 6*b^2*d^4*e^2*f*x^2 ...
```

3.38.8 Giac [F]

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)^3(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^3*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*(b*arctanh(d*x + c) + a)^2, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3(a+b\text{arctanh}(c+dx))^2\,dx = \int (e+f\,x)^3\,(a+b\,\text{atanh}(c+d\,x))^2\,dx$$

input
```
int((e + f*x)^3*(a + b*atanh(c + d*x))^2,x)
```

output
```
int((e + f*x)^3*(a + b*atanh(c + d*x))^2, x)
```

3.39 $\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 374

$$
\begin{aligned}
&\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx \\
&= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\text{arctanh}(c+dx)}{3d^3} \\
&\quad + \frac{2b^2f(de-cf)(c+dx)\text{arctanh}(c+dx)}{d^3} + \frac{bf^2(c+dx)^2(a+b\text{arctanh}(c+dx))}{3d^3} \\
&\quad - \frac{(de-cf)\left(d^2e^2-2cdef+(3+c^2)\,f^2\right)(a+b\text{arctanh}(c+dx))^2}{3d^3f} \\
&\quad + \frac{\left(3d^2e^2-6cdef+(1+3c^2)\,f^2\right)(a+b\text{arctanh}(c+dx))^2}{3d^3} \\
&\quad + \frac{(e+fx)^3(a+b\text{arctanh}(c+dx))^2}{3f} \\
&\quad - \frac{2b\left(3d^2e^2-6cdef+(1+3c^2)\,f^2\right)(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{3d^3} \\
&\quad + \frac{b^2f(de-cf)\log\left(1-(c+dx)^2\right)}{d^3} \\
&\quad - \frac{b^2\left(3d^2e^2-6cdef+(1+3c^2)\,f^2\right)\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{3d^3}
\end{aligned}
$$

output
```
1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctanh(d*x+c)/d^3+
2*b^2*f*(-c*f+d*e)*(d*x+c)*arctanh(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arc
tanh(d*x+c))/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arcta
nh(d*x+c))^2/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*
x+c))^2/d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x+c))^2/f-2/3*b*(3*d^2*e^2-6*c*d*
e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^3+b^2*f*(-c*f+d
*e)*ln(1-(d*x+c)^2)/d^3-1/3*b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylo
g(2,(-d*x-c-1)/(-d*x-c+1))/d^3
```

3.39.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 795 vs. $2(374) = 748$.

Time = 2.52 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.13

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx$$

$$= a^2e^2x + a^2efx^2 + \frac{1}{3}a^2f^2x^3 + \frac{1}{3}ab\Big(2x\left(3e^2+3efx+f^2x^2\right)\text{arctanh}(c+dx)$$

$$+ \frac{dfx(6de-4cf+dfx) - (-1+c)\left(3d^2e^2-3(-1+c)def+(-1+c)^2f^2\right)\log(1-c-dx) + (1+c)\left(3c\right.}{d^3}$$

$$+ \frac{b^2e^2\left(\text{arctanh}(c+dx)\left((-1+c+dx)\text{arctanh}(c+dx) - 2\log\left(1+e^{-2\text{arctanh}(c+dx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2}\right.\right.}{d}$$

$$+ \frac{b^2ef\Big(\left(-1+2c-c^2+d^2x^2\right)\text{arctanh}(c+dx)^2 + 2\text{arctanh}(c+dx)\left(c+dx+2c\log\left(1+e^{-2\text{arctanh}(c+dx)}\right.\right.}{d^2}$$

$$- \frac{b^2f^2\left(1-(c+dx)^2\right)^{3/2}\left(-\frac{c+dx}{\sqrt{1-(c+dx)^2}} + \frac{6c(c+dx)\text{arctanh}(c+dx)}{\sqrt{1-(c+dx)^2}} + \frac{3(c+dx)\text{arctanh}(c+dx)^2}{\sqrt{1-(c+dx)^2}} - \frac{3c^2(c+dx)\text{arctanh}}{\sqrt{1-(c+dx)^2}}\right.}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*ArcTanh[c + d*x] + (d*f*x*(6*d*e - 4*c*f + d*f*x) - (-1 + c)*(3*d^
2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + (1 + c)*(3*d
^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/d^3))/3 + (b^2
*e^2*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*
ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d + (b^2*e*f*
((-1 + 2*c - c^2 + d^2*x^2)*ArcTanh[c + d*x]^2 + 2*ArcTanh[c + d*x]*(c + d
*x + 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 2*Log[1/Sqrt[1 - (c + d*x)^2]
] - 2*c*PolyLog[2, -E^(-2*ArcTanh[c + d*x])]))/d^2 - (b^2*f^2*(1 - (c + d*
x)^2)^(3/2)*(-((c + d*x)/Sqrt[1 - (c + d*x)^2]) + (6*c*(c + d*x)*ArcTanh[c
 + d*x])/Sqrt[1 - (c + d*x)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 -
 (c + d*x)^2] - (3*c^2*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2]
 + ArcTanh[c + d*x]^2*Cosh[3*ArcTanh[c + d*x]] + 3*c^2*ArcTanh[c + d*x]^2*
Cosh[3*ArcTanh[c + d*x]] + 2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c + d*x]]*Log
[1 + E^(-2*ArcTanh[c + d*x])] + 6*c^2*ArcTanh[c + d*x]*Cosh[3*ArcTanh[c +
d*x]]*Log[1 + E^(-2*ArcTanh[c + d*x])] - 6*c*Cosh[3*ArcTanh[c + d*x]]*Log[
1/Sqrt[1 - (c + d*x)^2]] + (3*(1 - 4*c + 3*c^2)*ArcTanh[c + d*x]^2 + 2*Arc
Tanh[c + d*x]*(2 + (3 + 9*c^2)*Log[1 + E^(-2*ArcTanh[c + d*x])]) - 18*c*Lo
g[1/Sqrt[1 - (c + d*x)^2]])/Sqrt[1 - (c + d*x)^2] - (4*(1 + 3*c^2)*PolyLog
[2, -E^(-2*ArcTanh[c + d*x])])/(1 - (c + d*x)^2)^(3/2) - Sinh[3*ArcTanh...
```

3.39.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 6661}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\text{arctanh}(c+dx))^2}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 6480}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))^2}{3f}-\frac{2b\int\left(-\left((c+dx)(a+b\text{arctanh}(c+dx))f^3\right)-3(de-cf)(a+b\text{arctanh}(c+dx))f^2+\frac{\left((de-cf)\left(d^2e^2\right.\right.}{3f}}{}}{d^3}$$

$\Big\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))^2}{3f}-\frac{2b\left(-\frac{f\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)(a+b\text{arctanh}(c+dx))^2}{2b}+\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)(a+b\text{ar}}{2b}\right.}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTanh[c + d*x])^2)/(3*f) - (2*b*(-1
/2*(b*f^3*(c + d*x)) - 3*a*f^2*(d*e - c*f)*(c + d*x) + (b*f^3*ArcTanh[c +
d*x])/2 - 3*b*f^2*(d*e - c*f)*(c + d*x)*ArcTanh[c + d*x] - (f^3*(c + d*x)^
2*(a + b*ArcTanh[c + d*x]))/2 + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c
^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(2*b) - (f*(3*d^2*e^2 - 6*c*d*e*f + (
1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])^2)/(2*b) + f*(3*d^2*e^2 - 6*c*d*e
*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)] - (3*b
*f^2*(d*e - c*f)*Log[1 - (c + d*x)^2])/2 + (b*f*(3*d^2*e^2 - 6*c*d*e*f + (
1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/2))/(3*f))/d^3
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(360) = 720.

Time = 0.19 (sec) , antiderivative size = 1411, normalized size of antiderivative = 3.77

method	result	size
parts	Expression too large to display	1411
derivativedivides	Expression too large to display	1412
default	Expression too large to display	1412
risch	Expression too large to display	1993

input
```
int((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arctanh(d*x+c)^2*(d*x+c)^3-1/d^2*f^
2*arctanh(d*x+c)^2*(d*x+c)^2*c+1/d*f*arctanh(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^
2*arctanh(d*x+c)^2*(d*x+c)*c^2-2/d*f*arctanh(d*x+c)^2*(d*x+c)*c*e+arctanh(
d*x+c)^2*(d*x+c)*e^2-1/3/d^2*f^2*arctanh(d*x+c)^2*c^3+1/d*f*arctanh(d*x+c)
^2*c^2*e-arctanh(d*x+c)^2*c*e^2+1/3*d/f*arctanh(d*x+c)^2*e^3-2/3/d^2/f*(-1
/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e+f)*ln(d*x+c-1)-1/2*(6*c*f-6*d*e+f)*ln(
d*x+c+1))-1/2*(-c^3*f^3+3*c^2*d*e*f^2-3*c*d^2*e^2*f+d^3*e^3+3*c^2*f^3-6*c*
d*e*f^2+3*d^2*e^2*f-3*c*f^3+3*d*e*f^2+f^3)*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/
2*d*x+1/2*c+1/2)-1/2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2))-1/2*(c^3*f^3-3*c^2
*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3+3*c^2*f^3-6*c*d*e*f^2+3*d^2*e^2*f+3*c*f^3-3
*d*e*f^2+f^3)*(-1/4*ln(d*x+c+1)^2+1/2*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*
ln(-1/2*d*x-1/2*c+1/2)-1/2*dilog(1/2*d*x+1/2*c+1/2))-1/2*arctanh(d*x+c)*f^
3*(d*x+c)^2-1/2*arctanh(d*x+c)*ln(d*x+c-1)*f^3-1/2*arctanh(d*x+c)*ln(d*x+c
+1)*f^3-3/2*arctanh(d*x+c)*ln(d*x+c-1)*c^2*d*e*f^2+3/2*arctanh(d*x+c)*ln(d
*x+c-1)*c*d^2*e^2*f+3*arctanh(d*x+c)*ln(d*x+c-1)*c*d*e*f^2+3/2*arctanh(d*x
+c)*ln(d*x+c+1)*c^2*d*e*f^2-3/2*arctanh(d*x+c)*ln(d*x+c+1)*c*d^2*e^2*f+3*a
rctanh(d*x+c)*ln(d*x+c+1)*c*d*e*f^2+3*arctanh(d*x+c)*c*f^3*(d*x+c)+1/2*arc
tanh(d*x+c)*ln(d*x+c-1)*c^3*f^3-1/2*arctanh(d*x+c)*ln(d*x+c-1)*d^3*e^3-3/2
*arctanh(d*x+c)*ln(d*x+c-1)*c^2*f^3+3/2*arctanh(d*x+c)*ln(d*x+c-1)*c*f^3-1
/2*arctanh(d*x+c)*ln(d*x+c+1)*c^3*f^3+1/2*arctanh(d*x+c)*ln(d*x+c+1)*d^...
```

3.39.5 Fricas [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)^2(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arctanh(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*ar
ctanh(d*x + c), x)
```

3.39.6 Sympy [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx = \int (a+b\,\text{atanh}\,(c+dx))^2\,(e+fx)^2\,\,dx$$

input
```
integrate((f*x+e)**2*(a+b*atanh(d*x+c))**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2*(e + f*x)**2, x)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(350) = 700$.

Time = 0.39 (sec) , antiderivative size = 806, normalized size of antiderivative = 2.16

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx = \frac{1}{3}\,a^2f^2x^3 + a^2efx^2$$
$$+\left(2\,x^2\,\text{artanh}\,(dx+c) + d\left(\frac{2\,x}{d^2} - \frac{(c^2+2\,c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2\,c+1)\log(dx+c-1)}{d^3}\right)\right)ab$$
$$+\frac{1}{3}\left(2\,x^3\,\text{artanh}\,(dx+c) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{(c^3+3\,c^2+3\,c+1)\log(dx+c+1)}{d^4} - \frac{(c^3-3\,c^2+3\,c-1)}{d^4}\right.\right.$$
$$+a^2e^2x + \frac{\left(2\,(dx+c)\,\text{artanh}\,(dx+c) + \log\left(-(dx+c)^2+1\right)\right)abe^2}{d}$$
$$+\frac{(3\,d^2e^2-6\,cdef+3\,c^2f^2+f^2)\left(\log(dx+c+1)\log\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}\,dx+\frac{1}{2}\,c+\frac{1}{2}\right)\right)b^2}{3\,d^3}$$
$$-\frac{(5\,c^2f^2-6\,def-6\,(def-f^2)c+f^2)b^2\log(dx+c+1)}{6\,d^3}$$
$$+\frac{(5\,c^2f^2+6\,def-6\,(def+f^2)c+f^2)b^2\log(dx+c-1)}{6\,d^3}$$
$$+\frac{4\,b^2df^2x + (b^2d^3f^2x^3+3\,b^2d^3efx^2+3\,b^2d^3e^2x+(c^3f^2+3\,d^2e^2-3\,(def-f^2)c^2-3\,def+3\,(d^2e^2-2}{}$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + (2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^
2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))
*a*b*e*f + 1/3*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3
*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c
 - 1)/d^4))*a*b*f^2 + a^2*e^2*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*
x + c)^2 + 1))*a*b*e^2/d + 1/3*(3*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2 + f^2)*(
log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x + 1/2*c + 1/2
))*b^2/d^3 - 1/6*(5*c^2*f^2 - 6*d*e*f - 6*(d*e*f - f^2)*c + f^2)*b^2*log(d
*x + c + 1)/d^3 + 1/6*(5*c^2*f^2 + 6*d*e*f - 6*(d*e*f + f^2)*c + f^2)*b^2*
log(d*x + c - 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*
e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3
*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*log(d*x + c + 1)^2 + (b
^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^
2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b
^2)*log(-d*x - c + 1)^2 + 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b
^2*x)*log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b^
2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 +
3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c
+ f^2)*b^2)*log(d*x + c + 1))*log(-d*x - c + 1))/d^3
```

3.39.8 Giac [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)^2(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arctanh(d*x + c) + a)^2, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^2\,dx = \int (e+f\,x)^2\,(a+b\,\text{atanh}(c+d\,x))^2\,dx$$

input
```
int((e + f*x)^2*(a + b*atanh(c + d*x))^2,x)
```

output
```
int((e + f*x)^2*(a + b*atanh(c + d*x))^2, x)
```

3.40 $\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx$

3.40.1 Optimal result

Integrand size = 18, antiderivative size = 221

$$\begin{aligned}\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx &= \frac{abfx}{d}+\frac{b^2f(c+dx)\text{arctanh}(c+dx)}{d^2}+\frac{(de-cf)(a+b\text{arctanh}(c+dx))^2}{d^2}\\ &\quad-\frac{(d^2e^2-2cdef+(1+c^2)f^2)(a+b\text{arctanh}(c+dx))^2}{2d^2f}\\ &\quad+\frac{(e+fx)^2(a+b\text{arctanh}(c+dx))^2}{2f}-\frac{2b(de-cf)(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d^2}\\ &\quad+\frac{b^2f\log(1-(c+dx)^2)}{2d^2}-\frac{b^2(de-cf)\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d^2}\end{aligned}$$

output

```
a*b*f*x/d+b^2*f*(d*x+c)*arctanh(d*x+c)/d^2+(-c*f+d*e)*(a+b*arctanh(d*x+c))
^2/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arctanh(d*x+c))^2/d^2/f+1/
2*(f*x+e)^2*(a+b*arctanh(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arctanh(d*x+c))*l
n(2/(-d*x-c+1))/d^2+1/2*b^2*f*ln(1-(d*x+c)^2)/d^2-b^2*(-c*f+d*e)*polylog(2
,(-d*x-c-1)/(-d*x-c+1))/d^2
```

3.40.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx$$
$$= \frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(-1+c+dx)(2de+f-cf+dfx)\text{arctanh}(}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d
^2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcTanh[c + d*x]^2
 + 2*b*ArcTanh[c + d*x]*(-((c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) -
 2*b*(d*e - c*f)*Log[1 + E^(-2*ArcTanh[c + d*x])]) + a*b*f*Log[1 - c - d*x
] - a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/Sqrt[1 - (c + d*x)^2]] - 2*b^
2*f*Log[1/Sqrt[1 - (c + d*x)^2]] + 4*a*b*c*f*Log[1/Sqrt[1 - (c + d*x)^2]]
+ 2*b^2*(d*e - c*f)*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/(2*d^2)
```

3.40.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx$$
$$\Big\downarrow \text{6661}$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\text{arctanh}(c+dx))^2}{d}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (de-cf+f(c+dx))(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d^2}$$
$$\Big\downarrow \text{6480}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))^2}{2f}-\frac{b\int\left(\frac{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2+2f(de-cf)(c+dx)\right)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}-f^2(a+b\text{arctanh}(c+dx))\right.}{f}}{d^2}$$

$\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))^2}{2f}-\frac{b\left(\frac{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)(a+b\text{arctanh}(c+dx))^2}{2b}-\frac{f(de-cf)(a+b\text{arctanh}(c+dx))^2}{b}+2f(de-cf)\right.}{}}{d}$$

input
```
Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTanh[c + d*x])^2)/(2*f) - (b*(-(a*
f^2*(c + d*x)) - b*f^2*(c + d*x)*ArcTanh[c + d*x] - (f*(d*e - c*f)*(a + b*
ArcTanh[c + d*x])^2)/b + ((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*Arc
Tanh[c + d*x])^2)/(2*b) + 2*f*(d*e - c*f)*(a + b*ArcTanh[c + d*x])*Log[2/(
1 - c - d*x)] - (b*f^2*Log[1 - (c + d*x)^2])/2 + b*f*(d*e - c*f)*PolyLog[2
, -((1 + c + d*x)/(1 - c - d*x))]))/f)/d^2
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(217) = 434$.

Time = 0.14 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.05

method	result
parts	$a^2\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b^2\left(\frac{\operatorname{arctanh}(dx+c)^2(dx+c)^2 f}{2d}-\frac{\operatorname{arctanh}(dx+c)^2 cf(dx+c)}{d}+\operatorname{arctanh}(dx+c)^2 e(dx+c)-\frac{-\operatorname{arctanh}(dx\ldots}{\ldots}\right.}{\ldots}$
derivativedivides	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arctanh}(dx+c)^2 fc(dx+c)-\operatorname{arctanh}(dx+c)^2 ed(dx+c)-\frac{\operatorname{arctanh}(dx+c)^2 f(dx+c)^2}{2}-\operatorname{arc}\ldots\right.}{\ldots}$
default	$-\frac{a^2\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b^2\left(\operatorname{arctanh}(dx+c)^2 fc(dx+c)-\operatorname{arctanh}(dx+c)^2 ed(dx+c)-\frac{\operatorname{arctanh}(dx+c)^2 f(dx+c)^2}{2}-\operatorname{arc}\ldots\right.}{\ldots}$
risch	$\frac{abfx}{d}+\left(-\frac{b^2x(fx+2e)\ln(-dx-c+1)}{4}+\frac{b(2a\,d^2f\,x^2+4a\,d^2ex+\ln(-dx-c+1)bc^2f-2\ln(-dx-c+1)bcde-2\ln(-dx\ldots}{4d^2}\right.$

input
```
int((f*x+e)*(a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arctanh(d*x+c)^2*(d*x+c)^2*f-1/d*arctanh(
d*x+c)^2*c*f*(d*x+c)+arctanh(d*x+c)^2*e*(d*x+c)-1/d*(-arctanh(d*x+c)*(d*x+
c)*f+arctanh(d*x+c)*ln(d*x+c-1)*c*f-arctanh(d*x+c)*ln(d*x+c-1)*d*e-1/2*arc
tanh(d*x+c)*ln(d*x+c-1)*f+arctanh(d*x+c)*ln(d*x+c+1)*c*f-arctanh(d*x+c)*ln
(d*x+c+1)*d*e+1/2*arctanh(d*x+c)*ln(d*x+c+1)*f-1/2*ln(d*x+c-1)*f-1/2*ln(d*
x+c+1)*f-1/2*(-2*c*f+2*d*e+f)*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1/2*c+1
/2)-1/2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2))-1/2*(-2*c*f+2*d*e-f)*(-1/4*ln(d
*x+c+1)^2+1/2*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1
/2*dilog(1/2*d*x+1/2*c+1/2))))+2*a*b/d*(1/2/d*arctanh(d*x+c)*(d*x+c)^2*f-1
/d*arctanh(d*x+c)*c*f*(d*x+c)+arctanh(d*x+c)*e*(d*x+c)-1/2/d*(-f*(d*x+c)-1
/2*(-2*c*f+2*d*e+f)*ln(d*x+c-1)+1/2*(2*c*f-2*d*e+f)*ln(d*x+c+1)))
```

3.40.5 Fricas [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctanh(d*x + c)^2 + 2*(a*b*f
*x + a*b*e)*arctanh(d*x + c), x)
```

3.40.6 Sympy [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx = \int (a+b\,\text{atanh}\,(c+dx))^2\,(e+fx)\,\,dx$$

input
```
integrate((f*x+e)*(a+b*atanh(d*x+c))**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2*(e + f*x), x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(207) = 414$.

Time = 0.42 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.88

$$\begin{aligned}\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx &= \frac{1}{2}\,a^2fx^2\\ &+\frac{1}{2}\left(2\,x^2\,\text{artanh}\,(dx+c)+d\left(\frac{2\,x}{d^2}-\frac{(c^2+2\,c+1)\log{(dx+c+1)}}{d^3}+\frac{(c^2-2\,c+1)\log{(dx+c-1)}}{d^3}\right)\right)\\ &+a^2ex+\frac{\left(2\,(dx+c)\,\text{artanh}\,(dx+c)+\log\left(-(dx+c)^2+1\right)\right)abe}{d}\\ &+\frac{(de-cf)\left(\log{(dx+c+1)}\log\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}\,dx+\frac{1}{2}\,c+\frac{1}{2}\right)\right)b^2}{d^2}\\ &+\frac{(cf+f)b^2\log{(dx+c+1)}}{2\,d^2}-\frac{(cf-f)b^2\log{(dx+c-1)}}{2\,d^2}\\ &+\frac{4\,b^2dfx\log{(dx+c+1)}+(b^2d^2fx^2+2\,b^2d^2ex-(c^2f-2\,(de-f)c-2\,de+f)b^2)\log{(dx+c+1)}^2+}{}\end{aligned}$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*f*x^2 + 1/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)
*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2
*e*x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d + (d
*e - c*f)*(log(d*x + c + 1)*log(-1/2*d*x - 1/2*c + 1/2) + dilog(1/2*d*x +
1/2*c + 1/2))*b^2/d^2 + 1/2*(c*f + f)*b^2*log(d*x + c + 1)/d^2 - 1/2*(c*f
- f)*b^2*log(d*x + c - 1)/d^2 + 1/8*(4*b^2*d*f*x*log(d*x + c + 1) + (b^2*d
^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(d*
x + c + 1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2
*d*e + f)*b^2)*log(-d*x - c + 1)^2 - 2*(2*b^2*d*f*x + (b^2*d^2*f*x^2 + 2*b
^2*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(d*x + c + 1))*lo
g(-d*x - c + 1))/d^2
```

3.40.8 Giac [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx = \int (fx+e)(b\,\text{artanh}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arctanh(d*x + c) + a)^2, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^2\,dx = \int (e+f\,x)\,\left(a+b\,\text{atanh}(c+d\,x)\right)^2\,dx$$

input
```
int((e + f*x)*(a + b*atanh(c + d*x))^2,x)
```

output
```
int((e + f*x)*(a + b*atanh(c + d*x))^2, x)
```

3.41 $\int (a+b\text{arctanh}(c+dx))^2\,dx$

3.41.1 Optimal result

Integrand size = 12, antiderivative size = 97

$$\int (a+b\text{arctanh}(c+dx))^2\,dx=\frac{(a+b\text{arctanh}(c+dx))^2}{d}+\frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{d}-\frac{2b(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{b^2\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d}$$

output
```
(a+b*arctanh(d*x+c))^2/d+(d*x+c)*(a+b*arctanh(d*x+c))^2/d-2*b*(a+b*arctanh
(d*x+c))*ln(2/(-d*x-c+1))/d-b^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int (a+b\text{arctanh}(c+dx))^2\,dx=\frac{b^2(-1+c+dx)\text{arctanh}(c+dx)^2+2b\text{arctanh}(c+dx)\left(ac+adx-b\log\left(1+e^{-2\text{arctanh}(c+dx)}\right)\right)+a\Big(ac+}{d}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
(b^2*(-1 + c + d*x)*ArcTanh[c + d*x]^2 + 2*b*ArcTanh[c + d*x]*(a*c + a*d*x
 - b*Log[1 + E^(-2*ArcTanh[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/Sqrt[1
 - (c + d*x)^2]]) + b^2*PolyLog[2, -E^(-2*ArcTanh[c + d*x])])/d
```

3.41.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6653, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \; 6653$$

$$\frac{\int (a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 6436$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^2-2b\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 6546$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^2-2b\left(\int \frac{a+b\text{arctanh}(c+dx)}{-c-dx+1}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}\right)}{d}$$

$$\Big\downarrow \; 6470$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^2-2b\left(-b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+ba$$

$$\Big\downarrow \; 2849$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^2-2b\left(b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-\frac{2}{-c-dx+1}}d\frac{1}{-c-dx+1}-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+bar$$

$$\Big\downarrow \; 2752$$

$$\frac{(c+dx)(a+b\operatorname{arctanh}(c+dx))^2-2b\left(-\frac{(a+b\operatorname{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+b\operatorname{arctanh}(c+dx))+\frac{1}{2}b\operatorname{PolyLo}\right.}{d}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcTanh[c + d*x])^2 - 2*b*(-1/2*(a + b*ArcTanh[c + d*x])
^2/b + (a + b*ArcTanh[c + d*x])*Log[2/(1 - c - d*x)] + (b*PolyLog[2, 1 - 2
/(1 - c - d*x)])/2))/d
```

3.41.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6653
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.41.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

method	result
parts	$a^2x + \frac{b^2\left(\operatorname{arctanh}(dx+c)^2(dx+c-1)+2\operatorname{arctanh}(dx+c)^2-2\operatorname{arctanh}(dx+c)\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-\operatorname{polylog}\left(2,-\frac{(dx+c+}{1-(dx+}\right.\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\operatorname{arctanh}(dx+c)^2(dx+c-1)+2\operatorname{arctanh}(dx+c)^2-2\operatorname{arctanh}(dx+c)\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-\operatorname{polylog}\left(2,-\frac{(dx+}{1-(}\right.\right.}{d}$
default	$\frac{(dx+c)a^2+b^2\left(\operatorname{arctanh}(dx+c)^2(dx+c-1)+2\operatorname{arctanh}(dx+c)^2-2\operatorname{arctanh}(dx+c)\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-\operatorname{polylog}\left(2,-\frac{(dx+}{1-(}\right.\right.}{d}$
risch	$\left(-\frac{b^2x\ln(-dx-c+1)}{2}-\frac{b(-2adx+b\ln(-dx-c+1)c-b\ln(-dx-c+1))}{2d}\right)\ln(dx+c+1)-\frac{a^2}{d}+\frac{b^2\operatorname{dilog}\left(-\right.}{d}$

input
```
int((a+b*arctanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(arctanh(d*x+c)^2*(d*x+c-1)+2*arctanh(d*x+c)^2-2*arctanh(d*x+c
)*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))-polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2)))+2
*a*b/d*((d*x+c)*arctanh(d*x+c)+1/2*ln(1-(d*x+c)^2))
```

3.41.5 Fricas [F]

$$\int (a + b\operatorname{arctanh}(c+dx))^2\,dx = \int (b\operatorname{artanh}(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2, x)
```

3.41.6 Sympy [F]

$$\int (a + b\text{arctanh}(c + dx))^2 \, dx = \int (a + b\,\text{atanh}\,(c + dx))^2 \, dx$$

input
```
integrate((a+b*atanh(d*x+c))**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2, x)
```

3.41.7 Maxima [F]

$$\int (a + b\text{arctanh}(c + dx))^2 \, dx = \int (b\,\text{artanh}\,(dx + c) + a)^2 \, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(c*d*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2) +
d^2*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(
d*x + c - 1)/d^3) - 2*c*d*integrate(x*log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 - 1), x) - 2*c^2*integrate(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2
 - 1), x) + d*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2
) - 6*d*integrate(x*log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x) - 4
*c*integrate(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x) - (d*x + c
 - 1)*(log(-d*x - c + 1)^2 - 2*log(-d*x - c + 1) + 2)/d - (d*x*log(d*x + c
 + 1)^2 + 2*(d*x - (d*x + c + 1)*log(d*x + c + 1))*log(-d*x - c + 1))/d -
2*integrate(log(d*x + c + 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x))*b^2 + a^2*
x + (2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d
```

3.41.8 Giac [F]

$$\int (a + b\text{arctanh}(c + dx))^2 \, dx = \int (b\,\text{artanh}\,(dx + c) + a)^2 \, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arctanh}(c + dx))^2 \, dx = \int (a + b \operatorname{atanh}(c + d x))^2 \, dx$$

input
```
int((a + b*atanh(c + d*x))^2,x)
```

output
```
int((a + b*atanh(c + d*x))^2, x)
```

3.42 $\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 214

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\ &+\frac{b(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{f}\\ &-\frac{b(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\ &+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+c+dx}\right)}{2f}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\end{aligned}$$

output

```
-(a+b*arctanh(d*x+c))^2*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))^2*ln(2*d*(f
*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+b*(a+b*arctanh(d*x+c))*polylog(2,1-2/(d*x+
c+1))/f-b*(a+b*arctanh(d*x+c))*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c
+1))/f+1/2*b^2*polylog(3,1-2/(d*x+c+1))/f-1/2*b^2*polylog(3,1-2*d*(f*x+e)/
(-c*f+d*e+f)/(d*x+c+1))/f
```

3.42.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 102.08 (sec) , antiderivative size = 3806, normalized size of antiderivative = 17.79

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x),x]
```

output
```
(a^2*Log[e + f*x])/f - ((2*I)*a*b*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c
+ d*x)^2]] + Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-
I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*A
rcTanh[c + d*x])^2 + 2*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log
[1 - E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2
*I)*ArcTanh[c + d*x])*Log[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi -
 (2*I)*ArcTanh[c + d*x])*Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(
I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e
 - c*f)/f] + ArcTanh[c + d*x]]] - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e -
c*f)/f] + I*ArcTanh[c + d*x]))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c
+ d*x]))])/2))/f + (b^2*(d*e - c*f + f*(c + d*x))*((Sqrt[1 - (c + d*x)^2]*
((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))
/Sqrt[1 - (c + d*x)^2])*ArcTanh[c + d*x]^3)/(3*(d*e - c*f)*(d*e - c*f + f*
(c + d*x))) - (Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/
Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])*(ArcTanh[c +
d*x]^3/3 + ArcTanh[c + d*x]^2*Log[1 + E^(-2*ArcTanh[c + d*x])] - ArcTanh[c
 + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] - PolyLog[3, -E^(-2*ArcTanh[c
 + d*x])]/2))/(f*(d*e - c*f + f*(c + d*x))) - (-6*d*e*ArcTanh[c + d*x]^3 +
 2*f*ArcTanh[c + d*x]^3 + 6*c*f*ArcTanh[c + d*x]^3 - 4*E^ArcTanh[c - (d*e)
/f]*Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]^3 - ...
```

3.42.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6661, 27, 6474}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}dx$$
$$\Big\downarrow \text{ 6661}$$
$$\frac{\int \frac{d(a+b\text{arctanh}(c+dx))^2}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{f(c+dx)-cf+de}d(c+dx)$$
$$\Big\downarrow \text{ 6474}$$
$$-\frac{b(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{f}+\frac{(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\text{arctanh}(c+dx))^2}{f}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)}{2f}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x),x]
```

output
```
-(((a + b*ArcTanh[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c
 + d*x])^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x
))])/f + (b*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f -
(b*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/(
(d*e + f - c*f)*(1 + c + d*x))])/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])
/(2*f) - (b^2*PolyLog[3, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f
)*(1 + c + d*x))])/(2*f)
```

3.42. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}dx$

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6474
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcTanh[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcT
anh[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.42.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.36 (sec) , antiderivative size = 1700, normalized size of antiderivative = 7.94

method	result	size
derivativedivides	Expression too large to display	1700
default	Expression too large to display	1700
parts	Expression too large to display	1800

input
```
int((a+b*arctanh(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctanh
(d*x+c)^2+2/f*(1/2*arctanh(d*x+c)^2*ln(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(
-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)-1/4*I*
Pi*csgn(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)
*e*d+((d*x+c+1)^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*(csgn
(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((
d*x+c+1)^2/((d*x+c)^2-1)+1)*f))*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))-csgn
(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((
d*x+c+1)^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I/(1-(d
*x+c+1)^2/((d*x+c)^2-1)))-csgn(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+
c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)^2/((d*x+c)^2-1)+1)*f))*csgn(I*(f*c*
(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)
^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))+csgn(I*(f*c*(1-(d*x+
c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)^2/((d*x
+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2)*arctanh(d*x+c)^2+1/2*arct
anh(d*x+c)*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))-1/4*polylog(3,-(d*x+c+1)^
2/(1-(d*x+c)^2))-1/2/(c*f-d*e-f)*f*c*arctanh(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*
x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))-1/2/(c*f-d*e-f)*f*c*arctanh(d*x+c)*po
lylog(2,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/4/(c*f-d*e-f
)*f*c*polylog(3,(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/2...
```

3.42.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f*x + e)
, x)
```

3.42.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\operatorname{atanh}(c+dx))^2}{e+fx}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**2/(f*x+e),x)
```

output
```
Integral((a + b*atanh(c + d*x))**2/(e + f*x), x)
```

3.42.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(d*x + c + 1) - log(-d*x - c +
1))^2/(f*x + e) + a*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)
```

3.42.8 Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^2}{e+fx}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(f*x + e), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{e+fx}\,dx=\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^2}{e+f\,x}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(e + f*x),x)
```

output
```
int((a + b*atanh(c + d*x))^2/(e + f*x), x)
```

3.43 $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 480

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx =& -\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)} \\ &+ \frac{b^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} \\ &- \frac{abd\log(1-c-dx)}{f(de+f-cf)} - \frac{b^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{f(de-f-cf)} \\ &+ \frac{2b^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\ &+ \frac{abd\log(1+c+dx)}{f(de-f-cf)} + \frac{2abd\log(e+fx)}{f^2-(de-cf)^2} \\ &- \frac{2b^2d\text{arctanh}(c+dx)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)} \\ &+ \frac{b^2d\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2f(de+f-cf)} + \frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)} \\ &- \frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)} \\ &+ \frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}\end{aligned}$$

output
```
-(a+b*arctanh(d*x+c))^2/f/(f*x+e)+b^2*d*arctanh(d*x+c)*ln(2/(-d*x-c+1))/f/
(-c*f+d*e+f)-a*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*arctanh(d*x+c)*ln(2/(
d*x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*arctanh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-
f)/(-c*f+d*e+f)+a*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*ln(f*x+e)/(f^2-(-
c*f+d*e)^2)-2*b^2*d*arctanh(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/
(-c*f+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c
*f+d*e+f)+1/2*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*polylog(
2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*polylog(2,1-2*d*(f*x+e)/(
-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.35 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.89

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(e + fx)^2}\,dx$$

$$= \frac{-\frac{a^2}{f} + \frac{2ab\left(\left(f-c^2f+d^2ex+cd(e-fx)\right)\text{arctanh}(c+dx)-d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{1-(c+dx)^2}}\right)\right)}{(de+f-cf)(de-(1+c)f)} + \frac{b^2d(e+fx)\left(-\frac{e^{-\text{arctanh}\left(\frac{de-cf}{f}\right)}\text{arctanh}(c-}{f\sqrt{1-\frac{(de-cf)^2}{f^2}}}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2,x]
```

output
```
(-(a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*ArcTanh[c + d*x]
 - d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 - (c + d*x)^2]]))/((d*e + f - c*f)
*(d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(-(ArcTanh[c + d*x]^2/(E^ArcTanh[(d
*e - c*f)/f]*f*Sqrt[1 - (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTanh[c + d*x]
^2)/(d*(e + f*x)) + ((d*e - c*f)*(I*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - 2
*ArcTanh[c + d*x]*Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]
))] - I*Pi*(ArcTanh[c + d*x] + Log[1/Sqrt[1 - (c + d*x)^2]]) - 2*ArcTanh[(
d*e - c*f)/f]*(ArcTanh[c + d*x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] +
ArcTanh[c + d*x]))] - Log[I*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]
]]) + PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]))/(d^
2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*e - c*f))/(e + f*x)
```

3.43.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6659, 7292, 6671, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}dx$$

$$\Big\downarrow \ 6659$$

$$\frac{2bd\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \ 7292$$

$$\frac{2bd\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \ 6671$$

$$\frac{2b\int \frac{d(a+b\text{arctanh}(c+dx))}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \ 27$$

$$\frac{2bd\int \frac{a+b\text{arctanh}(c+dx)}{(de-cf+f(c+dx))(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \ 7276$$

$$\frac{2bd\int \left(-\frac{a}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{b\text{arctanh}(c+dx)}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}\right)d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \ 2009$$

$$\frac{2bd\left(-\frac{a\log(-c-dx+1)}{2(-cf+de+f)}+\frac{a\log(c+dx+1)}{2(de-(c+1)f)}-\frac{af\log(f(c+dx)-cf+de)}{(-cf+de+f)(de-(c+1)f)}+\frac{b\text{arctanh}(c+dx)\log\left(\frac{2}{-c-dx+1}\right)}{2(-cf+de+f)}-\frac{b\text{arctanh}(c+dx)\log\left(\frac{\ldots}{c+d\ldots}\right.}{2(-cf+de-f)}\ldots}{\ldots}$$

$$\frac{(a+b\text{arctanh}(c+dx))^2}{f(e+fx)}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a + b*ArcTanh[c + d*x])^2/(f*(e + f*x))) + (2*b*d*((b*ArcTanh[c + d*x]*
Log[2/(1 - c - d*x)])/(2*(d*e + f - c*f)) - (a*Log[1 - c - d*x])/(2*(d*e +
 f - c*f)) - (b*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(2*(d*e - f - c*f))
 + (b*f*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1
+ c)*f)) + (a*Log[1 + c + d*x])/(2*(d*e - (1 + c)*f)) - (a*f*Log[d*e - c*f
 + f*(c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (b*f*ArcTanh[c + d*
x]*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d
*e + f - c*f)*(d*e - (1 + c)*f)) + (b*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/(4*(d*e + f - c*f)) + (b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(4*(d*e
- f - c*f)) - (b*f*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*
e - (1 + c)*f)) + (b*f*PolyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e
+ f - c*f)*(1 + c + d*x))])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f))))/f
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.43.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\operatorname{arctanh}(dx+c)^2}{(f(dx+c)-cf+de)f}+\frac{2d^2\left(-\frac{\operatorname{arctanh}(dx+c)f\ln(f(dx+c)-cf+de)}{(cf-de-f)(cf-de+f)}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2cf-2de-2f}-\frac{\operatorname{arctanh}(\cdots}{2c\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arctanh}(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\left(-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2cf-2de-2f}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2cf-2de+2f}+\frac{\operatorname{arctanh}(dx+c)f}{(cf-de-\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\operatorname{arctanh}(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\left(-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2cf-2de-2f}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2cf-2de+2f}+\frac{\operatorname{arctanh}(dx+c)f}{(cf-de-\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$

input
```
int((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arctanh(d*x+c)^2+2*d^2/f*
(-arctanh(d*x+c)*f/(c*f-d*e-f)/(c*f-d*e+f)*ln(f*(d*x+c)-c*f+d*e)+arctanh(d
*x+c)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-arctanh(d*x+c)/(2*c*f-2*d*e+2*f)*ln(d*
x+c+1)-1/(c*f-d*e-f)/(c*f-d*e+f)*(1/2*f*(dilog((f*(d*x+c)-f)/(c*f-d*e-f))+
ln(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)-f)/(c*f-d*e-f)))-1/2*f*(dilog((f*(d*x+
c)+f)/(c*f-d*e+f))+ln(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)+f)/(c*f-d*e+f))))+1
/2/(c*f-d*e-f)*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1/2*c+1/2)-1/2*ln(d*x+
c-1)*ln(1/2*d*x+1/2*c+1/2))-1/2/(c*f-d*e+f)*(-1/4*ln(d*x+c+1)^2+1/2*(ln(d*
x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/2*dilog(1/2*d*x+1/2
*c+1/2))))-2*a*b*d/(d*f*x+d*e)/f*arctanh(d*x+c)-2*a*b*d/(c*f-d*e-f)/(c*f-d
*e+f)*ln(f*(d*x+c)-c*f+d*e)+2*a*b*d/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*a*b*
d/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)
```

3.43.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(dx+c\right)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^2*x^2
+ 2*e*f*x + e^2), x)
```

3.43.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c+dx\right)\right)^2}{\left(e+fx\right)^2}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2/(e + f*x)**2, x)
```

3.43.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f - (c
- 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*arct
anh(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(log(-d*x - c + 1)^2/(f^2*x + e*
f) + integrate(-((d*f*x + c*f - f)*log(d*x + c + 1)^2 + 2*(d*f*x + d*e - (
d*f*x + c*f - f)*log(d*x + c + 1))*log(-d*x - c + 1))/(d*f^3*x^3 + c*e^2*f
 - e^2*f + (2*d*e*f^2 + c*f^3 - f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 - 2*e*f^2)
*x), x)) - a^2/(f^2*x + e*f)
```

3.43.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^2, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(a+b\,\text{atanh}(c+d\,x))^2}{(e+f\,x)^2}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*atanh(c + d*x))^2/(e + f*x)^2, x)
```

3.44 $\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^2}{(e+fx)^3}\,dx$

3.44.1 Optimal result

Integrand size = 20, antiderivative size = 750

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx = -\frac{abd}{(f^2-(de-cf)^2)(e+fx)} + \frac{b^2d\text{arctanh}(c+dx)}{(de+f-cf)(de-(1+c)f)(e+fx)} - \frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2} + \frac{b^2d^2\text{arctanh}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{2f(de+f-cf)^2} - \frac{abd^2\log(1-c-dx)}{2f(de+f-cf)^2} + \frac{b^2d^2\log(1-c-dx)}{2(de+f-cf)^2(de-(1+c)f)} - \frac{b^2d^2\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{2f(de-f-cf)^2} + \frac{2b^2d^2(de-cf)\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)^2(de-(1+c)f)^2} + \frac{abd^2\log(1+c+dx)}{2f(de-f-cf)^2} - \frac{b^2d^2\log(1+c+dx)}{2(de+f-cf)(de-(1+c)f)^2} + \frac{b^2d^2f\log(e+fx)}{(de+f-cf)^2(de-(1+c)f)^2} - \frac{2abd^2(de-cf)\log(e+fx)}{(de+f-cf)^2(de-(1+c)f)^2} - \frac{2b^2d^2(de-cf)\text{arctanh}(c+dx)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)^2(de-(1+c)f)^2} + \frac{b^2d^2\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{4f(de+f-cf)^2} + \frac{b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{4f(de-f-cf)^2} - \frac{b^2d^2(de-cf)\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)^2(de-(1+c)f)^2} + \frac{b^2d^2(de-cf)\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)^2(de-(1+c)f)^2}$$

output
```
-a*b*d/(f^2-(-c*f+d*e)^2)/(f*x+e)+b^2*d*arctanh(d*x+c)/(-c*f+d*e-f)/(-c*f+
d*e+f)/(f*x+e)-1/2*(a+b*arctanh(d*x+c))^2/f/(f*x+e)^2+1/2*b^2*d^2*arctanh(
d*x+c)*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)^2-1/2*a*b*d^2*ln(-d*x-c+1)/f/(-c*f+
d*e+f)^2+1/2*b^2*d^2*ln(-d*x-c+1)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)-1/2*b^2*d^2
*arctanh(d*x+c)*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)^2+2*b^2*d^2*(-c*f+d*e)*arct
anh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+1/2*a*b*d^2*ln(d
*x+c+1)/f/(-c*f+d*e-f)^2-1/2*b^2*d^2*ln(d*x+c+1)/(-c*f+d*e+f)/(d*e-(1+c)*f
)^2+b^2*d^2*f*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*a*b*d^2*(-c*f+d*e
)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2-2*b^2*d^2*(-c*f+d*e)*arctanh(d*
x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
+1/4*b^2*d^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)^2+1/4*b^2*d^2
*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)^2-b^2*d^2*(-c*f+d*e)*polylog(2,1-
2/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2+b^2*d^2*(-c*f+d*e)*polylog(2,1
-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

3.44.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 1318, normalized size of antiderivative = 1.76

$$\int \frac{(a + b\text{arctanh}(c + dx))^2}{(e + fx)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3,x]
```

output
```
-1/2*a^2/(f*(e + f*x)^2) + (a*b*(d*e - c*f + f*(c + d*x))^3*((f*(2 + ((d*e
 + f - c*f)*(d*e - (1 + c)*f))/((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c
+ d*x))/Sqrt[1 - (c + d*x)^2])^2)*ArcTanh[c + d*x])/((d*e + f - c*f)^2*(-(
d*e) + f + c*f)^2) - ((c + d*x)*(f - 2*d*e*ArcTanh[c + d*x] + 2*c*f*ArcTan
h[c + d*x]))/((d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Sqrt[1 - (c +
d*x)^2]*((d*e - c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d
*x)^2])) - (2*(d*e - c*f)*Log[(d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)/Sqrt[1 -
 (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2]])/(d^2*e^2 - 2*c*d*e*f
 + (-1 + c^2)*f^2)^2))/(d*(e + f*x)^3) + (b^2*(d*e - c*f + f*(c + d*x))^3*
((d^2*(-(f^2*ArcTanh[c + d*x]) - I*d^2*e^2*Pi*ArcTanh[c + d*x] + (2*I)*c*d
*e*f*Pi*ArcTanh[c + d*x] - I*c^2*f^2*Pi*ArcTanh[c + d*x] + d*e*E^ArcTanh[c
 - (d*e)/f]*Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*f*ArcTanh[c + d*x]
^2 - c*E^ArcTanh[c - (d*e)/f]*Sqrt[1 - c^2 - (d^2*e^2)/f^2 + (2*c*d*e)/f]*
f^2*ArcTanh[c + d*x]^2 - 2*d^2*e^2*ArcTanh[c + d*x]*Log[1 - E^(2*ArcTanh[c
 - (d*e)/f] - 2*ArcTanh[c + d*x])] + 4*c*d*e*f*ArcTanh[c + d*x]*Log[1 - E^
(2*ArcTanh[c - (d*e)/f] - 2*ArcTanh[c + d*x])] - 2*c^2*f^2*ArcTanh[c + d*x
]*Log[1 - E^(2*ArcTanh[c - (d*e)/f] - 2*ArcTanh[c + d*x])] + I*d^2*e^2*Pi*
Log[1 + E^(2*ArcTanh[c + d*x])] - (2*I)*c*d*e*f*Pi*Log[1 + E^(2*ArcTanh[c
+ d*x])] + I*c^2*f^2*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - I*d^2*e^2*Pi*Log
[1/Sqrt[1 - (c + d*x)^2]] + (2*I)*c*d*e*f*Pi*Log[1/Sqrt[1 - (c + d*x)^2...
```

3.44.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 739, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6659, 7292, 6671, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx$$

$$\Big\downarrow \; 6659$$

$$\frac{bd\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2(1-(c+dx)^2)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

$$\Big\downarrow \; 7292$$

$$\frac{bd\int \frac{a+b\text{arctanh}(c+dx)}{(e+fx)^2(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

$$\Big\downarrow \text{6671}$$

$$\frac{b\int \frac{d^2(a+b\text{arctanh}(c+dx))}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^2\int \frac{a+b\text{arctanh}(c+dx)}{(de-cf+f(c+dx))^2(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

$$\Big\downarrow \text{7276}$$

$$\frac{bd^2\int \left(-\frac{a}{(de-cf+f(c+dx))^2((c+dx)^2-1)}-\frac{b\text{arctanh}(c+dx)}{(de-cf+f(c+dx))^2((c+dx)^2-1)}\right)d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

$$\Big\downarrow \text{2009}$$

$$bd^2\left(\frac{af}{(-cf+de+f)(de-(c+1)f)(f(c+dx)-cf+de)}-\frac{2af(de-cf)\log(f(c+dx)-cf+de)}{(-cf+de+f)^2(de-(c+1)f)^2}-\frac{a\log(-c-dx+1)}{2(-cf+de+f)^2}+\frac{a\log(c+dx+1)}{2(de-(c+1)f)^2}+\frac{}{(-cf+de\cdots}\right.$$

$$\frac{(a+b\text{arctanh}(c+dx))^2}{2f(e+fx)^2}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^2/(e + f*x)^3,x]
```

3.44. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx$

output
```
-1/2*(a + b*ArcTanh[c + d*x])^2/(f*(e + f*x)^2) + (b*d^2*((a*f)/((d*e + f
- c*f)*(d*e - (1 + c)*f)*(d*e - c*f + f*(c + d*x))) + (b*f*ArcTanh[c + d*x
])/((d*e + f - c*f)*(d*e - (1 + c)*f)*(d*e - c*f + f*(c + d*x))) + (b*ArcT
anh[c + d*x]*Log[2/(1 - c - d*x)])/(2*(d*e + f - c*f)^2) - (a*Log[1 - c -
d*x])/(2*(d*e + f - c*f)^2) + (b*f*Log[1 - c - d*x])/(2*(d*e + f - c*f)^2*
(d*e - (1 + c)*f)) - (b*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(2*(d*e - (
1 + c)*f)^2) + (2*b*f*(d*e - c*f)*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(
(d*e + f - c*f)^2*(d*e - (1 + c)*f)^2) + (a*Log[1 + c + d*x])/(2*(d*e - (1
 + c)*f)^2) - (b*f*Log[1 + c + d*x])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)^
2) + (b*f^2*Log[d*e - c*f + f*(c + d*x)])/((d*e + f - c*f)^2*(d*e - (1 + c
)*f)^2) - (2*a*f*(d*e - c*f)*Log[d*e - c*f + f*(c + d*x)])/((d*e + f - c*f
)^2*(d*e - (1 + c)*f)^2) - (2*b*f*(d*e - c*f)*ArcTanh[c + d*x]*Log[(2*(d*e
 - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)^
2*(d*e - (1 + c)*f)^2) + (b*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(4
*(d*e + f - c*f)^2) + (b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(4*(d*e - (1 + c
)*f)^2) - (b*f*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*
f)^2*(d*e - (1 + c)*f)^2) + (b*f*(d*e - c*f)*PolyLog[2, 1 - (2*(d*e - c*f
+ f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)^2*(d*e
- (1 + c)*f)^2)))/f
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
+ 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.44.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.16

method	result
parts	$-\frac{a^2}{2(fx+e)^2 f}+\frac{b^2\left(-\frac{d^3\operatorname{arctanh}(dx+c)^2}{2(f(dx+c)-cf+de)^2 f}+\frac{d^3\left(\frac{\operatorname{arctanh}(dx+c)f}{(cf-de-f)(cf-de+f)(f(dx+c)-cf+de)}+\frac{2\operatorname{arctanh}(dx+c)f^2\ln(f(dx+c)-}{(cf-de-f)^2(cf-de+f)^2}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a^2 d^3}{2(cf-de-f(dx+c))^2 f}-b^2 d^3\left(\frac{\operatorname{arctanh}(dx+c)^2}{2(cf-de-f(dx+c))^2 f}-\frac{-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2(cf-de-f)^2}-\frac{\operatorname{arctanh}(dx+c)f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}}{}\right.}{}$
default	$\frac{-\frac{a^2 d^3}{2(cf-de-f(dx+c))^2 f}-b^2 d^3\left(\frac{\operatorname{arctanh}(dx+c)^2}{2(cf-de-f(dx+c))^2 f}-\frac{-\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2(cf-de-f)^2}-\frac{\operatorname{arctanh}(dx+c)f}{(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}}{}\right.}{}$

input
```
int((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/(f*x+e)^2/f+b^2/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arctanh(d*x+c
)^2+d^3/f*(arctanh(d*x+c)*f/(c*f-d*e-f)/(c*f-d*e+f)/(f*(d*x+c)-c*f+d*e)+2*
arctanh(d*x+c)*f^2/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(f*(d*x+c)-c*f+d*e)*c-2*a
rctanh(d*x+c)*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(f*(d*x+c)-c*f+d*e)*d*e-1/2*
arctanh(d*x+c)/(c*f-d*e-f)^2*ln(d*x+c-1)+1/2*arctanh(d*x+c)/(c*f-d*e+f)^2*
ln(d*x+c+1)-1/2/(c*f-d*e-f)^2*(1/4*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1/2*c+1
/2)-1/2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2))+1/2/(c*f-d*e+f)^2*(-1/4*ln(d*x+
c+1)^2+1/2*(ln(d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)-1/2*
dilog(1/2*d*x+1/2*c+1/2))+f/(c*f-d*e-f)/(c*f-d*e+f)*(f/(c*f-d*e-f)/(c*f-d*
e+f)*ln(f*(d*x+c)-c*f+d*e)-1/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)+1/(2*c*f-2*d*e+
2*f)*ln(d*x+c+1))+2*(c*f-d*e)/(c*f-d*e-f)^2/(c*f-d*e+f)^2*(1/2*f*(dilog((f
*(d*x+c)-f)/(c*f-d*e-f))+ln(f*(d*x+c)-c*f+d*e)*ln((f*(d*x+c)-f)/(c*f-d*e-f
)))-1/2*f*(dilog((f*(d*x+c)+f)/(c*f-d*e+f))+ln(f*(d*x+c)-c*f+d*e)*ln((f*(d
*x+c)+f)/(c*f-d*e+f))))))+2*a*b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arctan
h(d*x+c)+1/2*d^3/f*(f/(c*f-d*e-f)/(c*f-d*e+f)/(f*(d*x+c)-c*f+d*e)+2*f*(c*f
-d*e)/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(f*(d*x+c)-c*f+d*e)-1/2/(c*f-d*e-f)^2*
ln(d*x+c-1)+1/2/(c*f-d*e+f)^2*ln(d*x+c+1)))
```

3.44.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(fx+e)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)/(f^3*x^3
+ 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x)
```

3.44.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx=\int \frac{(a+b\,\text{atanh}\,(c+dx))^2}{(e+fx)^3}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**2/(f*x+e)**3,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2/(e + f*x)**3, x)
```

3.44.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(fx+e)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/2*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1
)*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c +
1)*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c
^2 - 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^
2*e^3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 -
1)*f^3)*x)) - 2*arctanh(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*a*b - 1/8*
b^2*(log(-d*x - c + 1)^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) + 2*integrate(-((d*
f*x + c*f - f)*log(d*x + c + 1)^2 + (d*f*x + d*e - 2*(d*f*x + c*f - f)*log
(d*x + c + 1))*log(-d*x - c + 1))/(d*f^4*x^4 + c*e^3*f - e^3*f + (3*d*e*f^
3 + c*f^4 - f^4)*x^3 + 3*(d*e^2*f^2 + c*e*f^3 - e*f^3)*x^2 + (d*e^3*f + 3*
c*e^2*f^2 - 3*e^2*f^2)*x), x)) - 1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

3.44.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^2}{(fx+e)^3}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^2/(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2/(f*x + e)^3, x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3}\,dx=\int \frac{(a+b\,\text{atanh}(c+d\,x))^2}{(e+f\,x)^3}\,dx$$

input
```
int((a + b*atanh(c + d*x))^2/(e + f*x)^3,x)
```

output
```
int((a + b*atanh(c + d*x))^2/(e + f*x)^3, x)
```

3.44. $\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)^3} dx$

3.45 $\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx$

3.45.1 Optimal result

Integrand size = 20, antiderivative size = 546

$$\begin{aligned}
&\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx \\
&= \frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\text{arctanh}(c+dx)}{d^3}-\frac{bf^2(a+b\text{arctanh}(c+dx))^2}{2d^3} \\
&\quad+\frac{3bf(de-cf)(a+b\text{arctanh}(c+dx))^2}{d^3} \\
&\quad+\frac{3bf(de-cf)(c+dx)(a+b\text{arctanh}(c+dx))^2}{d^3}+\frac{bf^2(c+dx)^2(a+b\text{arctanh}(c+dx))^2}{2d^3} \\
&\quad-\frac{(de-cf)\left(d^2e^2-2cdef+(3+c^2)f^2\right)(a+b\text{arctanh}(c+dx))^3}{3d^3f} \\
&\quad+\frac{\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)(a+b\text{arctanh}(c+dx))^3}{3d^3} \\
&\quad+\frac{(e+fx)^3(a+b\text{arctanh}(c+dx))^3}{3f}-\frac{6b^2f(de-cf)(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d^3} \\
&\quad-\frac{b\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2}{1-c-dx}\right)}{d^3} \\
&\quad+\frac{b^3f^2\log\left(1-(c+dx)^2\right)}{2d^3}-\frac{3b^3f(de-cf)\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{d^3} \\
&\quad-\frac{b^2\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d^3} \\
&\quad+\frac{b^3\left(3d^2e^2-6cdef+(1+3c^2)f^2\right)\text{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d^3}
\end{aligned}$$

output
```
a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctanh(d*x+c)/d^3-1/2*b*f^2*(a+b*arctanh(
d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(a+b*arctanh(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)
*(d*x+c)*(a+b*arctanh(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arctanh(d*x+c
))^2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arctanh(d*x+c
))^3/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))^3/
d^3+1/3*(f*x+e)^3*(a+b*arctanh(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctanh
(d*x+c))*ln(2/(-d*x-c+1))/d^3-b*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*a
rctanh(d*x+c))^2*ln(2/(-d*x-c+1))/d^3+1/2*b^3*f^2*ln(1-(d*x+c)^2)/d^3-3*b^
3*f*(-c*f+d*e)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^3-b^2*(3*d^2*e^2-6*c*d*e
*f+(3*c^2+1)*f^2)*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^3+1/2*b
^3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(3,1-2/(-d*x-c+1))/d^3
```

3.45.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1646 vs. 2(546) = 1092.

Time = 6.96 (sec) , antiderivative size = 1646, normalized size of antiderivative = 3.01

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx=\text{Too large to display}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
a^2*(a*e^2 + (b*f*(3*d*e - 2*c*f))/d^2)*x + (a^2*f*(2*a*d*e + b*f)*x^2)/(2
*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[c + d*
x] - (a^2*b*(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1
 - c - d*x])/(2*d^3) + (a^2*b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 +
c)^2*f^2)*Log[1 + c + d*x])/(2*d^3) + (3*a*b^2*e^2*(ArcTanh[c + d*x]*((-1
+ c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLo
g[2, -E^(-2*ArcTanh[c + d*x])]))/d - (3*a*b^2*e*f*((1 - 2*c + c^2 - d^2*x^
2)*ArcTanh[c + d*x]^2 - 2*ArcTanh[c + d*x]*(c + d*x + 2*c*Log[1 + E^(-2*Ar
cTanh[c + d*x])]) + 2*Log[1/Sqrt[1 - (c + d*x)^2]] + 2*c*PolyLog[2, -E^(-2
*ArcTanh[c + d*x])]))/d^2 + (b^3*e^2*(2*ArcTanh[c + d*x]^2*((-1 + c + d*x)
*ArcTanh[c + d*x] - 3*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 6*ArcTanh[c + d*
x]*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c +
d*x])]))/(2*d) - (b^3*e*f*(ArcTanh[c + d*x]*((1 - 2*c + c^2 - d^2*x^2)*Arc
Tanh[c + d*x]^2 + 6*Log[1 + E^(-2*ArcTanh[c + d*x])] - 3*ArcTanh[c + d*x]*
(-1 + c + d*x + 2*c*Log[1 + E^(-2*ArcTanh[c + d*x])])) + (-3 + 6*c*ArcTanh
[c + d*x])*PolyLog[2, -E^(-2*ArcTanh[c + d*x])] + 3*c*PolyLog[3, -E^(-2*Ar
cTanh[c + d*x])]))/d^2 - (a*b^2*f^2*(1 - (c + d*x)^2)^(3/2)*(-((c + d*x)/S
qrt[1 - (c + d*x)^2]) + (6*c*(c + d*x)*ArcTanh[c + d*x])/Sqrt[1 - (c + d*x
)^2] + (3*(c + d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] - (3*c^2*(c
+ d*x)*ArcTanh[c + d*x]^2)/Sqrt[1 - (c + d*x)^2] + ArcTanh[c + d*x]^2*C...
```

3.45.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 533, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\text{arctanh}(c+dx))^3}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d^3}$$

$$\Big\downarrow \; 6480$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))^3}{3f}-\frac{b\int\left(-\left((c+dx)(a+b\text{arctanh}(c+dx))^2f^3\right)-3(de-cf)(a+b\text{arctanh}(c+dx))^2f^2+\frac{((de-cf)(d^2e}{f}}{d^3}$$

$\Big\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^3(a+b\text{arctanh}(c+dx))^3}{3f}-\frac{b\left(bf\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))-\frac{f\left(\left(3c^2+1\right)f\right.}{}\right.}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTanh[c + d*x])^3)/(3*f) - (b*(-(a*
b*f^3*(c + d*x)) - b^2*f^3*(c + d*x)*ArcTanh[c + d*x] + (f^3*(a + b*ArcTan
h[c + d*x])^2)/2 - 3*f^2*(d*e - c*f)*(a + b*ArcTanh[c + d*x])^2 - 3*f^2*(d
*e - c*f)*(c + d*x)*(a + b*ArcTanh[c + d*x])^2 - (f^3*(c + d*x)^2*(a + b*A
rcTanh[c + d*x])^2)/2 + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)
*(a + b*ArcTanh[c + d*x])^3)/(3*b) - (f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^
2)*f^2)*(a + b*ArcTanh[c + d*x])^3)/(3*b) + 6*b*f^2*(d*e - c*f)*(a + b*Arc
Tanh[c + d*x])*Log[2/(1 - c - d*x)] + f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^
2)*f^2)*(a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)] - (b^2*f^3*Log[1 -
 (c + d*x)^2])/2 + 3*b^2*f^2*(d*e - c*f)*PolyLog[2, -((1 + c + d*x)/(1 - c
 - d*x))] + b*f*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcTanh[c
 + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)] - (b^2*f*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x)])/2))/f)/d^3
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.45.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.71 (sec) , antiderivative size = 10013, normalized size of antiderivative = 18.34

method	result	size
derivativedivides	Expression too large to display	10013
default	Expression too large to display	10013
parts	Expression too large to display	10025

input
```
int((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.45.5 Fricas [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx = \int (fx+e)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x
+ b^3*e^2)*arctanh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e
^2)*arctanh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arc
tanh(d*x + c), x)
```

3.45.6 Sympy [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx = \int (a+b\,\text{atanh}\,(c+dx))^3\,(e+fx)^2\,\,dx$$

input
```
integrate((f*x+e)**2*(a+b*atanh(d*x+c))**3,x)
```

output
```
Integral((a + b*atanh(c + d*x))**3*(e + f*x)**2, x)
```

3.45.7 Maxima [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx = \int (fx+e)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*f^2*x^3 + a^3*e*f*x^2 + 3/2*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 -
 (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d
^3))*a^2*b*e*f + 1/2*(2*x^3*arctanh(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c
^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d
*x + c - 1)/d^4))*a^2*b*f^2 + a^3*e^2*x + 3/2*(2*(d*x + c)*arctanh(d*x + c
) + log(-(d*x + c)^2 + 1))*a^2*b*e^2/d - 1/24*((b^3*d^3*f^2*x^3 + 3*b^3*d^
3*e*f*x^2 + 3*b^3*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 -
 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^3)*log(-d*x - c + 1)^3 -
 3*(2*a*b^2*d^3*f^2*x^3 + (6*a*b^2*d^3*e*f + b^3*d^2*f^2)*x^2 + 2*(3*a*b^2
*d^3*e^2 + (3*d^2*e*f - 2*c*d*f^2)*b^3)*x + (b^3*d^3*f^2*x^3 + 3*b^3*d^3*e
*f*x^2 + 3*b^3*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*
d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^3)*log(d*x + c + 1))*log(-d
*x - c + 1)^2)/d^3 - integrate(-1/8*((b^3*d^3*f^2*x^3 + (2*d^3*e*f + c*d^2
*f^2 - d^2*f^2)*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f - 2*d^2*e*f)*b^3*x + (c*d
^2*e^2 - d^2*e^2)*b^3)*log(d*x + c + 1)^3 + 6*(a*b^2*d^3*f^2*x^3 + (2*d^3*
e*f + c*d^2*f^2 - d^2*f^2)*a*b^2*x^2 + (d^3*e^2 + 2*c*d^2*e*f - 2*d^2*e*f)
*a*b^2*x + (c*d^2*e^2 - d^2*e^2)*a*b^2)*log(d*x + c + 1)^2 - (4*a*b^2*d^3*
f^2*x^3 + 2*(6*a*b^2*d^3*e*f + b^3*d^2*f^2)*x^2 + 3*(b^3*d^3*f^2*x^3 + (2*
d^3*e*f + c*d^2*f^2 - d^2*f^2)*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f - 2*d^2*e*
f)*b^3*x + (c*d^2*e^2 - d^2*e^2)*b^3)*log(d*x + c + 1)^2 + 4*(3*a*b^2*d...
```

3.45.8 Giac [F]

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx = \int (fx+e)^2(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arctanh(d*x + c) + a)^3, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\text{arctanh}(c+dx))^3\,dx = \int (e+f\,x)^2\,(a+b\,\text{atanh}(c+d\,x))^3\,dx$$

input
```
int((e + f*x)^2*(a + b*atanh(c + d*x))^3,x)
```

output
```
int((e + f*x)^2*(a + b*atanh(c + d*x))^3, x)
```

3.46 $\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx$

3.46.1 Optimal result

Integrand size = 18, antiderivative size = 326

$$\begin{aligned}
&\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx \\
&= \frac{3bf(a+b\text{arctanh}(c+dx))^2}{2d^2}+\frac{3bf(c+dx)(a+b\text{arctanh}(c+dx))^2}{2d^2} \\
&\quad+\frac{(de-cf)(a+b\text{arctanh}(c+dx))^3}{d^2} \\
&\quad-\frac{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(a+b\text{arctanh}(c+dx))^3}{2d^2f} \\
&\quad+\frac{(e+fx)^2(a+b\text{arctanh}(c+dx))^3}{2f}-\frac{3b^2f(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d^2} \\
&\quad-\frac{3b(de-cf)(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2}{1-c-dx}\right)}{d^2}-\frac{3b^3f\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2d^2} \\
&\quad-\frac{3b^2(de-cf)(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d^2} \\
&\quad+\frac{3b^3(de-cf)\,\text{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d^2}
\end{aligned}$$

output
```
3/2*b*f*(a+b*arctanh(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arctanh(d*x+c))^2/
d^2+(-c*f+d*e)*(a+b*arctanh(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f
^2)*(a+b*arctanh(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctanh(d*x+c))^3/f-3*
b^2*f*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*arctan
h(d*x+c))^2*ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*polylog(2,(-d*x-c-1)/(-d*x-c+1)
)/d^2-3*b^2*(-c*f+d*e)*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^2+
3/2*b^3*(-c*f+d*e)*polylog(3,1-2/(-d*x-c+1))/d^2
```

3.46.2 Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.74

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx$$
$$= \frac{2a^2(2ade+3bf-2acf)(c+dx)+2a^3f(c+dx)^2-6a^2b(c+dx)(cf-d(2e+fx))\text{arctanh}(c+dx)+3a}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2
*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTanh[c + d*x] + 3*a^2*b*(2*d*e + f -
 2*c*f)*Log[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*Log[1 + c + d*x]
+ 12*a*b^2*f*((c + d*x)*ArcTanh[c + d*x] - ((1 - (c + d*x)^2)*ArcTanh[c +
d*x]^2)/2 - Log[1/Sqrt[1 - (c + d*x)^2]]) + 12*a*b^2*d*e*(ArcTanh[c + d*x]
*((-1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) +
PolyLog[2, -E^(-2*ArcTanh[c + d*x])]) - 12*a*b^2*c*f*(ArcTanh[c + d*x]*((-
1 + c + d*x)*ArcTanh[c + d*x] - 2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + Poly
Log[2, -E^(-2*ArcTanh[c + d*x])]) + 2*b^3*f*(ArcTanh[c + d*x]*(3*(-1 + c +
 d*x)*ArcTanh[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTanh[c + d*x]^2
 - 6*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*PolyLog[2, -E^(-2*ArcTanh[c + d
*x])]) + 4*b^3*d*e*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] -
3*Log[1 + E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2
*ArcTanh[c + d*x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/2) - 4*b^3*
c*f*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] - 3*Log[1 + E^(-2
*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[c + d*
x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/2))/(4*d^2)
```

3.46.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6661, 27, 6480, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx$$

$\downarrow$ 6661

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\text{arctanh}(c+dx))^3}{d}d(c+dx)}{d}$$

$\downarrow$ 27

$$\frac{\int (de-cf+f(c+dx))(a+b\text{arctanh}(c+dx))^3d(c+dx)}{d^2}$$

$\downarrow$ 6480

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))^3}{2f}-\frac{3b\int \left(\frac{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2+2f(de-cf)(c+dx)\right)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}-f^2(a+b\text{arctanh}(c+dx)\right.}{2f}}{d^2}$$

$\downarrow$ 2009

$$\frac{(f(c+dx)-cf+de)^2(a+b\text{arctanh}(c+dx))^3}{2f}-\frac{3b\left(\frac{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)(a+b\text{arctanh}(c+dx))^3}{3b}+2bf(de-cf)\,\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+b\right.}{}$$

input
```
Int[(e + f*x)*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTanh[c + d*x])^3)/(2*f) - (3*b*(-(
f^2*(a + b*ArcTanh[c + d*x])^2) - f^2*(c + d*x)*(a + b*ArcTanh[c + d*x])^2
 - (2*f*(d*e - c*f)*(a + b*ArcTanh[c + d*x])^3)/(3*b) + ((d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)*(a + b*ArcTanh[c + d*x])^3)/(3*b) + 2*b*f^2*(a + b*Ar
cTanh[c + d*x])*Log[2/(1 - c - d*x)] + 2*f*(d*e - c*f)*(a + b*ArcTanh[c +
d*x])^2*Log[2/(1 - c - d*x)] + b^2*f^2*PolyLog[2, -((1 + c + d*x)/(1 - c -
 d*x))] + 2*b*f*(d*e - c*f)*(a + b*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 -
 c - d*x)] - b^2*f*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)]))/(2*f))/d^
2
```

3.46. $\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx$

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6480
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])^p/(e*(q + 1))), x] -
 Simp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1
), (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
 && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.46.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 10314, normalized size of antiderivative = 31.64

method	result	size
parts	Expression too large to display	10314
derivativedivides	Expression too large to display	10374
default	Expression too large to display	10374

input
```
int((f*x+e)*(a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.46.5 Fricas [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx = \int (fx+e)(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctanh(d*x + c)^3 + 3*(a*b^2
*f*x + a*b^2*e)*arctanh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctanh(d*x +
 c), x)
```

3.46.6 Sympy [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx = \int (a+b\,\text{atanh}\,(c+dx))^3\,(e+fx)\,\,dx$$

input
```
integrate((f*x+e)*(a+b*atanh(d*x+c))**3,x)
```

output
```
Integral((a + b*atanh(c + d*x))**3*(e + f*x), x)
```

3.46.7 Maxima [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx = \int (fx+e)(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*f*x^2 + 3/4*(2*x^2*arctanh(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)
*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a
^3*e*x + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*
e/d - 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*
e + f)*b^3)*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e
+ b^3*d*f)*x + (b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2
*d*e + f)*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)^2)/d^2 - integrate(-1/8
*((b^3*d^2*f*x^2 + (d^2*e + c*d*f - d*f)*b^3*x + (c*d*e - d*e)*b^3)*log(d*
x + c + 1)^3 + 6*(a*b^2*d^2*f*x^2 + (d^2*e + c*d*f - d*f)*a*b^2*x + (c*d*e
 - d*e)*a*b^2)*log(d*x + c + 1)^2 - 3*(2*a*b^2*d^2*f*x^2 + (b^3*d^2*f*x^2
+ (d^2*e + c*d*f - d*f)*b^3*x + (c*d*e - d*e)*b^3)*log(d*x + c + 1)^2 + 2*
(2*a*b^2*d^2*e + b^3*d*f)*x + (4*(c*d*e - d*e)*a*b^2 - (c^2*f - 2*(d*e - f
)*c - 2*d*e + f)*b^3 + (4*a*b^2*d^2*f + b^3*d^2*f)*x^2 + 2*(b^3*d^2*e + 2*
(d^2*e + c*d*f - d*f)*a*b^2)*x)*log(d*x + c + 1))*log(-d*x - c + 1))/(d^2*
x + c*d - d), x)
```

3.46.8 Giac [F]

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx=\int (fx+e)(b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arctanh(d*x + c) + a)^3, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\text{arctanh}(c+dx))^3\,dx=\int (e+f\,x)\,\left(a+b\,\text{atanh}(c+d\,x)\right)^3\,dx$$

input
```
int((e + f*x)*(a + b*atanh(c + d*x))^3,x)
```

output
```
int((e + f*x)*(a + b*atanh(c + d*x))^3, x)
```

3.47 $\int (a+b\text{arctanh}(c+dx))^3\,dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 132

$$\int (a+b\text{arctanh}(c+dx))^3\,dx = \frac{(a+b\text{arctanh}(c+dx))^3}{d}+\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3}{d} - \frac{3b(a+b\text{arctanh}(c+dx))^2\log\left(\frac{2}{1-c-dx}\right)}{d} - \frac{3b^2(a+b\text{arctanh}(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{d} + \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-c-dx}\right)}{2d}$$

output
```
(a+b*arctanh(d*x+c))^3/d+(d*x+c)*(a+b*arctanh(d*x+c))^3/d-3*b*(a+b*arctanh
(d*x+c))^2*ln(2/(-d*x-c+1))/d-3*b^2*(a+b*arctanh(d*x+c))*polylog(2,1-2/(-d
*x-c+1))/d+3/2*b^3*polylog(3,1-2/(-d*x-c+1))/d
```

3.47.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.47

$$\int (a+b\text{arctanh}(c+dx))^3\,dx = \frac{2a^3(c+dx)+6a^2b(c+dx)\text{arctanh}(c+dx)+3a^2b\log\left(1-(c+dx)^2\right)+6ab^2\left(\text{arctanh}(c+dx)\left((-1+c\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
(2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcTanh[c + d*x] + 3*a^2*b*Log[1 - (c
 + d*x)^2] + 6*a*b^2*(ArcTanh[c + d*x]*((-1 + c + d*x)*ArcTanh[c + d*x] -
2*Log[1 + E^(-2*ArcTanh[c + d*x])]) + PolyLog[2, -E^(-2*ArcTanh[c + d*x])]
) + 2*b^3*(ArcTanh[c + d*x]^2*((-1 + c + d*x)*ArcTanh[c + d*x] - 3*Log[1 +
 E^(-2*ArcTanh[c + d*x])]) + 3*ArcTanh[c + d*x]*PolyLog[2, -E^(-2*ArcTanh[
c + d*x])] + (3*PolyLog[3, -E^(-2*ArcTanh[c + d*x])])/2))/(2*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6653, 6436, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(c+dx))^3\,dx$$

$$\Big\downarrow \text{6653}$$

$$\frac{\int (a+b\text{arctanh}(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{6436}$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3-3b\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))^2}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{6546}$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3-3b\left(\int \frac{(a+b\text{arctanh}(c+dx))^2}{-c-dx+1}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{3b}\right)}{d}$$

$$\Big\downarrow \text{6470}$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3-3b\left(-2b\int \frac{(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^3}{3b}+\text{lo}\right.}{d}$$

$$\Big\downarrow \text{6620}$$

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3-3b\left(-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a\right.\right.}{d}$$

$\big\downarrow$ 7164

$$\frac{(c+dx)(a+b\text{arctanh}(c+dx))^3-3b\left(-2b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)(a+\text{barc}\right.\right.}{d}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcTanh[c + d*x])^3 - 3*b*(-1/3*(a + b*ArcTanh[c + d*x])
^3/b + (a + b*ArcTanh[c + d*x])^2*Log[2/(1 - c - d*x)] - 2*b*(-1/2*((a + b
*ArcTanh[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)]) + (b*PolyLog[3, 1 - 2/
(1 - c - d*x)])/4)))/d
```

3.47.3.1 Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6653
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(130) = 260$.

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arctanh}(dx+c)^3(dx+c-1)+2\operatorname{arctanh}(dx+c)^3-3\operatorname{arctanh}(dx+c)^2\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-3\operatorname{arctanh}(dx+c\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arctanh}(dx+c)^3(dx+c-1)+2\operatorname{arctanh}(dx+c)^3-3\operatorname{arctanh}(dx+c)^2\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-3\operatorname{arctanh}(dx+c\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arctanh}(dx+c)^3(dx+c-1)+2\operatorname{arctanh}(dx+c)^3-3\operatorname{arctanh}(dx+c)^2\ln\left(1+\frac{(dx+c+1)^2}{1-(dx+c)^2}\right)-3\operatorname{arctanh}(dx+c)\,\mathrm{p}\right.}{d}$

input
```
int((a+b*arctanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*(arctanh(d*x+c)^3*(d*x+c-1)+2*arctanh(d*x+c)^3-3*arct
anh(d*x+c)^2*ln(1+(d*x+c+1)^2/(1-(d*x+c)^2))-3*arctanh(d*x+c)*polylog(2,-(
d*x+c+1)^2/(1-(d*x+c)^2))+3/2*polylog(3,-(d*x+c+1)^2/(1-(d*x+c)^2)))+3*a*b
^2*(arctanh(d*x+c)^2*(d*x+c-1)+2*arctanh(d*x+c)^2-2*arctanh(d*x+c)*ln(1+(d
*x+c+1)^2/(1-(d*x+c)^2))-polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2)))+3*a^2*b*((
d*x+c)*arctanh(d*x+c)+1/2*ln(1-(d*x+c)^2)))
```

3.47.5 Fricas [F]

$$\int (a+b\text{arctanh}(c+dx))^3\,dx=\int (b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*arc
tanh(d*x + c) + a^3, x)
```

3.47.6 Sympy [F]

$$\int (a+b\text{arctanh}(c+dx))^3\,dx=\int (a+b\,\text{atanh}\,(c+dx))^3\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**3,x)
```

output
```
Integral((a + b*atanh(c + d*x))**3, x)
```

3.47.7 Maxima [F]

$$\int (a+b\text{arctanh}(c+dx))^3\,dx=\int (b\,\text{artanh}\,(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 3/2*(2*(d*x + c)*arctanh(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d
 - 1/8*((b^3*d*x + b^3*(c - 1))*log(-d*x - c + 1)^3 - 3*(2*a*b^2*d*x + (b^
3*d*x + b^3*(c + 1))*log(d*x + c + 1))*log(-d*x - c + 1)^2)/d - integrate(
-1/8*((b^3*d*x + b^3*(c - 1))*log(d*x + c + 1)^3 + 6*(a*b^2*d*x + a*b^2*(c
 - 1))*log(d*x + c + 1)^2 - 3*(4*a*b^2*d*x + (b^3*d*x + b^3*(c - 1))*log(d
*x + c + 1)^2 + 2*(b^3*(c + 1) + 2*a*b^2*(c - 1) + (2*a*b^2*d + b^3*d)*x)*
log(d*x + c + 1))*log(-d*x - c + 1))/(d*x + c - 1), x)
```

3.47.8 Giac [F]

$$\int (a + b\text{arctanh}(c + dx))^3 \, dx = \int (b \operatorname{artanh}(dx + c) + a)^3 \, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arctanh}(c + dx))^3 \, dx = \int (a + b \operatorname{atanh}(c + d x))^3 \, dx$$

input
```
int((a + b*atanh(c + d*x))^3,x)
```

output
```
int((a + b*atanh(c + d*x))^3, x)
```

3.48 $\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^3}{e+fx}\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}
\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^3}{e+fx}\,dx
&= -\frac{(a+b\,\mathrm{arctanh}(c+dx))^3\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{(a+b\,\mathrm{arctanh}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}\\
&\quad+\frac{3b(a+b\,\mathrm{arctanh}(c+dx))^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f}\\
&\quad-\frac{3b(a+b\,\mathrm{arctanh}(c+dx))^2\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\\
&\quad+\frac{3b^2(a+b\,\mathrm{arctanh}(c+dx))\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+c+dx}\right)}{2f}\\
&\quad-\frac{3b^2(a+b\,\mathrm{arctanh}(c+dx))\,\mathrm{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{2f}\\
&\quad+\frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2}{1+c+dx}\right)}{4f}-\frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{4f}
\end{aligned}$$

output
```
-(a+b*arctanh(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arctanh(d*x+c))^3*ln(2*d*(f
*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2
/(d*x+c+1))/f-3/2*b*(a+b*arctanh(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d
*e+f)/(d*x+c+1))/f+3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2/(d*x+c+1))/f
-3/2*b^2*(a+b*arctanh(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+
1))/f+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(
-c*f+d*e+f)/(d*x+c+1))/f
```

3.48.2 Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{e+fx}\,dx=\int \frac{(a+b\text{arctanh}(c+dx))^3}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x), x]
```

3.48.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6661, 27, 6476}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{e+fx}\,dx$$

$$\Big\downarrow \; 6661$$

$$\frac{\int \frac{d(a+b\text{arctanh}(c+dx))^3}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \; 6476$$

$$-\frac{3b^2(a+b\,\mathrm{arctanh}(c+dx))\,\mathrm{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{3b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)(a+b\,\mathrm{arctanh}(c+dx))}{2f}-\frac{3b(a+b\,\mathrm{arctanh}(c+dx))^2\,\mathrm{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{(a+b\,\mathrm{arctanh}(c+dx))^3\log\left(\frac{2(f(c+dx)-cf+de)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{3b\,\mathrm{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)(a+b\,\mathrm{arctanh}(c+dx))^2}{2f}-\frac{\log\left(\frac{2}{c+dx+1}\right)(a+b\,\mathrm{arctanh}(c+dx))^3}{f}-\frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2(de-cf+f(c+dx))}{(de-cf+f)(c+dx+1)}\right)}{4f}+\frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2}{c+dx+1}\right)}{4f}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x),x]
```

output
```
-(((a + b*ArcTanh[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcTanh[c
 + d*x])^3*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x
))])/f + (3*b*(a + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/
(2*f) - (3*b*(a + b*ArcTanh[c + d*x])^2*PolyLog[2, 1 - (2*(d*e - c*f + f*(
c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTanh
[c + d*x])*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b^2*(a + b*ArcTanh[
c + d*x])*PolyLog[3, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1
 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1 + c + d*x)])/(4*f) - (3*
b^3*PolyLog[4, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c +
 d*x))])/(4*f)
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6476
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcTanh[c*x])^3)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Tanh[c*x])^3*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[3*b*
(a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b*(
a + b*ArcTanh[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))
]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTanh[c*x])*(PolyLog[3, 1 - 2/(1 + c*x)]
/(2*e)), x] - Simp[3*b^2*(a + b*ArcTanh[c*x])*(PolyLog[3, 1 - 2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(2*e)), x] + Simp[3*b^3*(PolyLog[4, 1 - 2/(1 + c*
x)]/(4*e)), x] - Simp[3*b^3*(PolyLog[4, 1 - 2*c*((d + e*x)/((c*d + e)*(1 +
c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.48.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 3441, normalized size of antiderivative = 11.17

method	result	size
derivativedivides	Expression too large to display	3441
default	Expression too large to display	3441
parts	Expression too large to display	3656

input
```
int((a+b*arctanh(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctanh
(d*x+c)^3+3/f*(1/3*arctanh(d*x+c)^3*ln(f*c*(1+(d*x+c+1)^2/(1-(d*x+c)^2))+(
-(d*x+c+1)^2/(1-(d*x+c)^2)-1)*e*d+(-(d*x+c+1)^2/(1-(d*x+c)^2)+1)*f)-1/6*I*
Pi*csgn(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)
*e*d+((d*x+c+1)^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*(csgn
(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((
d*x+c+1)^2/((d*x+c)^2-1)+1)*f))*csgn(I/(1-(d*x+c+1)^2/((d*x+c)^2-1)))-csgn
(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((
d*x+c+1)^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))*csgn(I/(1-(d
*x+c+1)^2/((d*x+c)^2-1)))-csgn(I*(f*c*(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+
c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)^2/((d*x+c)^2-1)+1)*f))*csgn(I*(f*c*
(1-(d*x+c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)
^2/((d*x+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))+csgn(I*(f*c*(1-(d*x+
c+1)^2/((d*x+c)^2-1))+((d*x+c+1)^2/((d*x+c)^2-1)-1)*e*d+((d*x+c+1)^2/((d*x
+c)^2-1)+1)*f)/(1-(d*x+c+1)^2/((d*x+c)^2-1)))^2)*arctanh(d*x+c)^3+1/2*arct
anh(d*x+c)^2*polylog(2,-(d*x+c+1)^2/(1-(d*x+c)^2))-1/2*arctanh(d*x+c)*poly
log(3,-(d*x+c+1)^2/(1-(d*x+c)^2))+1/4*polylog(4,-(d*x+c+1)^2/(1-(d*x+c)^2)
)-1/3*f*c/(c*f-d*e-f)*arctanh(d*x+c)^3*ln(1-(c*f-d*e-f)*(d*x+c+1)^2/(1-(d*
x+c)^2)/(-c*f+d*e-f))-1/2*f*c/(c*f-d*e-f)*arctanh(d*x+c)^2*polylog(2,(c*f-
d*e-f)*(d*x+c+1)^2/(1-(d*x+c)^2)/(-c*f+d*e-f))+1/2*f*c/(c*f-d*e-f)*arct...
```

3.48.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{e+fx}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(f*x + e), x)
```

3.48.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{e+fx}\,dx = \int \frac{(a+b\operatorname{atanh}(c+dx))^3}{e+fx}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**3/(f*x+e),x)
```

output
```
Integral((a + b*atanh(c + d*x))**3/(e + f*x), x)
```

3.48.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\operatorname{artanh}(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

output
```
a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(d*x + c + 1) - log(-d*x - c +
1))^3/(f*x + e) + 3/4*a*b^2*(log(d*x + c + 1) - log(-d*x - c + 1))^2/(f*x
+ e) + 3/2*a^2*b*(log(d*x + c + 1) - log(-d*x - c + 1))/(f*x + e), x)
```

3.48.8 Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{e+fx}\,dx = \int \frac{(b\operatorname{artanh}(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(f*x + e), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{e+fx}\,dx=\int \frac{{\left(a+b\,\mathrm{atanh}\left(c+d\,x\right)\right)}^3}{e+f\,x}\,dx$$

input
```
int((a + b*atanh(c + d*x))^3/(e + f*x),x)
```

output
```
int((a + b*atanh(c + d*x))^3/(e + f*x), x)
```

3.49 $\int \frac{(a+b\,\mathrm{arctanh}(c+dx))^3}{(e+fx)^2}\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 1089

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}\,dx = -\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$+\frac{3ab^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}$$

$$+\frac{3b^3d\text{arctanh}(c+dx)^2\log\left(\frac{2}{1-c-dx}\right)}{2f(de+f-cf)}$$

$$-\frac{3a^2bd\log(1-c-dx)}{2f(de+f-cf)}$$

$$-\frac{3ab^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{f(de-f-cf)}$$

$$+\frac{6ab^2d\text{arctanh}(c+dx)\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$-\frac{3b^3d\text{arctanh}(c+dx)^2\log\left(\frac{2}{1+c+dx}\right)}{2f(de-f-cf)}$$

$$+\frac{3b^3d\text{arctanh}(c+dx)^2\log\left(\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$+\frac{3a^2bd\log(1+c+dx)}{2f(de-f-cf)}+\frac{3a^2bd\log(e+fx)}{f^2-(de-cf)^2}$$

$$-\frac{6ab^2d\text{arctanh}(c+dx)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$-\frac{3b^3d\text{arctanh}(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$+\frac{3ab^2d\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2f(de+f-cf)}$$

$$+\frac{3b^3d\text{arctanh}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1-c-dx}\right)}{2f(de+f-cf)}$$

$$+\frac{3ab^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)}$$

$$-\frac{3ab^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$+\frac{3b^3d\text{arctanh}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f(de-f-cf)}$$

$$-\frac{3b^3d\text{arctanh}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$+\frac{3ab^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}$$

$$+\frac{3b^3d\text{arctanh}(c+dx)\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{(de+f-cf)(de-(1+c)f)}$$

output
```
-(a+b*arctanh(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*arctanh(d*x+c)*ln(2/(-d*x-c+1)
)/f/(-c*f+d*e+f)+3/2*b^3*d*arctanh(d*x+c)^2*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f
)-3/2*a^2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*arctanh(d*x+c)*ln(2/(d
*x+c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*arctanh(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e
-f)/(-c*f+d*e+f)-3/2*b^3*d*arctanh(d*x+c)^2*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)
+3*b^3*d*arctanh(d*x+c)^2*ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^
2*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*
a*b^2*d*arctanh(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)
/(-c*f+d*e+f)-3*b^3*d*arctanh(d*x+c)^2*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+
1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))
/f/(-c*f+d*e+f)+3/2*b^3*d*arctanh(d*x+c)*polylog(2,1-2/(-d*x-c+1))/f/(-c*f
+d*e+f)+3/2*a*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*poly
log(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*arctanh(d*x+c)*po
lylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*arctanh(d*x+c)*polylog(2,1-2
/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(-
c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*arctanh(d*x+c)*pol
ylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4
*b^3*d*polylog(3,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*polylog(3,1-2/(d
*x+c+1))/f/(-c*f+d*e-f)-3/2*b^3*d*polylog(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-
c*f+d*e+f)+3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-...
```

3.49.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 87.00 (sec) , antiderivative size = 3976, normalized size of antiderivative = 3.65

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{(e + fx)^2} dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2,x]
```

output
```
-(a^3/(f*(e + f*x))) - (3*a^2*b*ArcTanh[c + d*x])/(f*(e + f*x)) + (3*a^2*b
*d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x
])/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*
f - f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*((d*e - c*f)/Sqrt[1 - (c +
 d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*(-(ArcTanh[c + d*x]^2/(E
^ArcTanh[(d*e - c*f)/f]*f*Sqrt[1 - (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTa
nh[c + d*x]^2)/(Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 - (c + d*x)^2] - (c*f)
/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])) + ((d*e - c
*f)*(I*Pi*Log[1 + E^(2*ArcTanh[c + d*x])] - 2*ArcTanh[c + d*x]*Log[1 - E^(
-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - I*Pi*(ArcTanh[c + d*x]
+ Log[1/Sqrt[1 - (c + d*x)^2]]) - 2*ArcTanh[(d*e - c*f)/f]*(ArcTanh[c + d*
x] + Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))] - Log[I*S
inh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + PolyLog[2, E^(-2*(ArcTa
nh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2
)*f^2)))/(d*(d*e - c*f)*(e + f*x)^2) + (b^3*(1 - (c + d*x)^2)*((d*e - c*f)
/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c + d*x)^2])^2*((d*(c + d
*x)*ArcTanh[c + d*x]^3)/((d*e - c*f)*Sqrt[1 - (c + d*x)^2]*((d*e)/Sqrt[1 -
 (c + d*x)^2] - (c*f)/Sqrt[1 - (c + d*x)^2] + (f*(c + d*x))/Sqrt[1 - (c +
d*x)^2])) - (d*(-6*d*e*ArcTanh[c + d*x]^3 + 2*f*ArcTanh[c + d*x]^3 + 6*c*f
*ArcTanh[c + d*x]^3 - 4*E^ArcTanh[c - (d*e)/f]*Sqrt[1 - c^2 - (d^2*e^2)...
```

3.49.3 Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 1085, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6659, 7292, 6671, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}dx$$

$$\Big\downarrow \text{6659}$$

$$\frac{3bd\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)(1-(c+dx)^2)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$\frac{3bd\int \frac{(a+b\text{arctanh}(c+dx))^2}{(e+fx)(-c^2-2dxc-d^2x^2+1)}dx}{f}-\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \; 6671$$

$$\frac{3b\int \frac{d(a+b\text{arctanh}(c+dx))^2}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \; 27$$

$$\frac{3bd\int \frac{(a+b\text{arctanh}(c+dx))^2}{(de-cf+f(c+dx))(1-(c+dx)^2)}d(c+dx)}{f}-\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \; 7276$$

$$\frac{3bd\int \left(-\frac{a^2}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{2b\text{arctanh}(c+dx)a}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}-\frac{b^2\text{arctanh}(c+dx)^2}{(c+dx-1)(c+dx+1)(de-cf+f(c+dx))}\right)d(c+}{f}$$

$$\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \; 2009$$

$$3bd\left(-\frac{\log(-c-dx+1)a^2}{2(de-cf+f)}+\frac{\log(c+dx+1)a^2}{2(de-(c+1)f)}-\frac{f\log(de-cf+f(c+dx))a^2}{(de-cf+f)(de-(c+1)f)}+\frac{b\text{arctanh}(c+dx)\log\left(\frac{2}{-c-dx+1}\right)a}{de-cf+f}-\frac{b\text{arctanh}(c+dx)\log\left(\frac{}{c}\right.}{de-cf-f}\right.$$

$$\frac{(a+b\text{arctanh}(c+dx))^3}{f(e+fx)}$$

input
```
Int[(a + b*ArcTanh[c + d*x])^3/(e + f*x)^2,x]
```

3.49. $\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}\,dx$

output
```
-((a + b*ArcTanh[c + d*x])^3/(f*(e + f*x))) + (3*b*d*((a*b*ArcTanh[c + d*x
]*Log[2/(1 - c - d*x)])/(d*e + f - c*f) + (b^2*ArcTanh[c + d*x]^2*Log[2/(1
 - c - d*x)])/(2*(d*e + f - c*f)) - (a^2*Log[1 - c - d*x])/(2*(d*e + f - c
*f)) - (a*b*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/(d*e - f - c*f) + (2*a*
b*f*ArcTanh[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)
*f)) - (b^2*ArcTanh[c + d*x]^2*Log[2/(1 + c + d*x)])/(2*(d*e - f - c*f)) +
 (b^2*f*ArcTanh[c + d*x]^2*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (
1 + c)*f)) + (a^2*Log[1 + c + d*x])/(2*(d*e - (1 + c)*f)) - (a^2*f*Log[d*e
 - c*f + f*(c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*a*b*f*ArcT
anh[c + d*x]*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d
*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (b^2*f*ArcTanh[c + d*x]^2*Log
[(2*(d*e - c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f)) + (a*b*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x)
)])/(2*(d*e + f - c*f)) + (b^2*ArcTanh[c + d*x]*PolyLog[2, 1 - 2/(1 - c -
d*x)])/(2*(d*e + f - c*f)) + (a*b*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*(d*e
 - f - c*f)) - (a*b*f*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d
*e - (1 + c)*f)) + (b^2*ArcTanh[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*x)])/
(2*(d*e - f - c*f)) - (b^2*f*ArcTanh[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*
x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (a*b*f*PolyLog[2, 1 - (2*(d*e -
 c*f + f*(c + d*x)))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)...
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6659
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTanh[c + d*x])^p/(f*(m
 + 1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTa
nh[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 6671
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Sub
st[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[
x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x
] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.49.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 5109, normalized size of antiderivative = 4.69

method	result	size
derivativedivides	Expression too large to display	5109
default	Expression too large to display	5109
parts	Expression too large to display	5242

input
```
int((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.49.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(b\,\text{artanh}\,(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

3.49.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(a+b\operatorname{atanh}(c+dx))^3}{(e+fx)^2}\,dx$$

input
```
integrate((a+b*atanh(d*x+c))**3/(f*x+e)**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**3/(e + f*x)**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(b\operatorname{artanh}(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

output
```
3/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
 (c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*
arctanh(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/8*(((d^2*e*f
 - c*d*f^2 - d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 - d*e*f + f^2)*b^3)*log(-d*
x - c + 1)^3 + 3*(2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2)*a*b^2 - ((d^2*e*
f - c*d*f^2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*
x + c + 1))*log(-d*x - c + 1)^2)/(d^2*e^3*f - 2*c*d*e^2*f^2 + c^2*e*f^3 -
e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + c^2*f^4 - f^4)*x) - integrate(-1/8*((
(d^2*e*f - c*d*f^2 - d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 - d*e*f + f^2)*b^3)
*log(d*x + c + 1)^3 + 6*((d^2*e*f - c*d*f^2 - d*f^2)*a*b^2*x + (c*d*e*f -
c^2*f^2 - d*e*f + f^2)*a*b^2)*log(d*x + c + 1)^2 + 3*(4*(d^2*e*f - c*d*f^2
 - d*f^2)*a*b^2*x + 4*(d^2*e^2 - c*d*e*f - d*e*f)*a*b^2 - ((d^2*e*f - c*d*
f^2 - d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 - d*e*f + f^2)*b^3)*log(d*x + c +
1)^2 - 2*(b^3*d^2*f^2*x^2 + 2*(c*d*e*f - c^2*f^2 - d*e*f + f^2)*a*b^2 + (c
*d*e*f + d*e*f)*b^3 + (2*(d^2*e*f - c*d*f^2 - d*f^2)*a*b^2 + (d^2*e*f + c*
d*f^2 + d*f^2)*b^3)*x)*log(d*x + c + 1))*log(-d*x - c + 1))/(c*d*e^3*f - c
^2*e^2*f^2 - d*e^3*f + e^2*f^2 + (d^2*e*f^3 - c*d*f^4 - d*f^4)*x^3 + (2*d^
2*e^2*f^2 - c*d*e*f^3 - c^2*f^4 - 3*d*e*f^3 + f^4)*x^2 + (d^2*e^3*f + c*d*
e^2*f^2 - 2*c^2*e*f^3 - 3*d*e^2*f^2 + 2*e*f^3)*x), x)
```

3.49.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{(e + fx)^2} \, dx = \int \frac{(b \operatorname{artanh}(dx + c) + a)^3}{(fx + e)^2} \, dx$$

input
```
integrate((a+b*arctanh(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3/(f*x + e)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(c + dx))^3}{(e + fx)^2} \, dx = \int \frac{(a + b \operatorname{atanh}(c + d\,x))^3}{(e + f\,x)^2} \, dx$$

input
```
int((a + b*atanh(c + d*x))^3/(e + f*x)^2,x)
```

output
```
int((a + b*atanh(c + d*x))^3/(e + f*x)^2, x)
```

3.49. $\int \frac{(a+b\operatorname{arctanh}(c+dx))^3}{(e+fx)^2}\,dx$

3.50 $\int (e+fx)^m(a+b\text{arctanh}(c+dx))^3\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^3\,dx = \text{Int}\left((e+fx)^m(a+b\text{arctanh}(c+dx))^3,x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

3.50.2 Mathematica [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^3\,dx = \int (e+fx)^m(a+b\text{arctanh}(c+dx))^3\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3, x]
```

3.50.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6661, 6651}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m (a+b\text{arctanh}(c+dx))^3 \, dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m (a+b\text{arctanh}(c+dx))^3 d(c+dx)}{d}$$

$$\Big\downarrow \text{6651}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m (a+b\text{arctanh}(c+dx))^3 d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^3,x]
```

output
```
$Aborted
```

3.50.3.1 Defintions of rubi rules used

rule 6651
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcTanh[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.50.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\operatorname{arctanh}(dx+c))^3\,dx$$

input
```
int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

output
```
int((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x)
```

3.50.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m(a+b\operatorname{arctanh}(c+dx))^3\,dx = \int (b\operatorname{artanh}(dx+c)+a)^3(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(d*x + c)^3 + 3*a*b^2*arctanh(d*x + c)^2 + 3*a^2*b*ar
ctanh(d*x + c) + a^3)*(f*x + e)^m, x)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\operatorname{arctanh}(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*atanh(d*x+c))**3,x)
```

output
```
Timed out
```

3.50.7 Maxima [N/A]

Not integrable

Time = 4.00 (sec) , antiderivative size = 432, normalized size of antiderivative = 21.60

$$\int (e+fx)^m (a+b\text{arctanh}(c+dx))^3 \, dx = \int (b \,\text{artanh}\,(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/8*(b^3*f*x + b^3*e)*(f*x + e)^m*log(-d*x - c + 1)^3/(f*(m + 1)) + (f*x
+ e)^(m + 1)*a^3/(f*(m + 1)) + integrate(1/8*((b^3*d*f*(m + 1)*x + (c*f*(m
 + 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^3 + 6*(a*b^2*d*f*(m + 1)*x + (c*f
*(m + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1)^2 + 3*(b^3*d*e + 2*(c*f*(m +
 1) - f*(m + 1))*a*b^2 + (2*a*b^2*d*f*(m + 1) + b^3*d*f)*x + (b^3*d*f*(m +
 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1))*log(-d*x - c + 1)
^2 + 12*(a^2*b*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a^2*b)*log(d*x +
c + 1) - 3*(4*a^2*b*d*f*(m + 1)*x + 4*(c*f*(m + 1) - f*(m + 1))*a^2*b + (b
^3*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*b^3)*log(d*x + c + 1)^2 + 4*(
a*b^2*d*f*(m + 1)*x + (c*f*(m + 1) - f*(m + 1))*a*b^2)*log(d*x + c + 1))*l
og(-d*x - c + 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x
)
```

3.50.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m (a+b\text{arctanh}(c+dx))^3 \, dx = \int (b \,\text{artanh}\,(dx+c)+a)^3 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^3*(f*x + e)^m, x)
```

3.50.9 Mupad [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^3\,dx=\int (e+f\,x)^m\,\left(a+b\,\text{atanh}(c+d\,x)\right)^3\,dx$$

input
```
int((e + f*x)^m*(a + b*atanh(c + d*x))^3,x)
```

output
```
int((e + f*x)^m*(a + b*atanh(c + d*x))^3, x)
```

3.51 $\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx = \text{Int}\big((e+fx)^m(a+b\text{arctanh}(c+dx))^2,x\big)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
```

3.51.2 Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx = \int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2, x]
```

3.51.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6661, 6651}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{6651}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\text{arctanh}(c+dx))^2d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x])^2,x]
```

output
```
$Aborted
```

3.51.3.1 Defintions of rubi rules used

rule 6651
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcTanh[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] ||
 MatchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f
_.)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || Match
Q[u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*
x)^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\ \mathrm{arctanh}\,(dx+c))^2\,dx$$

input
```
int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
```

output
```
int((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x)
```

3.51.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m(a+b\mathrm{arctanh}(c+dx))^2\,dx = \int (b\,\mathrm{artanh}\,(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(d*x + c)^2 + 2*a*b*arctanh(d*x + c) + a^2)*(f*x + e)
^m, x)
```

3.51.6 Sympy [N/A]

Not integrable

Time = 93.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (e+fx)^m(a+b\mathrm{arctanh}(c+dx))^2\,dx = \int (a+b\,\mathrm{atanh}\,(c+dx))^2\,(e+fx)^m\,dx$$

input
```
integrate((f*x+e)**m*(a+b*atanh(d*x+c))**2,x)
```

output
```
Integral((a + b*atanh(c + d*x))**2*(e + f*x)**m, x)
```

3.51.7 Maxima [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 13.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx = \int (b\,\text{artanh}\,(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*f*x + b^2*e)*(f*x + e)^m*log(-d*x - c + 1)^2/(f*(m + 1)) + (f*x +
 e)^(m + 1)*a^2/(f*(m + 1)) - integrate(-1/4*((b^2*d*f*(m + 1)*x + (c*f*(m
 + 1) - f*(m + 1))*b^2)*log(d*x + c + 1)^2 + 4*(a*b*d*f*(m + 1)*x + (c*f*(
m + 1) - f*(m + 1))*a*b)*log(d*x + c + 1) - 2*(b^2*d*e + 2*(c*f*(m + 1) -
f*(m + 1))*a*b + (2*a*b*d*f*(m + 1) + b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c
*f*(m + 1) - f*(m + 1))*b^2)*log(d*x + c + 1))*log(-d*x - c + 1))*(f*x + e
)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)
```

3.51.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx = \int (b\,\text{artanh}\,(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)^2*(f*x + e)^m, x)
```

3.51.9 Mupad [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx = \int (e+f\,x)^m\,(a+b\,\text{atanh}(c+d\,x))^2\,dx$$

input
```
int((e + f*x)^m*(a + b*atanh(c + d*x))^2,x)
```

output
```
int((e + f*x)^m*(a + b*atanh(c + d*x))^2, x)
```

3.51. $\int (e+fx)^m(a+b\text{arctanh}(c+dx))^2\,dx$

3.52 $\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx$

3.52.1 Optimal result

Integrand size = 18, antiderivative size = 162

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx = \frac{(e+fx)^{1+m}(a+b\text{arctanh}(c+dx))}{f(1+m)} + \frac{bd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-f-cf}\right)}{2f(de-(1+c)f)(1+m)(2+m)} - \frac{bd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+f-cf}\right)}{2f(de+f-cf)(1+m)(2+m)}$$

output
```
(f*x+e)^(1+m)*(a+b*arctanh(d*x+c))/f/(1+m)+1/2*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e-f))/f/(d*e-(1+c)*f)/(1+m)/(2+m)-1/2*b*
d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e+f))/f/(-c*f+d
*e+f)/(1+m)/(2+m)
```

3.52.2 Mathematica [F]

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx=\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]),x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcTanh[c + d*x]), x]
```

3.52.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6661, 6478, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\text{arctanh}(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{6478}$$

$$\frac{\frac{d(a+b\text{arctanh}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int \frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{1-(c+dx)^2}d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{485}$$

$$\frac{\frac{d(a+b\text{arctanh}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int \left(\frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(-c-dx+1)}+\frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(c+dx+1)}\right)d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d(a+b\text{arctanh}(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\left(\frac{d\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{de-cf+f(c+dx)}{de-cf+f}\right)}{2(m+2)(-cf+de+f)}-\frac{d\left(\frac{f(c+dx)}{d}-\frac{cf}{d}\cdots\right.}{\cdots}\right.}{f(m+1)}}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTanh[c + d*x]),x]
```

output
```
((d*(e - (c*f)/d + (f*(c + d*x))/d)^(1 + m)*(a + b*ArcTanh[c + d*x]))/(f*(
1 + m)) - (b*d*(-1/2*(d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hypergeome
tric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e - f - c*f)])/((d*e
 - (1 + c)*f)*(2 + m)) + (d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e + f - c*f)])/(
2*(d*e + f - c*f)*(2 + m))))/(f*(1 + m)))/d
```

3.52.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6478
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTanh[c*x])/(e*(q + 1))), x] - Simp[b
*(c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
 b, c, d, e, q}, x] && NeQ[q, -1]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.52.4 Maple [F]

$$\int (fx+e)^m (a+b \operatorname{arctanh}(dx+c))\, dx$$

input
```
int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)
```

output
```
int((f*x+e)^m*(a+b*arctanh(d*x+c)),x)
```

3.52.5 Fricas [F]

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\, dx = \int (b \operatorname{artanh}(dx+c)+a)(fx+e)^m\, dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)
```

3.52.6 Sympy [F]

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\, dx = \int (a+b \operatorname{atanh}(c+dx))\,(e+fx)^m\, dx$$

input
```
integrate((f*x+e)**m*(a+b*atanh(d*x+c)),x)
```

output
```
Integral((a + b*atanh(c + d*x))*(e + f*x)**m, x)
```

3.52.7 Maxima [F]

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx = \int (b\,\text{artanh}\,(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*b*((f*x + e)*(f*x + e)^m*log(-d*x - c + 1)/(f*(m + 1)) - integrate((d
*f*x + d*e + (d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1))*log(d*x + c + 1))*(
f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) - f*(m + 1)), x)) + (f*x + e)^(m +
 1)*a/(f*(m + 1))
```

3.52.8 Giac [F]

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx = \int (b\,\text{artanh}\,(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctanh(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(d*x + c) + a)*(f*x + e)^m, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\text{arctanh}(c+dx))\,dx = \int (e+f\,x)^m\,(a+b\,\text{atanh}(c+d\,x))\,dx$$

input
```
int((e + f*x)^m*(a + b*atanh(c + d*x)),x)
```

output
```
int((e + f*x)^m*(a + b*atanh(c + d*x)), x)
```

3.53 $\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx$

3.53.1 Optimal result

Integrand size = 16, antiderivative size = 780

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx = -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3}\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{2/3}\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{-1}\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{-1}\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(-1)^{2/3}(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{d}(1-a-bx)}{b\sqrt[3]{c}+(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(1-a-bx)}{b\sqrt[3]{c}-\sqrt[3]{-1}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{d}(1-a-bx)}{b\sqrt[3]{c}+(-1)^{2/3}(1-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\operatorname{PolyLog}\left(2,-\frac{\sqrt[3]{d}(1+a+bx)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{2/3}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(1+a+bx)}{b\sqrt[3]{c}+\sqrt[3]{-1}(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{-1}\operatorname{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{d}(1+a+bx)}{b\sqrt[3]{c}-(-1)^{2/3}(1+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

output
```
-1/6*ln(-b*x-a+1)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(1-a)*d^(1/3)))/c^(2
/3)/d^(1/3)+1/6*ln(b*x+a+1)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)-(1+a)*d^(1
/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(2/3)*ln(-b*x-a+1)*ln(b*(c^(1/3)-(-1)^(1/3)
*d^(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)
^(2/3)*ln(b*x+a+1)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/
3)*(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/3)*ln(-b*x-a+1)*ln(b*(c^(1/
3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(
1/3)-1/6*(-1)^(1/3)*ln(b*x+a+1)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(
1/3)-(-1)^(2/3)*(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*polylog(2,d^(1/3)*(-b*
x-a+1)/(b*c^(1/3)+(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(2/3)*polylog(2
,-(-1)^(1/3)*d^(1/3)*(-b*x-a+1)/(b*c^(1/3)-(-1)^(1/3)*(1-a)*d^(1/3)))/c^(2
/3)/d^(1/3)+1/6*(-1)^(1/3)*polylog(2,(-1)^(2/3)*d^(1/3)*(-b*x-a+1)/(b*c^(1
/3)+(-1)^(2/3)*(1-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*polylog(2,-d^(1/3)*(b*x
+a+1)/(b*c^(1/3)-(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(2/3)*polylog(2,
(-1)^(1/3)*d^(1/3)*(b*x+a+1)/(b*c^(1/3)+(-1)^(1/3)*(1+a)*d^(1/3)))/c^(2/3)
/d^(1/3)-1/6*(-1)^(1/3)*polylog(2,-(-1)^(2/3)*d^(1/3)*(b*x+a+1)/(b*c^(1/3)
-(-1)^(2/3)*(1+a)*d^(1/3)))/c^(2/3)/d^(1/3)
```

3.53.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx$$

$$= \frac{-\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(-1+a)\sqrt[3]{d}}\right)+\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(1+a)\sqrt[3]{d}}\right)-(-1)^{2/3}\log(1-a-b}{}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d*x^3),x]
```

output
```
(-(Log[1 - a - b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-1 + a)*d^
(1/3))]) + Log[1 + a + b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (1
+ a)*d^(1/3))] - (-1)^(2/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*
d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-1 + a)*d^(1/3))] + (-1)^(2/3)*Log[1
+ a + b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3
)*(1 + a)*d^(1/3))] + (-1)^(1/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) + (-1)^(
2/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-1 + a)*d^(1/3))] - (-1)^(1/3)*L
og[1 + a + b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)
^(2/3)*(1 + a)*d^(1/3))] - PolyLog[2, -((d^(1/3)*(-1 + a + b*x))/(b*c^(1/3
) - (-1 + a)*d^(1/3)))] - (-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-1 +
a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(-1 + a)*d^(1/3))] + (-1)^(1/3)*PolyLog[
2, ((-1)^(2/3)*d^(1/3)*(-1 + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(-1 + a)
*d^(1/3))] + PolyLog[2, -((d^(1/3)*(1 + a + b*x))/(b*c^(1/3) - (1 + a)*d^(
1/3)))] + (-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(1 + a + b*x))/(b*c^(1
/3) + (-1)^(1/3)*(1 + a)*d^(1/3))] - (-1)^(1/3)*PolyLog[2, ((-1)^(2/3)*d^(
1/3)*(1 + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(1 + a)*d^(1/3))])/(6*c^(2/
3)*d^(1/3))
```

3.53.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6665, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx$$

$$\Big\downarrow \text{6665}$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{dx^3+c}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{dx^3+c}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}\int \left(-\frac{\log(a+bx+1)}{3c^{2/3}\left(-\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(a+bx+1)}{3c^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(a+bx+1)}{3c^{2/3}\left(-(-1)^{2/3}\sqrt[3]{d}x-\sqrt[3]{c}\right)}\right)dx-\frac{1}{2}\int \left(-\frac{\log(-a-bx+1)}{3c^{2/3}\left(-\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(-a-bx+1)}{3c^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(-a-bx+1)}{3c^{2/3}\left(-(-1)^{2/3}\sqrt[3]{d}x-\sqrt[3]{c}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log(-a-bx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(1-a)+b\sqrt[3]{c}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{(-1)^{2/3}\log(-a-bx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(1-a)\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[3]{-1}\log(\ldots}{\ldots}\right.$$

$$\frac{1}{2}\left(\frac{\log(a+bx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+1)\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{(-1)^{2/3}\log(a+bx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{\sqrt[3]{-1}\sqrt[3]{d}(a+1)+b\sqrt[3]{c}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{-1}\log(a+b\ldots}{\ldots}\right.$$

input

```
Int[ArcTanh[a + b*x]/(c + d*x^3),x]
```

output

```
(-1/3*(Log[1 - a - b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (1 - a)
*d^(1/3))])/(c^(2/3)*d^(1/3)) - ((-1)^(2/3)*Log[1 - a - b*x]*Log[(b*(c^(1/
3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(1/3)*(1 - a)*d^(1/3))])/(3*
c^(2/3)*d^(1/3)) + ((-1)^(1/3)*Log[1 - a - b*x]*Log[(b*(c^(1/3) + (-1)^(2/
3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(1 - a)*d^(1/3))])/(3*c^(2/3)*d^(1/
3)) - PolyLog[2, (d^(1/3)*(1 - a - b*x))/(b*c^(1/3) + (1 - a)*d^(1/3))]/(3
*c^(2/3)*d^(1/3)) - ((-1)^(2/3)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(1 - a -
b*x))/(b*c^(1/3) - (-1)^(1/3)*(1 - a)*d^(1/3)))])/(3*c^(2/3)*d^(1/3)) + ((
-1)^(1/3)*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(1 - a - b*x))/(b*c^(1/3) + (-1)^
(2/3)*(1 - a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)))/2 + ((Log[1 + a + b*x]*Log[(
b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (1 + a)*d^(1/3))])/(3*c^(2/3)*d^(1/3
)) + ((-1)^(2/3)*Log[1 + a + b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))
/(b*c^(1/3) + (-1)^(1/3)*(1 + a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - ((-1)^(1
/3)*Log[1 + a + b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) -
 (-1)^(2/3)*(1 + a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) + PolyLog[2, -((d^(1/3)
*(1 + a + b*x))/(b*c^(1/3) - (1 + a)*d^(1/3)))]/(3*c^(2/3)*d^(1/3)) + ((-1
)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(1 + a + b*x))/(b*c^(1/3) + (-1)^(1
/3)*(1 + a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) - ((-1)^(1/3)*PolyLog[2, -((d^(
1/3)*((-1)^(2/3) + (-1)^(2/3)*a + (-1)^(2/3)*b*x))/(b*c^(1/3) - (-1)^(2/3)
*(1 + a)*d^(1/3)))])/(3*c^(2/3)*d^(1/3)))/2
```

3.53. $\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.53.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.33

method	result
risch	$-\frac{b^2\left(\sum_{\_R1=\text{RootOf}\left(d\_Z^3+(3ad-3d)\_Z^2+\left(3a^2d-6ad+3d\right)\_Z+d\,a^3-b^3c-3a^2d+3ad-d\right)}\frac{\ln(-bx-a+1)\ln\left(\frac{bx+\_R1+a-1}{\_R1}\right)}{\_R1^2+2\_R1a+a^2}\right)}{6d}$
derivativedivides	$-\frac{b^3\left(\sum_{\_R=\text{RootOf}\left(d\_Z^3-3\_Z^2ad+3\_Za^2d-d\,a^3+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\text{arctanh}(bx+a)}{3d}+\frac{b^3\left(\text{arctanh}(bx+a)\left(\sum_{\_R=\text{Ro}}\right.\right.}{}$
default	$-\frac{b^3\left(\sum_{\_R=\text{RootOf}\left(d\_Z^3-3\_Z^2ad+3\_Za^2d-d\,a^3+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\text{arctanh}(bx+a)}{3d}+\frac{b^3\left(\text{arctanh}(bx+a)\left(\sum_{\_R=\text{Ro}}\right.\right.}{}$

input
```
int(arctanh(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^2/d*sum(1/(_R1^2+2*_R1*a+a^2-2*_R1-2*a+1)*(ln(-b*x-a+1)*ln((b*x+_R1
+a-1)/_R1)+dilog((b*x+_R1+a-1)/_R1)),_R1=RootOf(d*_Z^3+(3*a*d-3*d)*_Z^2+(3
*a^2*d-6*a*d+3*d)*_Z+d*a^3-b^3*c-3*a^2*d+3*a*d-d))+1/6*b^2/d*sum(1/(_R1^2-
2*_R1*a+a^2-2*_R1+2*a+1)*(ln(b*x+a+1)*ln((-b*x+_R1-a-1)/_R1)+dilog((-b*x+_
R1-a-1)/_R1)),_R1=RootOf(d*_Z^3+(-3*a*d-3*d)*_Z^2+(3*a^2*d+6*a*d+3*d)*_Z-d
*a^3+b^3*c-3*a^2*d-3*a*d-d))
```

3.53. $\int \frac{\text{arctanh}(a+bx)}{c+dx^3}\,dx$

3.53.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)}{dx^3+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)/(d*x^3 + c), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.53.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)}{dx^3+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(arctanh(b*x + a)/(d*x^3 + c), x)
```

3.53.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{dx^3+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^3}\,dx=\int \frac{\operatorname{atanh}(a+b\,x)}{d\,x^3+c}\,dx$$

input
```
int(atanh(a + b*x)/(c + d*x^3),x)
```

output
```
int(atanh(a + b*x)/(c + d*x^3), x)
```

3.54 $\int \frac{\text{arctanh}(a+bx)}{c+dx^2} \, dx$

3.54.1 Optimal result

Integrand size = 16, antiderivative size = 481

$$\int \frac{\text{arctanh}(a+bx)}{c+dx^2} \, dx = -\frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

output
```
-1/4*ln(-b*x-a+1)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(1-a)*d^(1/2))
)/(-c)^(1/2)/d^(1/2)+1/4*ln(b*x+a+1)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(
1/2)+(1+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln(-b*x-a+1)*ln(b*((-c)^(1/2)+
x*d^(1/2))/(b*(-c)^(1/2)+(1-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*ln(b*x+a+1
)*ln(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(1+a)*d^(1/2)))/(-c)^(1/2)/d^(
1/2)-1/4*polylog(2,-(-b*x-a+1)*d^(1/2)/(b*(-c)^(1/2)-(1-a)*d^(1/2)))/(-c)^
(1/2)/d^(1/2)+1/4*polylog(2,(-b*x-a+1)*d^(1/2)/(b*(-c)^(1/2)+(1-a)*d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(b*x+a+1)*d^(1/2)/(b*(-c)^(1/2)-(1+a)
*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,(b*x+a+1)*d^(1/2)/(b*(-c)^(1/2
)+(1+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx$$
$$= \frac{-\log(1-a-bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right)+\log(1+a+bx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)+\log(1-a-bx)\log\left(\frac{b(}{b\sqrt{-}}\right.}{}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d*x^2),x]
```

output
```
(-(Log[1 - a - b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-1 + a)*
Sqrt[d])]) + Log[1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] +
 (1 + a)*Sqrt[d])] + Log[1 - a - b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sq
rt[-c] - (-1 + a)*Sqrt[d])] - Log[1 + a + b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*
x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])] + PolyLog[2, -((Sqrt[d]*(-1 + a + b*x)
)/(b*Sqrt[-c] - (-1 + a)*Sqrt[d]))] - PolyLog[2, (Sqrt[d]*(-1 + a + b*x))/
(b*Sqrt[-c] + (-1 + a)*Sqrt[d])] - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b
*Sqrt[-c] - (1 + a)*Sqrt[d]))] + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(b*Sqr
t[-c] + (1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d])
```

3.54.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6665, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \; 6665$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{dx^2+c}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{dx^2+c}dx$$

$$\Big\downarrow \; 2856$$

$$\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(a+bx+1)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(a+bx+1)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx-\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(-a-bx+1)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(-a-bx+1)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+1)}{\sqrt{d}(1-a)+b\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\log(-a-bx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log\cdots}{\cdots}\right.$$
$$\frac{1}{2}\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+1)}{\sqrt{d}(a+1)+b\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log(a+bx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\log(\cdots}{\cdots}\right.$$

input
```
Int[ArcTanh[a + b*x]/(c + d*x^2),x]
```

output
```
(-1/2*(Log[1 - a - b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (1 -
a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + (Log[1 - a - b*x]*Log[(b*(Sqrt[-c] + Sq
rt[d]*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[
2, -((Sqrt[d]*(1 - a - b*x))/(b*Sqrt[-c] - (1 - a)*Sqrt[d]))]/(2*Sqrt[-c]*
Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 - a - b*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d
])]/(2*Sqrt[-c]*Sqrt[d]))/2 + ((Log[1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[d
]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d]) - (Log[1 + a +
 b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])])/(2*S
qrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 +
 a)*Sqrt[d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(
b*Sqrt[-c] + (1 + a)*Sqrt[d])]/(2*Sqrt[-c]*Sqrt[d]))/2
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.54.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\ln(-bx-a+1)\ln\left(\frac{b\sqrt{-cd}-(-bx-a+1)d-ad+d}{b\sqrt{-cd}-ad+d}\right)}{4\sqrt{-cd}}-\frac{\ln(-bx-a+1)\ln\left(\frac{b\sqrt{-cd}+(-bx-a+1)d+ad-d}{b\sqrt{-cd}+ad-d}\right)}{4\sqrt{-cd}}+\frac{\operatorname{dilog}\left(\frac{b\sqrt{-cd}-(-b}{b\sqrt{-c}}\right.}{4\sqrt{-}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctanh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-b*x-a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(-b*x-a+1)*d-a*d+d)/(b*(-
c*d)^(1/2)-a*d+d))-1/4*ln(-b*x-a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(-b*x-
a+1)*d+a*d-d)/(b*(-c*d)^(1/2)+a*d-d))+1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/
2)-(-b*x-a+1)*d-a*d+d)/(b*(-c*d)^(1/2)-a*d+d))-1/4/(-c*d)^(1/2)*dilog((b*(
-c*d)^(1/2)+(-b*x-a+1)*d+a*d-d)/(b*(-c*d)^(1/2)+a*d-d))+1/4*ln(b*x+a+1)/(-
c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a+1)*d+a*d+d)/(b*(-c*d)^(1/2)+a*d+d))-1
/4*ln(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a+1)*d-a*d-d)/(b*(-c*d
)^(1/2)-a*d-d))+1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)-(b*x+a+1)*d+a*d+d)/
(b*(-c*d)^(1/2)+a*d+d))-1/4/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)+(b*x+a+1)*d
-a*d-d)/(b*(-c*d)^(1/2)-a*d-d))
```

3.54.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)/(d*x^2 + c), x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx=\text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.54.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx = \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)\operatorname{artanh}(bx+a)}{\sqrt{cd}} + \frac{\left(\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d}, \frac{(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right) - \arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d}, \frac{(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)\right)\log(dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
arctan(d*x/sqrt(c*d))*arctanh(b*x + a)/sqrt(c*d) + 1/4*((arctan2((b^2*x +
(a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (
a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*
b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*
a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/
sqrt(c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a
^2 + 2*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b
*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*
b*d*x + b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a
 + I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b
^2*c - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a + I)*b*s
qrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I
*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - I)*b*sqrt(c)*sq
rt(d) - (a^2 + 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x +
(-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (
a^2 + 2*a + 1)*d)))/sqrt(c*d)
```

3.54.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(d*x^2 + c), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(atanh(a + b*x)/(c + d*x^2),x)
```

output
```
int(atanh(a + b*x)/(c + d*x^2), x)
```

3.55 $\int \frac{\text{arctanh}(a+bx)}{c+dx}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 120

$$\int \frac{\text{arctanh}(a+bx)}{c+dx}\,dx = -\frac{\text{arctanh}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\text{arctanh}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2d} - \frac{\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d}$$

output
```
-arctanh(b*x+a)*ln(2/(b*x+a+1))/d+arctanh(b*x+a)*ln(2*b*(d*x+c)/(-a*d+b*c+
d)/(b*x+a+1))/d+1/2*polylog(2,1-2/(b*x+a+1))/d-1/2*polylog(2,1-2*b*(d*x+c)
/(-a*d+b*c+d)/(b*x+a+1))/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15

$$\int \frac{\text{arctanh}(a+bx)}{c+dx}\,dx = -\frac{\log(1-a-bx)\log\left(-\frac{b(c+dx)}{-bc-(1-a)d}\right)}{2d} + \frac{\log(1+a+bx)\log\left(\frac{b(c+dx)}{bc-(1+a)d}\right)}{2d} - \frac{\text{PolyLog}\left(2,-\frac{d(1-a-bx)}{-bc-d+ad}\right)}{2d} + \frac{\text{PolyLog}\left(2,\frac{d(1+a+bx)}{-bc+d+ad}\right)}{2d}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d*x),x]
```

output
```
-1/2*(Log[1 - a - b*x]*Log[-((b*(c + d*x))/(-(b*c) - (1 - a)*d))])/d + (Lo
g[1 + a + b*x]*Log[(b*(c + d*x))/(b*c - (1 + a)*d)])/(2*d) - PolyLog[2, -(
(d*(1 - a - b*x))/(-(b*c) - d + a*d))]/(2*d) + PolyLog[2, (d*(1 + a + b*x)
)/(-(b*c) + d + a*d)]/(2*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6661, 27, 6472, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{6661}$$

$$\frac{\int \frac{b\,\text{arctanh}(a+bx)}{b\left(c-\frac{ad}{b}\right)+d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\text{arctanh}(a+bx)}{d(a+bx)-ad+bc}d(a+bx)$$

$$\Big\downarrow \text{6472}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\text{arctanh}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\text{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{d}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\int \frac{\log\left(\frac{2}{a+bx+1}\right)}{1-\frac{2}{a+bx+1}}d\frac{1}{a+bx+1}}{d}+\frac{\text{arctanh}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\text{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{d}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{1-(a+bx)^2}d(a+bx)}{d}+\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\operatorname{arctanh}(a+bx)\log\left(\frac{2}{a+bx+1}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2(bc-ad+d(a+bx))}{(bc-ad+d)(a+bx+1)}\right)}{2d}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}$$

input
```
Int[ArcTanh[a + b*x]/(c + d*x),x]
```

output
```
-((ArcTanh[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcTanh[a + b*x]*Log[(2*(b
*c - a*d + d*(a + b*x)))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2,
1 - 2/(1 + a + b*x)]/(2*d) - PolyLog[2, 1 - (2*(b*c - a*d + d*(a + b*x)))/
((b*c + d - a*d)*(1 + a + b*x))]/(2*d)
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6472
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*ArcTanh
[c*x])*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(c/e)
Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*c*((d
 + e*x)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x]) /; FreeQ[{a, b, c, d
, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

rule 6661
```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*
ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IG
tQ[p, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\operatorname{dilog}\left(\frac{(-bx-a+1)d+ad-bc-d}{ad-bc-d}\right)}{2d}-\frac{\ln(-bx-a+1)\ln\left(\frac{(-bx-a+1)d+ad-bc-d}{ad-bc-d}\right)}{2d}+\frac{\operatorname{dilog}\left(\frac{(bx+a+1)d-ad+bc-d}{-ad+bc-d}\right)}{2d}+\frac{\ln(}{}$
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arctanh}(bx+a)}{d}+\frac{b\left(-\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)+\ln(ad-bc-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)\right)}{2}+\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a}{-ad+bc}\right.\right.}{}\right.}{d^2}}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arctanh}(bx+a)}{d}+\frac{b\left(-\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)+\ln(ad-bc-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+bc-d}\right)\right)}{2}+\frac{d\left(\operatorname{dilog}\left(\frac{-d(bx+a}{-ad+bc}\right.\right.}{}\right.}{d^2}}{b}$
parts	$\frac{\ln(dx+c)\operatorname{arctanh}(bx+a)}{d}-\frac{b\left(\frac{d\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx+c)+d}{ad-bc+d}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-bc+b(dx+c)+d}{ad-bc+d}\right)}{b}\right)}{2}-\frac{d\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx-}{ad-bc-}\right.}{b}\right.}{}\right.}{d^2}$

input
```
int(arctanh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*dilog(((-b*x-a+1)*d+a*d-b*c-d)/(a*d-b*c-d))/d-1/2*ln(-b*x-a+1)*ln(((-
b*x-a+1)*d+a*d-b*c-d)/(a*d-b*c-d))/d+1/2*dilog(((b*x+a+1)*d-a*d+b*c-d)/(-a
*d+b*c-d))/d+1/2*ln(b*x+a+1)*ln(((b*x+a+1)*d-a*d+b*c-d)/(-a*d+b*c-d))/d
```

3.55.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(arctanh(b*x + a)/(d*x + c), x)
```

3.55.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{atanh}(a+bx)}{c+dx}\,dx$$

input
```
integrate(atanh(b*x+a)/(d*x+c),x)
```

output
```
Integral(atanh(a + b*x)/(c + d*x), x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx}\,dx = -\frac{1}{2}b\left(\frac{\log(bx+a-1)\log\left(\frac{bdx+ad-d}{bc-ad+d}+1\right)+\operatorname{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd} - \frac{\log(bx+a+1)\log\left(\frac{bdx+ad+d}{bc-ad-d}+1\right)+\operatorname{Li}_2(}{bd}\right. - \frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(dx+c)}{2\,d} + \frac{\operatorname{artanh}(bx+a)\log(dx+c)}{d}$$

input
```
integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*b*((log(b*x + a - 1)*log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + dil
og(-(b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (log(b*x + a + 1)*log((b*d
*x + a*d + d)/(b*c - a*d - d) + 1) + dilog(-(b*d*x + a*d + d)/(b*c - a*d -
 d)))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + c
)/d + arctanh(b*x + a)*log(d*x + c)/d
```

3.55.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{dx+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(d*x + c), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(atanh(a + b*x)/(c + d*x),x)
```

output
```
int(atanh(a + b*x)/(c + d*x), x)
```

3.56 $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}} dx$

3.56.1 Optimal result

Integrand size = 16, antiderivative size = 186

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}} dx = \frac{(1-a-bx)\log(1-a-bx)}{2bc} + \frac{(1+a+bx)\log(1+a+bx)}{2bc} - \frac{d\log(1+a+bx)\log\left(-\frac{b(d+cx)}{c+ac-bd}\right)}{2c^2} + \frac{d\log(1-a-bx)\log\left(\frac{b(d+cx)}{c-ac+bd}\right)}{2c^2} + \frac{d\,\mathrm{PolyLog}\left(2,\frac{c(1-a-bx)}{c-ac+bd}\right)}{2c^2} - \frac{d\,\mathrm{PolyLog}\left(2,\frac{c(1+a+bx)}{c+ac-bd}\right)}{2c^2}$$

output
```
1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-1/2*d*ln(b*x
+a+1)*ln(-b*(c*x+d)/(a*c-b*d+c))/c^2+1/2*d*ln(-b*x-a+1)*ln(b*(c*x+d)/(-a*c
+b*d+c))/c^2+1/2*d*polylog(2,c*(-b*x-a+1)/(-a*c+b*d+c))/c^2-1/2*d*polylog(
2,c*(b*x+a+1)/(a*c-b*d+c))/c^2
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.46 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{2c(a+bx)\operatorname{arctanh}(a+bx) + \frac{bcd\operatorname{arctanh}(a+bx)^2}{ac-bd} - 2c\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right) + \frac{bd\left(c\sqrt{1-a^2+\frac{2abd}{c}-\frac{b^2d^2}{c^2}}\,e^{\operatorname{arctanh}\left(a-\frac{bd}{c}\right)}\operatorname{ar}\ldots\right.}{\ldots}}{\ldots}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d/x),x]
```

output
```
(2*c*(a + b*x)*ArcTanh[a + b*x] + (b*c*d*ArcTanh[a + b*x]^2)/(a*c - b*d) -
 2*c*Log[1/Sqrt[1 - (a + b*x)^2]] + (b*d*(c*Sqrt[1 - a^2 + (2*a*b*d)/c - (
b^2*d^2)/c^2]*E^ArcTanh[a - (b*d)/c]*ArcTanh[a + b*x]^2 + (a*c - b*d)*ArcT
anh[a + b*x]*(I*Pi - 2*ArcTanh[a - (b*d)/c] + 2*Log[1 - E^(2*(ArcTanh[a -
(b*d)/c] - ArcTanh[a + b*x]))]) - (a*c - b*d)*(I*Pi*(Log[1 + E^(2*ArcTanh[
a + b*x])] - Log[1/Sqrt[1 - (a + b*x)^2]]) + 2*ArcTanh[a - (b*d)/c]*(Log[1
 - E^(2*(ArcTanh[a - (b*d)/c] - ArcTanh[a + b*x]))] - Log[(-I)*Sinh[ArcTan
h[a - (b*d)/c] - ArcTanh[a + b*x]]])) + (-(a*c) + b*d)*PolyLog[2, E^(2*(Ar
cTanh[a - (b*d)/c] - ArcTanh[a + b*x]))]))/(-(a*c) + b*d) + b*d*(ArcTanh[a
 + b*x]*(ArcTanh[a + b*x] + 2*Log[1 + E^(-2*ArcTanh[a + b*x])]) - PolyLog[
2, -E^(-2*ArcTanh[a + b*x])]))/(2*b*c^2)
```

3.56.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6665, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx$$

$$\Big\downarrow \ 6665$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{c+\frac{d}{x}}dx - \frac{1}{2}\int \frac{\log(-a-bx+1)}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}\int \left(\frac{\log(a+bx+1)}{c}-\frac{d\log(a+bx+1)}{c(d+cx)}\right)dx-\frac{1}{2}\int \left(\frac{\log(-a-bx+1)}{c}-\frac{d\log(-a-bx+1)}{c(d+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{d\,\text{PolyLog}\left(2,\frac{c(-a-bx+1)}{-ac+c+bd}\right)}{c^2}+\frac{d\log(-a-bx+1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{c^2}+\frac{(-a-bx+1)\log(-a-bx+1)}{bc}+\frac{x}{c}\right)+\frac{1}{2}\left(-\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx+1)}{ac+c-bd}\right)}{c^2}-\frac{d\log(a+bx+1)\log\left(-\frac{b(cx+d)}{ac-bd+c}\right)}{c^2}+\frac{(a+bx+1)\log(a+bx+1)}{bc}-\frac{x}{c}\right)$$

input
```
Int[ArcTanh[a + b*x]/(c + d/x),x]
```

output
```
(x/c + ((1 - a - b*x)*Log[1 - a - b*x])/(b*c) + (d*Log[1 - a - b*x]*Log[(b
*(d + c*x))/(c - a*c + b*d)])/c^2 + (d*PolyLog[2, (c*(1 - a - b*x))/(c - a
*c + b*d)])/c^2)/2 + (-(x/c) + ((1 + a + b*x)*Log[1 + a + b*x])/(b*c) - (d
*Log[1 + a + b*x]*Log[-((b*(d + c*x))/(c + a*c - b*d))])/c^2 - (d*PolyLog[
2, (c*(1 + a + b*x))/(c + a*c - b*d)])/c^2)/2
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.56.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.39

method	result
parts	$\frac{\operatorname{arctanh}(bx+a)x}{c}-\frac{\operatorname{arctanh}(bx+a)d\ln(cx+d)}{c^2}-\frac{b\left(\frac{(-1-a)\ln(ac-db+b(cx+d)+c)}{2b^2}+\frac{(-1+a)\ln(ac-db+b(cx+d)-c)}{2b^2}+d\left(-\right.\right.}{}$
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2bc}-\frac{1}{bc}-\frac{d\operatorname{dilog}\left(\frac{(bx+a+1)c-ac+db-c}{-ac+db-c}\right)}{2c^2}-\frac{d\ln(bx+a+1)\ln\left(\frac{(bx+a+\right.}{2c^2}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(bx+a)(bx+a)}{c}-\frac{\operatorname{arctanh}(bx+a)db\ln(ac-db-c(bx+a))}{c^2}+\frac{bd\left(\frac{\operatorname{dilog}\left(\frac{-c(bx+a)-c}{-ac+db-c}\right)+\ln(ac-db-c(bx+a))\ln\left(\frac{-c(bx+a)-c}{-ac+db-c}\right)}{2c}-\right.}{}}{}$
default	$\frac{\frac{\operatorname{arctanh}(bx+a)(bx+a)}{c}-\frac{\operatorname{arctanh}(bx+a)db\ln(ac-db-c(bx+a))}{c^2}+\frac{bd\left(\frac{\operatorname{dilog}\left(\frac{-c(bx+a)-c}{-ac+db-c}\right)+\ln(ac-db-c(bx+a))\ln\left(\frac{-c(bx+a)-c}{-ac+db-c}\right)}{2c}-\right.}{}}{}$

input
```
int(arctanh(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
arctanh(b*x+a)*x/c-arctanh(b*x+a)/c^2*d*ln(c*x+d)-b/c*(1/2*(-1-a)/b^2*ln(a
*c-d*b+b*(c*x+d)+c)+1/2*(-1+a)/b^2*ln(a*c-d*b+b*(c*x+d)-c)+d*(-1/2/c*(dilo
g((a*c-d*b+b*(c*x+d)+c)/(a*c-b*d+c))/b+ln(c*x+d)*ln((a*c-d*b+b*(c*x+d)+c)/
(a*c-b*d+c))/b)+1/2/c*(dilog((a*c-d*b+b*(c*x+d)-c)/(a*c-b*d-c))/b+ln(c*x+d
)*ln((a*c-d*b+b*(c*x+d)-c)/(a*c-b*d-c))/b)))
```

3.56.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="fricas")
```

output
```
integral(x*arctanh(b*x + a)/(c*x + d), x)
```

3.56.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{x\operatorname{atanh}(a+bx)}{cx+d}\,dx$$

input
```
integrate(atanh(b*x+a)/(c+d/x),x)
```

output
```
Integral(x*atanh(a + b*x)/(c*x + d), x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx$$
$$= \frac{1}{2}b\left(\frac{\left(\log(cx+d)\log\left(\frac{bcx+bd}{ac-bd+c}+1\right)+\operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right)\right)d}{bc^2} - \frac{\left(\log(cx+d)\log\left(\frac{bcx+bd}{ac-bd-c}+1\right)+\operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right)\right.}{bc^2}\right.$$
$$+\left(\frac{x}{c}-\frac{d\log(cx+d)}{c^2}\right)\operatorname{artanh}(bx+a)$$

input
```
integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="maxima")
```

output
```
1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*
x + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*
c - b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a
+ 1)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c -
 d*log(c*x + d)/c^2)*arctanh(b*x + a)
```

3.56.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(c + d/x), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

input
```
int(atanh(a + b*x)/(c + d/x),x)
```

output
```
int(atanh(a + b*x)/(c + d/x), x)
```

3.57 $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx$

3.57.1 Optimal result

Integrand size = 16, antiderivative size = 545

$$\begin{aligned}\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx &= \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}\\
&+\frac{\sqrt{d}\log(1-a-bx)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&-\frac{\sqrt{d}\log(1+a+bx)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&+\frac{\sqrt{d}\log(1+a+bx)\log\left(-\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&-\frac{\sqrt{d}\log(1-a-bx)\log\left(\frac{b\left(\sqrt{d}+\sqrt{-c}x\right)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&+\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(1-a-bx)}{\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(1-a-bx)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\\
&+\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}\end{aligned}$$

output
```
1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c+1/4*ln(-b*x-
a+1)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(
-c)^(3/2)+1/4*ln(b*x+a+1)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)-b
*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(-b*x-a+1)*ln(b*(x*(-c)^(1/2)+d^(1/2))
/((1-a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(b*x+a+1)*ln(b*(-x
*(-c)^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*
polylog(2,(-b*x-a+1)*(-c)^(1/2)/((-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/
2)/(-c)^(3/2)+1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)-b*d^(1/
2)))*d^(1/2)/(-c)^(3/2)-1/4*polylog(2,(-b*x-a+1)*(-c)^(1/2)/((1-a)*(-c)^(1
/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+
a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx =$$

$$-\frac{2\sqrt{-c}\log(1-a-bx)-2a\sqrt{-c}\log(1-a-bx)-2b\sqrt{-c}x\log(1-a-bx)+2\sqrt{-c}\log(1+a+bx)+}{}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d/x^2),x]
```

output
```
-1/4*(2*Sqrt[-c]*Log[1 - a - b*x] - 2*a*Sqrt[-c]*Log[1 - a - b*x] - 2*b*Sq
rt[-c]*x*Log[1 - a - b*x] + 2*Sqrt[-c]*Log[1 + a + b*x] + 2*a*Sqrt[-c]*Log
[1 + a + b*x] + 2*b*Sqrt[-c]*x*Log[1 + a + b*x] - b*Sqrt[d]*Log[1 - a - b*
x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((-1 + a)*Sqrt[-c] + b*Sqrt[d])] + b*Sqr
t[d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b
*Sqrt[d])] - b*Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/(
(1 + a)*Sqrt[-c] - b*Sqrt[d]))] + b*Sqrt[d]*Log[1 - a - b*x]*Log[(b*(Sqrt[
d] + Sqrt[-c]*x))/(-((-1 + a)*Sqrt[-c]) + b*Sqrt[d])] + b*Sqrt[d]*PolyLog[
2, (Sqrt[-c]*(-1 + a + b*x))/(-Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])] - b*Sqr
t[d]*PolyLog[2, (Sqrt[-c]*(-1 + a + b*x))/(-Sqrt[-c] + a*Sqrt[-c] + b*Sqrt
[d])] - b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/(Sqrt[-c] + a*Sqrt[-
c] - b*Sqrt[d])] + b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/(Sqrt[-c]
 + a*Sqrt[-c] + b*Sqrt[d])])/(b*(-c)^(3/2))
```

3.57.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6665, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{ 6665}$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{c+\frac{d}{x^2}}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{ 2856}$$

$$\frac{1}{2}\int \left(\frac{\log(a+bx+1)}{c}-\frac{d\log(a+bx+1)}{c\left(cx^2+d\right)}\right)dx-\frac{1}{2}\int \left(\frac{\log(-a-bx+1)}{c}-\frac{d\log(-a-bx+1)}{c\left(cx^2+d\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{-\sqrt{-c}a+\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{\sqrt{-c}(1-a)+b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\log(-a-bx+1)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-\cdots}\right.}{(1-a)\sqrt{-c}-\cdots}\right.}{2(-c)^{3/2}}\right.$$

$$\frac{1}{2}\left(\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+1)}{\sqrt{-c}(a+1)+b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log(a+bx+1)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}x\right)}{(a+1)\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}\right.$$

input
```
Int[ArcTanh[a + b*x]/(c + d/x^2),x]
```

output
```
(x/c + ((1 - a - b*x)*Log[1 - a - b*x])/(b*c) + (Sqrt[d]*Log[1 - a - b*x]*
Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)
^(3/2)) - (Sqrt[d]*Log[1 - a - b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a
)*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(
1 - a - b*x))/(Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt
[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(
2*(-c)^(3/2)))/2 + (-(x/c) + ((1 + a + b*x)*Log[1 + a + b*x])/(b*c) - (Sqr
t[d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b
*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] +
 Sqrt[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2)) + (Sqrt[d]*
PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(2*(-
c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c]
 + b*Sqrt[d])])/(2*(-c)^(3/2)))/2
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.57.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.07

method	result
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2bc}-\frac{1}{bc}-\frac{d\ln(bx+a+1)\ln\left(\frac{b\sqrt{-cd}-(bx+a+1)c+ac+c}{b\sqrt{-cd}+ac+c}\right)}{4c\sqrt{-cd}}+\frac{d\ln(bx+a+$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctanh(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*ln(b*x+a+1)*x+1/2/b/c*ln(b*x+a+1)*a+1/2/b/c*ln(b*x+a+1)-1/b/c-1/4*d/
c*ln(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(b*x+a+1)*c+a*c+c)/(b*(-c*d)
^(1/2)+a*c+c))+1/4*d/c*ln(b*x+a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(b*x+a+
1)*c-a*c-c)/(b*(-c*d)^(1/2)-a*c-c))-1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(
1/2)-(b*x+a+1)*c+a*c+c)/(b*(-c*d)^(1/2)+a*c+c))+1/4*d/c/(-c*d)^(1/2)*dilog
((b*(-c*d)^(1/2)+(b*x+a+1)*c-a*c-c)/(b*(-c*d)^(1/2)-a*c-c))-1/2/c*ln(-b*x-
a+1)*x-1/2/b/c*ln(-b*x-a+1)*a+1/2/b/c*ln(-b*x-a+1)-1/4*d/c*ln(-b*x-a+1)/(-
c*d)^(1/2)*ln((b*(-c*d)^(1/2)-(-b*x-a+1)*c-a*c+c)/(b*(-c*d)^(1/2)-a*c+c))+
1/4*d/c*ln(-b*x-a+1)/(-c*d)^(1/2)*ln((b*(-c*d)^(1/2)+(-b*x-a+1)*c+a*c-c)/(
b*(-c*d)^(1/2)+a*c-c))-1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(1/2)-(-b*x-a+
1)*c-a*c+c)/(b*(-c*d)^(1/2)-a*c+c))+1/4*d/c/(-c*d)^(1/2)*dilog((b*(-c*d)^(
1/2)+(-b*x-a+1)*c+a*c-c)/(b*(-c*d)^(1/2)+a*c-c))
```

3.57.5 Fricas [F]

$$\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\text{artanh}\,(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*arctanh(b*x + a)/(c*x^2 + d), x)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx=\text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(c+d/x**2),x)
```

output
```
Timed out
```

3.57.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right)\operatorname{artanh}(bx+a) + \frac{2(a+1)c\log(bx+a+1) - 2(a-1)c\log(bx+a-1) + \left(b\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2(a+1)bcx+(a^2+2a+1}{b^2d+(a^2+2a+1)c}\right.\right.}{}$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctanh(b*x + a) + 1/4*(2*(
a + 1)*c*log(b*x + a + 1) - 2*(a - 1)*c*log(b*x + a - 1) + (b*arctan(sqrt(
c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d
 + (a^2 + 2*a + 1)*c)) - b*arctan(sqrt(c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a
 - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*dilog(
((a - 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*
a + I)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*a + 1)*c)) - I*b*dilog(-((a -
1)*b*c*x + b^2*d - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)
*b*sqrt(c)*sqrt(d) - b^2*d + (a^2 - 2*a + 1)*c)) - I*b*dilog(((a + 1)*b*c*
x + b^2*d + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt
(c)*sqrt(d) + b^2*d - (a^2 + 2*a + 1)*c)) + I*b*dilog(-((a + 1)*b*c*x + b^
2*d - (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sq
rt(d) - b^2*d + (a^2 + 2*a + 1)*c)) - (b*arctan2((b^2*x + (a + 1)*b)*sqrt(
c)*sqrt(d)/(b^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c
)/(b^2*d + (a^2 + 2*a + 1)*c)) - b*arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqr
t(d)/(b^2*d + (a^2 - 2*a + 1)*c), ((a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2
*d + (a^2 - 2*a + 1)*c)))*log(c*x^2 + d))*sqrt(c)*sqrt(d))/(b*c^2)
```

3.57.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(c + d/x^2), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\mathrm{atanh}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(atanh(a + b*x)/(c + d/x^2),x)
```

output
```
int(atanh(a + b*x)/(c + d/x^2), x)
```

3.58 $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}} \, dx$

3.58.1 Optimal result

Integrand size = 16, antiderivative size = 832

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx = \frac{(1-a-bx)\log(1-a-bx)}{2bc}+\frac{(1+a+bx)\log(1+a+bx)}{2bc}$$
$$-\frac{\sqrt[3]{d}\log(1+a+bx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(1+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{\sqrt[3]{d}\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(1-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{(-1)^{2/3}\sqrt[3]{d}\log(1-a-bx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(1-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$-\frac{(-1)^{2/3}\sqrt[3]{d}\log(1+a+bx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(1+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\sqrt[3]{d}\log(1+a+bx)\log\left(-\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(1+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sqrt[3]{d}\log(1-a-bx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(1-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{(-1)^{2/3}\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(1-a-bx)}{\sqrt[3]{-1}(1-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{c}(1-a-bx)}{(1-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$-\frac{\sqrt[3]{-1}\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{c}(1-a-bx)}{(-1)^{2/3}(1-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$-\frac{\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{c}(1+a+bx)}{(1+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$
$$+\frac{\sqrt[3]{-1}\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{c}(1+a+bx)}{(-1)^{2/3}(1+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

3.58. $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx$

$$-\frac{(-1)^{2/3}\sqrt[3]{d}\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(1+a+bx)}{\sqrt[3]{-1}(1+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}}$$

output
```
1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-1/6*d^(1/3)*
ln(b*x+a+1)*ln(-b*(d^(1/3)+c^(1/3)*x)/((1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1
/6*d^(1/3)*ln(-b*x-a+1)*ln(b*(d^(1/3)+c^(1/3)*x)/((1-a)*c^(1/3)+b*d^(1/3))
)/c^(4/3)+1/6*(-1)^(2/3)*d^(1/3)*ln(-b*x-a+1)*ln(-b*(d^(1/3)-(-1)^(1/3)*c^
(1/3)*x)/((-1)^(1/3)*(1-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(2/3)*d^(1
/3)*ln(b*x+a+1)*ln(b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(1+a)*c^(1
/3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/3)*d^(1/3)*ln(b*x+a+1)*ln(-b*(d^(1/3)+
(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-
1)^(1/3)*d^(1/3)*ln(-b*x-a+1)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2
/3)*(1-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(2/3)*d^(1/3)*polylog(2,(-1
)^(1/3)*c^(1/3)*(-b*x-a+1)/((-1)^(1/3)*(1-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1
/6*d^(1/3)*polylog(2,c^(1/3)*(-b*x-a+1)/((1-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)
-1/6*(-1)^(1/3)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(-b*x-a+1)/((-1)^(2/3
)*(1-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*d^(1/3)*polylog(2,c^(1/3)*(b*x+a+1
)/((1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/3)*d^(1/3)*polylog(2,(-1)
^(2/3)*c^(1/3)*(b*x+a+1)/((-1)^(2/3)*(1+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6
*(-1)^(2/3)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(b*x+a+1)/((-1)^(1/3)*(1+
a)*c^(1/3)+b*d^(1/3)))/c^(4/3)
```

3.58.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 791, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx$$

$$= \frac{3\sqrt[3]{c}\log(1-a-bx)-3a\sqrt[3]{c}\log(1-a-bx)-3b\sqrt[3]{c}x\log(1-a-bx)+3\sqrt[3]{c}\log(1+a+bx)+3a\sqrt[3]{c}\log}{}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d/x^3),x]
```

output
```
(3*c^(1/3)*Log[1 - a - b*x] - 3*a*c^(1/3)*Log[1 - a - b*x] - 3*b*c^(1/3)*x
*Log[1 - a - b*x] + 3*c^(1/3)*Log[1 + a + b*x] + 3*a*c^(1/3)*Log[1 + a + b
*x] + 3*b*c^(1/3)*x*Log[1 + a + b*x] + b*d^(1/3)*Log[1 - a - b*x]*Log[(b*(
d^(1/3) + c^(1/3)*x))/(-((-1 + a)*c^(1/3)) + b*d^(1/3))] - b*d^(1/3)*Log[1
 + a + b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/(-((1 + a)*c^(1/3)) + b*d^(1/3))
] + (-1)^(2/3)*b*d^(1/3)*Log[1 - a - b*x]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(
1/3)*x))/((-1)^(1/3)*(-1 + a)*c^(1/3) + b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)
*Log[1 + a + b*x]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(1
+ a)*c^(1/3) + b*d^(1/3))] - (-1)^(1/3)*b*d^(1/3)*Log[1 - a - b*x]*Log[(b*
(d^(1/3) + (-1)^(2/3)*c^(1/3)*x))/(-((-1)^(2/3)*(-1 + a)*c^(1/3)) + b*d^(1
/3))] + (-1)^(1/3)*b*d^(1/3)*Log[1 + a + b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)
*c^(1/3)*x))/(-((-1)^(2/3)*(1 + a)*c^(1/3)) + b*d^(1/3))] + b*d^(1/3)*Poly
Log[2, (c^(1/3)*(-1 + a + b*x))/((-1 + a)*c^(1/3) - b*d^(1/3))] - (-1)^(1/
3)*b*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(-1 + a + b*x))/((-1)^(2/3)*(-
1 + a)*c^(1/3) - b*d^(1/3))] + (-1)^(2/3)*b*d^(1/3)*PolyLog[2, ((-1)^(1/3)
*c^(1/3)*(-1 + a + b*x))/((-1)^(1/3)*(-1 + a)*c^(1/3) + b*d^(1/3))] - b*d^
(1/3)*PolyLog[2, (c^(1/3)*(1 + a + b*x))/((1 + a)*c^(1/3) - b*d^(1/3))] +
(-1)^(1/3)*b*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(1 + a + b*x))/((-1)^(
2/3)*(1 + a)*c^(1/3) - b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)*PolyLog[2, ((-1)
^(1/3)*c^(1/3)*(1 + a + b*x))/((-1)^(1/3)*(1 + a)*c^(1/3) + b*d^(1/3))]...
```

3.58.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6665, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{x^3}}dx$$

$$\Big\downarrow \text{6665}$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{c+\frac{d}{x^3}}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{c+\frac{d}{x^3}}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}\int \left(\frac{\log(a+bx+1)}{c}-\frac{d\log(a+bx+1)}{c\,(cx^3+d)}\right)dx- \frac{1}{2}\int \left(\frac{\log(-a-bx+1)}{c}-\frac{d\log(-a-bx+1)}{c\,(cx^3+d)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x}{c}+\frac{(-a-bx+1)\log(-a-bx+1)}{bc}+\frac{\sqrt[3]{d}\log(-a-bx+1)\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(1-a)+b\sqrt[3]{d}}\right)}{3c^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{d}\log(-a-bx\right.$$

$$\frac{1}{2}\left(-\frac{x}{c}+\frac{(a+bx+1)\log(a+bx+1)}{bc}-\frac{\sqrt[3]{d}\log(a+bx+1)\log\left(-\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{(a+1)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{3c^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{d}\log(a+bx+\right.$$

input
```
Int[ArcTanh[a + b*x]/(c + d/x^3),x]
```

output
```
(x/c + ((1 - a - b*x)*Log[1 - a - b*x])/(b*c) + (d^(1/3)*Log[1 - a - b*x]*
Log[(b*(d^(1/3) + c^(1/3)*x))/((1 - a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3))
+ ((-1)^(2/3)*d^(1/3)*Log[1 - a - b*x]*Log[-((b*(d^(1/3) - (-1)^(1/3)*c^(1
/3)*x))/((-1)^(1/3)*(1 - a)*c^(1/3) - b*d^(1/3)))])/(3*c^(4/3)) - ((-1)^(1
/3)*d^(1/3)*Log[1 - a - b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x))/((-1
)^(2/3)*(1 - a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3)) + ((-1)^(2/3)*d^(1/3)*P
olyLog[2, ((-1)^(1/3)*c^(1/3)*(1 - a - b*x))/((-1)^(1/3)*(1 - a)*c^(1/3) -
 b*d^(1/3))])/(3*c^(4/3)) + (d^(1/3)*PolyLog[2, (c^(1/3)*(1 - a - b*x))/((
1 - a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3)) - ((-1)^(1/3)*d^(1/3)*PolyLog[2,
 ((-1)^(2/3)*c^(1/3)*(1 - a - b*x))/((-1)^(2/3)*(1 - a)*c^(1/3) + b*d^(1/3
))])/(3*c^(4/3)))/2 + (-(x/c) + ((1 + a + b*x)*Log[1 + a + b*x])/(b*c) - (
d^(1/3)*Log[1 + a + b*x]*Log[-((b*(d^(1/3) + c^(1/3)*x))/((1 + a)*c^(1/3)
- b*d^(1/3)))])/(3*c^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[1 + a + b*x]*Log[(b*
(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(1 + a)*c^(1/3) + b*d^(1/3))
])/(3*c^(4/3)) + ((-1)^(1/3)*d^(1/3)*Log[1 + a + b*x]*Log[-((b*(d^(1/3) +
(-1)^(2/3)*c^(1/3)*x))/((-1)^(2/3)*(1 + a)*c^(1/3) - b*d^(1/3)))])/(3*c^(4
/3)) - (d^(1/3)*PolyLog[2, (c^(1/3)*(1 + a + b*x))/((1 + a)*c^(1/3) - b*d^
(1/3))])/(3*c^(4/3)) + ((-1)^(1/3)*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*
(1 + a + b*x))/((-1)^(2/3)*(1 + a)*c^(1/3) - b*d^(1/3))])/(3*c^(4/3)) - ((
-1)^(2/3)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(1 + a + b*x))/((-1)^(...
```

3.58. $\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.58.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2bc}-\frac{1}{bc}-\frac{b^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3+(-3ac-3c)\_Z^2+\left(3a^2c+6ac+3c\right)\_\right.}\right.}{}$
derivativedivides	$\frac{\frac{\operatorname{arctanh}(bx+a)(bx+a)}{c}+\frac{\operatorname{arctanh}(bx+a)\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln(bx-\_R+a)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}+\frac{\ln(bx+}{2}}{}$
default	$\frac{\frac{\operatorname{arctanh}(bx+a)(bx+a)}{c}+\frac{\operatorname{arctanh}(bx+a)\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln(bx-\_R+a)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}+\frac{\ln(bx+}{2}}{}$

input
```
int(arctanh(b*x+a)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*ln(b*x+a+1)*x+1/2/b/c*ln(b*x+a+1)*a+1/2/b/c*ln(b*x+a+1)-1/b/c-1/6*b^
2*d/c^2*sum(1/(_R1^2-2*_R1*a+a^2-2*_R1+2*a+1)*(ln(b*x+a+1)*ln((-b*x+_R1-a-
1)/_R1)+dilog((-b*x+_R1-a-1)/_R1)),_R1=RootOf(c*_Z^3+(-3*a*c-3*c)*_Z^2+(3*
a^2*c+6*a*c+3*c)*_Z-a^3*c+b^3*d-3*a^2*c-3*a*c-c))-1/2/c*ln(-b*x-a+1)*x-1/2
/b/c*ln(-b*x-a+1)*a+1/2/b/c*ln(-b*x-a+1)+1/6*b^2*d/c^2*sum(1/(_R1^2+2*_R1*
a+a^2-2*_R1-2*a+1)*(ln(-b*x-a+1)*ln((b*x+_R1+a-1)/_R1)+dilog((b*x+_R1+a-1)
/_R1)),_R1=RootOf(c*_Z^3+(3*a*c-3*c)*_Z^2+(3*a^2*c-6*a*c+3*c)*_Z+a^3*c-b^3
*d-3*a^2*c+3*a*c-c))
```

3.58. $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx$

3.58.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

output
```
integral(x^3*arctanh(b*x + a)/(c*x^3 + d), x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(c+d/x**3),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="maxima")
```

output
```
integrate(arctanh(b*x + a)/(c + d/x^3), x)
```

3.58.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx=\int \frac{\operatorname{artanh}(bx+a)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{x^3}}\,dx=\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

input
```
int(atanh(a + b*x)/(c + d/x^3),x)
```

output
```
int(atanh(a + b*x)/(c + d/x^3), x)
```

3.59 $\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 585

$$\begin{aligned}\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx &= \frac{2\sqrt{1+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d}\\
&+ \frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&- \frac{c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+ \frac{c\log\left(-\frac{d\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&- \frac{c\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&- \frac{\sqrt{x}\log(1-a-bx)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log(1-a-bx)}{d^2}\\
&+ \frac{\sqrt{x}\log(1+a+bx)}{d} - \frac{c\log\left(c+d\sqrt{x}\right)\log(1+a+bx)}{d^2}\\
&+ \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right)}{d^2} + \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right)}{d^2}\\
&- \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2} - \frac{c\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2}\end{aligned}$$

output
```
c*ln(-b*x-a+1)*ln(c+d*x^(1/2))/d^2-c*ln(b*x+a+1)*ln(c+d*x^(1/2))/d^2+c*ln(
c+d*x^(1/2))*ln(d*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-1-a)^(1/2)+c*b^(1/2)
))/d^2-c*ln(c+d*x^(1/2))*ln(d*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(1-a)^(1/2)
+c*b^(1/2)))/d^2+c*ln(c+d*x^(1/2))*ln(-d*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(-
d*(-1-a)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(-d*((1-a)^(1/2)+b^(1/2
)*x^(1/2))/(-d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)*(c+d*x^(1/2
))/(-d*(-1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(
-1-a)^(1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d*(1-a)^(1
/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(1-a)^(1/2)+c*b^(
1/2)))/d^2-2*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/d/b^(1/2)+2*
arctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/d/b^(1/2)-ln(-b*x-a+1)*x^(
1/2)/d+ln(b*x+a+1)*x^(1/2)/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx$$

$$= \frac{\frac{2\sqrt{1+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{1-a}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}} + c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{1-a}d}\right.}{}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
((2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b] - (2*Sqrt
[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b] + c*Log[(d*(Sqrt
[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d
)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt
[b]*c) + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[1 - a] + Sqr
t[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[1 - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt
[x]*Log[1 - a - b*x] + c*Log[c + d*Sqrt[x]]*Log[1 - a - b*x] + d*Sqrt[x]*L
og[1 + a + b*x] - c*Log[c + d*Sqrt[x]]*Log[1 + a + b*x] + c*PolyLog[2, (Sq
rt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)] + c*PolyLog[2, (Sqrt[
b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*
(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c +
 d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```

3.59.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6665, 2855, 2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(a+bx)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{6665}$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{c+d\sqrt{x}}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{2855}$$

$$\int \frac{\sqrt{x}\log(a+bx+1)}{c+d\sqrt{x}}d\sqrt{x}-\int \frac{\sqrt{x}\log(-a-bx+1)}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log(a+bx+1)}{d}-\frac{c\log(a+bx+1)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}-\int \left(\frac{\log(-a-bx+1)}{d}-\frac{c\log(-a-bx+1)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{a+1}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}d}-\frac{2\sqrt{1-a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-1}d}\right)}{d^2}-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}-\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-a}d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}d+\sqrt{b}c}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}+\frac{c\log(-a-bx+1)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log(a+bx+1)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{\sqrt{x}\log(-a-bx+1)}{d}+\frac{\sqrt{x}\log(a+bx+1)}{d}$$

input
```
Int[ArcTanh[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
(2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqr
t[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(
Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*S
qrt[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqr
t[1 - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]
*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log
[-((d*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c
 + d*Sqrt[x]])/d^2 - (Sqrt[x]*Log[1 - a - b*x])/d + (c*Log[c + d*Sqrt[x]]*
Log[1 - a - b*x])/d^2 + (Sqrt[x]*Log[1 + a + b*x])/d - (c*Log[c + d*Sqrt[x
]]*Log[1 + a + b*x])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b
]*c - Sqrt[-1 - a]*d)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqr
t[b]*c + Sqrt[-1 - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(
Sqrt[b]*c - Sqrt[1 - a]*d)])/d^2 - (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))
/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2855
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Simp[k   Subst
[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && I
GtQ[p, 0]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.59.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{2\,\mathrm{arctanh}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arctanh}(bx+a)c\ln(c+d\sqrt{x})}{d^2}-\frac{4b\left(-d^2\left(\frac{(1+a)\arctan\left(\frac{-2bc+2b(c+d\sqrt{x})}{2\sqrt{ab\,d^2+b\,d^2}}\right)}{2b\sqrt{ab\,d^2+b\,d^2}}+\frac{(1-a)\arctan\left(\frac{-\cdots}{\cdots}\right)}{2b\sqrt{ab\cdots}}\cdots\right.\right.}{\cdots}$
default	$\frac{2\,\mathrm{arctanh}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arctanh}(bx+a)c\ln(c+d\sqrt{x})}{d^2}-\frac{4b\left(-d^2\left(\frac{(1+a)\arctan\left(\frac{-2bc+2b(c+d\sqrt{x})}{2\sqrt{ab\,d^2+b\,d^2}}\right)}{2b\sqrt{ab\,d^2+b\,d^2}}+\frac{(1-a)\arctan\left(\frac{-\cdots}{\cdots}\right)}{2b\sqrt{ab\cdots}}\cdots\right.\right.}{\cdots}$

input
```
int(arctanh(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arctanh(b*x+a)/d*x^(1/2)-2*arctanh(b*x+a)*c/d^2*ln(c+d*x^(1/2))-4*b/d^2*
(-d^2*(1/2*(1+a)/b/(a*b*d^2+b*d^2)^(1/2)*arctan(1/2*(-2*b*c+2*b*(c+d*x^(1/
2)))/(a*b*d^2+b*d^2)^(1/2))+1/2*(1-a)/b/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(
-2*b*c+2*b*(c+d*x^(1/2)))/(a*b*d^2-b*d^2)^(1/2)))-c*d^2*(1/2/d^2*(1/2*ln(c
+d*x^(1/2))*(ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(b*c+(-a*b*d^
2-b*d^2)^(1/2)))+ln((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(-b*c+(-
a*b*d^2-b*d^2)^(1/2))))/b+1/2*(dilog((b*c-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)
^(1/2))/(b*c+(-a*b*d^2-b*d^2)^(1/2)))+dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^
2-b*d^2)^(1/2))/(-b*c+(-a*b*d^2-b*d^2)^(1/2))))/b)+1/2/d^2*(-1/2*ln(c+d*x^
(1/2))*(ln((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(b*c+(-a*b*d^2+b*d
^2)^(1/2)))+ln((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(-b*c+(-a*b*d
^2+b*d^2)^(1/2))))/b-1/2*(dilog((b*c-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2
))/(b*c+(-a*b*d^2+b*d^2)^(1/2)))+dilog((-b*c+b*(c+d*x^(1/2))+(-a*b*d^2+b*d
^2)^(1/2))/(-b*c+(-a*b*d^2+b*d^2)^(1/2))))/b)))
```

3.59.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}\left(bx+a\right)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((d*sqrt(x)*arctanh(b*x + a) - c*arctanh(b*x + a))/(d^2*x - c^2),
x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(c+d*x**(1/2)),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arctanh(b*x + a)/(d*sqrt(x) + c), x)
```

3.59.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{artanh}(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(d*sqrt(x) + c), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{atanh}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

input
```
int(atanh(a + b*x)/(c + d*x^(1/2)),x)
```

output
```
int(atanh(a + b*x)/(c + d*x^(1/2)), x)
```

3.60 $\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}} \, dx$

3.60.1 Optimal result

Integrand size = 18, antiderivative size = 661

$$\begin{aligned}\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = & -\frac{2\sqrt{1+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} \\ & -\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\ & +\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\ & -\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\ & +\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3} \\ & +\frac{d\sqrt{x}\log(1-a-bx)}{c^2}+\frac{(1-a-bx)\log(1-a-bx)}{2bc} \\ & -\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1-a-bx)}{c^3}-\frac{d\sqrt{x}\log(1+a+bx)}{c^2} \\ & +\frac{(1+a+bx)\log(1+a+bx)}{2bc}+\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1+a+bx)}{c^3} \\ & -\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3} \\ & -\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}\end{aligned}$$

output

```
1/2*(-b*x-a+1)*ln(-b*x-a+1)/b/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c-d^2*ln(-b*x-
a+1)*ln(d+c*x^(1/2))/c^3+d^2*ln(b*x+a+1)*ln(d+c*x^(1/2))/c^3-d^2*ln(d+c*x^
(1/2))*ln(c*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-1-a)^(1/2)+d*b^(1/2)))/c^3
+d^2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(1-a)^(1/2)+d*b
^(1/2)))/c^3-d^2*ln(d+c*x^(1/2))*ln(c*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-
1-a)^(1/2)-d*b^(1/2)))/c^3+d^2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)+b^(1/2)*x
^(1/2))/(c*(1-a)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2)
)/(c*(-1-a)^(1/2)-d*b^(1/2)))/c^3+d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*
(1-a)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-1-a)^
(1/2)+d*b^(1/2)))/c^3+d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1/2)+d
*b^(1/2)))/c^3+2*d*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/c^2/b^
(1/2)-2*d*arctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/c^2/b^(1/2)+d*ln
(-b*x-a+1)*x^(1/2)/c^2-d*ln(b*x+a+1)*x^(1/2)/c^2
```

3.60. $\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

3.60.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\frac{2\sqrt{1+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}$$
$$-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{d^2\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{d\sqrt{x}\log(1-a-bx)}{c^2}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1-a-bx)}{c^3}$$
$$+\frac{x+\frac{(1-a-bx)\log(1-a-bx)}{b}}{2c}-\frac{d\sqrt{x}\log(1+a+bx)}{c^2}$$
$$+\frac{d^2\log\left(d+c\sqrt{x}\right)\log(1+a+bx)}{c^3}-\frac{x-\frac{(1+a+bx)\log(1+a+bx)}{b}}{2c}$$
$$-\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}$$
$$-\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}$$

input
```
Integrate[ArcTanh[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
(-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (
2*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d
^2*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*
Log[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sq
rt[1 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 -
 a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/
c^3 + (d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b
]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d*Sqrt[x]*Log[1 - a - b*x])/c^2 - (d^2*Lo
g[d + c*Sqrt[x]]*Log[1 - a - b*x])/c^3 + (x + ((1 - a - b*x)*Log[1 - a - b
*x])/b)/(2*c) - (d*Sqrt[x]*Log[1 + a + b*x])/c^2 + (d^2*Log[d + c*Sqrt[x]]
*Log[1 + a + b*x])/c^3 - (x - ((1 + a + b*x)*Log[1 + a + b*x])/b)/(2*c) -
(d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d))]
)/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[
b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c +
 Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]
*c + Sqrt[b]*d)])/c^3
```

3.60.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6665, 2855, 2005, 2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{6665}$$

$$\frac{1}{2}\int \frac{\log(a+bx+1)}{c+\frac{d}{\sqrt{x}}}dx-\frac{1}{2}\int \frac{\log(-a-bx+1)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{2855}$$

$$\int \frac{\sqrt{x}\log(a+bx+1)}{c+\frac{d}{\sqrt{x}}}d\sqrt{x}-\int \frac{\sqrt{x}\log(-a-bx+1)}{c+\frac{d}{\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{x\log(a+bx+1)}{\sqrt{x}c+d}d\sqrt{x}-\int \frac{x\log(-a-bx+1)}{\sqrt{x}c+d}d\sqrt{x}$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log(a+bx+1)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log(a+bx+1)d}{c^2}+\frac{\sqrt{x}\log(a+bx+1)}{c}\right)d\sqrt{x}-\int \left(\frac{\log(-a-bx+1)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log(-a-bx+1)d}{c^2}+\frac{\sqrt{x}\log(-a-bx+1)}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{a+1}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}-\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\log(-a-bx+1)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d^2\log(a+bx+1)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d\sqrt{x}\log(-a-bx+1)}{c^2}-\frac{d\sqrt{x}\log(a+bx+1)}{c^2}+\frac{(-a-bx+1)\log(-a-bx+1)}{2bc}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}$$

input
```
Int[ArcTanh[a + b*x]/(c + d/Sqrt[x]),x]
```

3.60. $\int \frac{\text{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

output
```
(-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (
2*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d
^2*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*
Log[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sq
rt[1 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 -
 a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/
c^3 + (d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b
]*d)]*Log[d + c*Sqrt[x]])/c^3 + (d*Sqrt[x]*Log[1 - a - b*x])/c^2 + ((1 - a
 - b*x)*Log[1 - a - b*x])/(2*b*c) - (d^2*Log[d + c*Sqrt[x]]*Log[1 - a - b*
x])/c^3 - (d*Sqrt[x]*Log[1 + a + b*x])/c^2 + ((1 + a + b*x)*Log[1 + a + b*
x])/(2*b*c) + (d^2*Log[d + c*Sqrt[x]]*Log[1 + a + b*x])/c^3 - (d^2*PolyLog
[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2
*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3
 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]
)/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]
*d)])/c^3
```

3.60.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2855
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Simp[k   Subst
[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && I
GtQ[p, 0]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

rule 6665
```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Simp
[1/2   Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Simp[1/2   Int[Log[1 - c
- d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

3.60.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\operatorname{arctanh}(bx+a)x}{c} - \frac{2\operatorname{arctanh}(bx+a)d\sqrt{x}}{c^2} + \frac{2\operatorname{arctanh}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} - \frac{4b\left(cd^2\left(-\frac{\ln(d+c\sqrt{x})\left(\ln\left(\frac{db-b(d+\dots}{db+\dots}\right.\right.}{\dots}\right.\right.}{\dots}$
default	$\frac{\operatorname{arctanh}(bx+a)x}{c} - \frac{2\operatorname{arctanh}(bx+a)d\sqrt{x}}{c^2} + \frac{2\operatorname{arctanh}(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} - \frac{4b\left(cd^2\left(-\frac{\ln(d+c\sqrt{x})\left(\ln\left(\frac{db-b(d+\dots}{db+\dots}\right.\right.}{\dots}\right.\right.}{\dots}$

input
```
int(arctanh(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctanh(b*x+a)*x/c-2*arctanh(b*x+a)/c^2*d*x^(1/2)+2*arctanh(b*x+a)*d^2/c^3
*ln(d+c*x^(1/2))-4*b/c^2*(c*d^2*(1/2/c^2*(-1/2*ln(d+c*x^(1/2))*(ln((d*b-b*
(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(d*b+(-a*b*c^2+b*c^2)^(1/2)))+ln((-d
*b+b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(-d*b+(-a*b*c^2+b*c^2)^(1/2))))
/b-1/2*(dilog((d*b-b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(d*b+(-a*b*c^2+
b*c^2)^(1/2)))+dilog((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(-d*b+(
-a*b*c^2+b*c^2)^(1/2))))/b)+1/2/c^2*(1/2*ln(d+c*x^(1/2))*(ln((d*b-b*(d+c*x
^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(d*b+(-a*b*c^2-b*c^2)^(1/2)))+ln((-d*b+b*(
d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-d*b+(-a*b*c^2-b*c^2)^(1/2))))/b+1/2
*(dilog((d*b-b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(d*b+(-a*b*c^2-b*c^2)
^(1/2)))+dilog((-d*b+b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-d*b+(-a*b*c
^2-b*c^2)^(1/2))))/b))+1/2*c*(-1/2*(-1+a)/b*(-1/2/b*ln(a*c^2+b*d^2-2*b*d*(
d+c*x^(1/2))+b*(d+c*x^(1/2))^2-c^2)+2*d/(a*b*c^2-b*c^2)^(1/2)*arctan(1/2*(
-2*d*b+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2)))+1/2*(1+a)/b*(-1/2/b*ln(a
*c^2+b*d^2-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2+c^2)+2*d/(a*b*c^2+b*c^2)^
(1/2)*arctan(1/2*(-2*d*b+2*b*(d+c*x^(1/2)))/(a*b*c^2+b*c^2)^(1/2))))
```

3.60.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

output
```
integral((c*x*arctanh(b*x + a) - d*sqrt(x)*arctanh(b*x + a))/(c^2*x - d^2)
, x)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\text{Timed out}$$

input
```
integrate(atanh(b*x+a)/(c+d/x**(1/2)),x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)
```

3.60.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\operatorname{artanh}\left(bx+a\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arctanh(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arctanh(b*x + a)/(c + d/sqrt(x)), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx=\int \frac{\operatorname{atanh}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
int(atanh(a + b*x)/(c + d/x^(1/2)),x)
```

output
```
int(atanh(a + b*x)/(c + d/x^(1/2)), x)
```

3.61 $\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2} dx$

3.61.1 Optimal result

Integrand size = 19, antiderivative size = 335

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx=\frac{\operatorname{arctanh}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}}-\frac{\operatorname{arctanh}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}}-\frac{\operatorname{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}e\right)(1+d+ex)}\right)}{2\sqrt{b^2-4ac}}+\frac{\operatorname{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{2\sqrt{b^2-4ac}}$$

output
```
arctanh(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*
(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctanh(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a
*c+b^2)^(1/2)-1/2*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(1/2)
))/(e*x+d+1)/(2*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*
polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d+1)/(2*c*
(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{-\log\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2c(-1+d)+\left(-b+\sqrt{b^2-4ac}\right)e}\right)\log(1-d-ex)+\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c(-1+d)+\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(1-d-ex)+\log\left(\cdots\right.}{}$$

input
```
Integrate[ArcTanh[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(-(Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-1 + d) + (-b + Sqrt[b^2
 - 4*a*c])*e)]*Log[1 - d - e*x]) + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))
/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - d - e*x] + Log[(e*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)
]*Log[1 + d + e*x] - Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d)
 + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + d + e*x] - PolyLog[2, (2*c*(-1 + d
+ e*x))/(2*c*(-1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (2*c*(-1
 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (2*c
*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - PolyLog[2, (
2*c*(1 + d + e*x))/(2*c*(1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[b^2
 - 4*a*c])
```

3.61.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \ 7279$$
$$\int \left(\frac{2c\operatorname{arctanh}(d+ex)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\operatorname{arctanh}(d+ex)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}(d+ex)\log\left(-\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(-e\sqrt{b^2-4ac}+be-2cd+2c\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\operatorname{arctanh}(d+ex)\log\left(-\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\operatorname{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2c+be-\sqrt{b^2-4ac}e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\operatorname{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}$$

input
```
Int[ArcTanh[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(ArcTanh[d + e*x]*Log[(-2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 + d + e*x))]])/Sqrt[b^2
 - 4*a*c] - (ArcTanh[d + e*x]*Log[(-2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e -
2*c*(d + e*x)))/((2*c*(1 - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]
)/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
 - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 + d + e*x
))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 + d
 + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.61. $\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx$

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(307) = 614$.

Time = 0.69 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.29

method	result
risch	$\frac{e\ln(-ex-d+1)\ln\left(\frac{-2(-ex-d+1)c+be-2cd+\sqrt{-4ac\,e^2+b^2e^2}+2c}{be-2cd+2c+\sqrt{-4ac\,e^2+b^2e^2}}\right)}{2\sqrt{-4ac\,e^2+b^2e^2}} - \frac{e\ln(-ex-d+1)\ln\left(\frac{2(-ex-d+1)c-be+2cd+\sqrt{-4ac\,e}}{-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}}\right.}{2\sqrt{-4ac\,e^2+b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctanh(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*ln(-e*x-d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((-2*(-e*x-d+1)*c+b*e-2*c*
d+(-4*a*c*e^2+b^2*e^2)^(1/2)+2*c)/(b*e-2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2
)))-1/2*e*ln(-e*x-d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((2*(-e*x-d+1)*c-b*e+2
*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2
)-2*c))+1/2*e/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((-2*(-e*x-d+1)*c+b*e-2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2)+2*c)/(b*e-2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2))
)-1/2*e/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((2*(-e*x-d+1)*c-b*e+2*c*d+(-4*a*c
*e^2+b^2*e^2)^(1/2)-2*c)/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))+1/2*
e*ln(e*x+d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((-2*(e*x+d+1)*c-b*e+2*c*d+(-4*
a*c*e^2+b^2*e^2)^(1/2)+2*c)/(-b*e+2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1
/2*e*ln(e*x+d+1)/(-4*a*c*e^2+b^2*e^2)^(1/2)*ln((2*(e*x+d+1)*c+b*e-2*c*d+(-
4*a*c*e^2+b^2*e^2)^(1/2)-2*c)/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))+
1/2*e/(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((-2*(e*x+d+1)*c-b*e+2*c*d+(-4*a*c*e
^2+b^2*e^2)^(1/2)+2*c)/(-b*e+2*c*d+2*c+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/2*e/
(-4*a*c*e^2+b^2*e^2)^(1/2)*dilog((2*(e*x+d+1)*c+b*e-2*c*d+(-4*a*c*e^2+b^2*
e^2)^(1/2)-2*c)/(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)-2*c))
```

3.61.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{artanh}(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(arctanh(e*x + d)/(c*x^2 + b*x + a), x)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(atanh(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.61.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\operatorname{artanh}(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctanh(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(arctanh(e*x + d)/(c*x^2 + b*x + a), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(d+ex)}{a+bx+cx^2}\,dx=\int \frac{\operatorname{atanh}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(atanh(d + e*x)/(a + b*x + c*x^2),x)
```

output
```
int(atanh(d + e*x)/(a + b*x + c*x^2), x)
```

3.62 $\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx$

3.62.1 Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = -\frac{e(a+b\text{arctanh}(c+dx))^2}{2bd} + \frac{e(a+b\text{arctanh}(c+dx))\log\left(\frac{2}{1-c-dx}\right)}{d} + \frac{be\,\text{PolyLog}\left(2,-\frac{1+c+dx}{1-c-dx}\right)}{2d}$$

output
```
-1/2*e*(a+b*arctanh(d*x+c))^2/b/d+e*(a+b*arctanh(d*x+c))*ln(2/(-d*x-c+1))/
d+1/2*b*e*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.81

$$\int \frac{(ce+dex)(a+b\operatorname{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = e\left(-\frac{a\log(1-c-dx)}{2d}+\frac{b\log^2(1-c-dx)}{8d} - \frac{b\log(2)\log(-1+c+dx)}{4d}-\frac{a\log(1+c+dx)}{2d} + \frac{b\log(2)\log(1+c+dx)}{4d}-\frac{b\log^2(1+c+dx)}{8d} + \frac{b\operatorname{PolyLog}\left(2,\frac{1}{2}(1-c-dx)\right)}{4d} - \frac{b\operatorname{PolyLog}\left(2,\frac{1}{2}(1+c+dx)\right)}{4d}\right)$$

input
```
Integrate[((c*e + d*e*x)*(a + b*ArcTanh[c + d*x]))/(1 - (c + d*x)^2),x]
```

output
```
e*(-1/2*(a*Log[1 - c - d*x])/d + (b*Log[1 - c - d*x]^2)/(8*d) - (b*Log[2]*
Log[-1 + c + d*x])/(4*d) - (a*Log[1 + c + d*x])/(2*d) + (b*Log[2]*Log[1 +
c + d*x])/(4*d) - (b*Log[1 + c + d*x]^2)/(8*d) + (b*PolyLog[2, (1 - c - d*
x)/2])/(4*d) - (b*PolyLog[2, (1 + c + d*x)/2])/(4*d))
```

3.62.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7281, 27, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)(a+b\operatorname{arctanh}(c+dx))}{1-(c+dx)^2}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \frac{e(c+dx)(a+b\operatorname{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{(c+dx)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{6546}$$
$$\frac{e\left(\int \frac{a+b\text{arctanh}(c+dx)}{-c-dx+1}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}\right)}{d}$$
$$\Big\downarrow \text{6470}$$
$$\frac{e\left(-b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-(c+dx)^2}d(c+dx)-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))\right)}{d}$$
$$\Big\downarrow \text{2849}$$
$$\frac{e\left(b\int \frac{\log\left(\frac{2}{-c-dx+1}\right)}{1-\frac{2}{-c-dx+1}}d\frac{1}{-c-dx+1}-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))\right)}{d}$$
$$\Big\downarrow \text{2752}$$
$$\frac{e\left(-\frac{(a+b\text{arctanh}(c+dx))^2}{2b}+\log\left(\frac{2}{-c-dx+1}\right)(a+b\text{arctanh}(c+dx))+\frac{1}{2}b\,\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\right)}{d}$$

input
```
Int[((c*e + d*e*x)*(a + b*ArcTanh[c + d*x]))/(1 - (c + d*x)^2),x]
```

output
```
(e*(-1/2*(a + b*ArcTanh[c + d*x])^2/b + (a + b*ArcTanh[c + d*x])*Log[2/(1
- c - d*x)] + (b*PolyLog[2, 1 - 2/(1 - c - d*x)])/2))/d
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.62.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{-ae\left(\frac{\ln(dx+c-1)}{2}+\frac{\ln(dx+c+1)}{2}\right)-be\left(\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}+\frac{\ln(dx+c-1)^2}{8}-\frac{\operatorname{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2}\cdots\right)}{d}$
default	$\frac{-ae\left(\frac{\ln(dx+c-1)}{2}+\frac{\ln(dx+c+1)}{2}\right)-be\left(\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}+\frac{\ln(dx+c-1)^2}{8}-\frac{\operatorname{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2}\cdots\right)}{d}$
risch	$\frac{eb\ln(-dx-c+1)^2}{8d}+\frac{eb\ln\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)\ln(-dx-c+1)}{4d}-\frac{eb\operatorname{dilog}\left(-\frac{dx}{2}-\frac{c}{2}+\frac{1}{2}\right)}{4d}-\frac{ea\ln((-dx-c+1)(-dx-c-1))}{2d}-\cdots$
parts	$-\frac{ae\ln(d^2x^2+2cdx+c^2-1)}{2d}-\frac{be\left(\frac{\operatorname{arctanh}(dx+c)\ln(dx+c-1)}{2}+\frac{\operatorname{arctanh}(dx+c)\ln(dx+c+1)}{2}+\frac{\ln(dx+c-1)^2}{8}-\frac{\operatorname{dilog}\left(\frac{dx}{2}+\frac{c}{2}+\frac{1}{2}\right)}{2}\cdots\right)}{\cdots}$

input
```
int((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(-a*e*(1/2*ln(d*x+c-1)+1/2*ln(d*x+c+1))-b*e*(1/2*arctanh(d*x+c)*ln(d*x
+c-1)+1/2*arctanh(d*x+c)*ln(d*x+c+1)+1/8*ln(d*x+c-1)^2-1/2*dilog(1/2*d*x+1
/2*c+1/2)-1/4*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/8*ln(d*x+c+1)^2+1/4*(ln(
d*x+c+1)-ln(1/2*d*x+1/2*c+1/2))*ln(-1/2*d*x-1/2*c+1/2)))
```

3.62.5 Fricas [F]

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = \int -\frac{(dex+ce)(b\,\text{artanh}\,(dx+c)+a)}{(dx+c)^2-1}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="fri
cas")
```

output
```
integral(-(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arctanh(d*x + c))/(d^2*x^2
+ 2*c*d*x + c^2 - 1), x)
```

3.62.6 Sympy [F]

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = -e\left(\int \frac{ac}{c^2+2cdx+d^2x^2-1}\,dx + \int \frac{adx}{c^2+2cdx+d^2x^2-1}\,dx + \int \frac{bc\,\text{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2-1}\,dx + \int \frac{bdx\,\text{atanh}\,(c+dx)}{c^2+2cdx+d^2x^2-1}\,dx\right)$$

input
```
integrate((d*e*x+c*e)*(a+b*atanh(d*x+c))/(1-(d*x+c)**2),x)
```

output
```
-e*(Integral(a*c/(c**2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(a*d*x/(c*
*2 + 2*c*d*x + d**2*x**2 - 1), x) + Integral(b*c*atanh(c + d*x)/(c**2 + 2*
c*d*x + d**2*x**2 - 1), x) + Integral(b*d*x*atanh(c + d*x)/(c**2 + 2*c*d*x
 + d**2*x**2 - 1), x))
```

3.62.7 Maxima [F]

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = \int -\frac{(dex+ce)(b\,\text{artanh}\,(dx+c)+a)}{(dx+c)^2-1}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="max
ima")
```

output
```
1/2*b*c*e*(log(d*x + c + 1)/d - log(d*x + c - 1)/d)*arctanh(d*x + c) - 1/2
*a*d*e*((c + 1)*log(d*x + c + 1)/d^2 - (c - 1)*log(d*x + c - 1)/d^2) + 1/2
*a*c*e*(log(d*x + c + 1)/d - log(d*x + c - 1)/d) + 1/8*b*d*e*((2*(c + 1)*l
og(d*x + c + 1)*log(-d*x - c + 1) - (c - 1)*log(-d*x - c + 1)^2)/d^2 - 4*i
ntegrate(1/2*(c^2 + (c*d + 3*d)*x + 2*c + 1)*log(d*x + c + 1)/(d^3*x^2 + 2
*c*d^2*x + c^2*d - d), x)) - 1/8*(log(d*x + c + 1)^2 - 2*log(d*x + c + 1)*
log(d*x + c - 1) + log(d*x + c - 1)^2)*b*c*e/d
```

3.62.8 Giac [F]

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = \int -\frac{(dex+ce)(b\,\text{artanh}\,(dx+c)+a)}{(dx+c)^2-1}\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctanh(d*x+c))/(1-(d*x+c)^2),x, algorithm="gia
c")
```

output
```
integrate(-(d*e*x + c*e)*(b*arctanh(d*x + c) + a)/((d*x + c)^2 - 1), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ce+dex)(a+b\text{arctanh}(c+dx))}{1-(c+dx)^2}\,dx = \int -\frac{(c\,e+d\,e\,x)\,\left(a+b\,\text{atanh}(c+d\,x)\right)}{(c+d\,x)^2-1}\,dx$$

input
```
int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1),x)
```

output
```
int(-((c*e + d*e*x)*(a + b*atanh(c + d*x)))/((c + d*x)^2 - 1), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func):  #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func):  #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args)))  #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']  #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```